Special Session 10: Non-regular Dynamical Systems: Complementarity Systems, Sweeping Process and Applications

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In this session, we expect to have talks concerning complementarity systems, sweeping process, variational and hemivariational inequalities, non-regular dynamical systems, applications in Mechanics, Automatics, etc.

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Recent Advances in Lyapounov's stability of nonsmooth dynamical systems

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The stability of stationary solutions of dynamical systems constitutes a very important topic in Applied Mathematics and Engineering which has been considerably developed in the last decade. The mathematical formulation of unilateral dynamical systems involves inequality constraints and necessarily contains natural nonsmoothness. The formalism of evolution variational inequalities represents a large class of unilateral dynamical systems. Our aim in this talk is to present some recent results in this field. More precisely, we will discuss a mathematical approach that can be used to state sufficient conditions of stability and asymptotic stability of stationary solutions, necessary conditions of asymptotic stability of isolated stationary solutions and invariance results applicable to a large class of unilateral dynamical systems. The theoretical results will be discussed on some models in unilateral mechanics and non-regular electrical circuits theory.

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Well-posedness results for non-autonomous dissipative complementarity systems

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This paper deals with the well-posedness of a class of nonsmooth dynamical systems: dissipative complementarity systems. Both the linear and the nonlinear cases are treated, and the systems are non-autonomous which is important in view of control applications. The dissipativity property is used to perform a particular change of state vector which allows one to transform the dynamics into a perturbed Moreau's sweeping process. Global existence and uniqueness of AC (absolutely continuous) or LBV (of Local Bounded Variation) solutions is proved in the linear case, depending on the control input being itself AC or LBV. Local existence and uniqueness of AC solutions is proved in the nonlinear case. As an example an electrical circuit with ideal diodes is presented.

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A Mathematical Analysis of A Dynamical Frictional Contact Model in Thermoviscoelasticity

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In this paper, we study the dynamic evolution of a thermoviscoelastic body which is on frictional contact with a rigid foundation. The contact is modeled by a general normal damped response condition with friction law and heat exchange. Then we present a variational formulation of the mechanical problem, and establish the existence and uniqueness of the weak solution, under the condition that the viscosity is sufficiently strong. Finally, the numerical analysis of a fully discrete scheme is presented.

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Asymptotic derivable fields and complementarity problems

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The study of Nonlinear Complementarity Problems is a big chapter in Complementarity Theory. It is known that the complementarity problems represent a class of mathematical models considered in optimization, in the study of economical equilibrium and in engineering among others.

In this talk we will present some interesting applications of asymptotical differentiability along a cone to the study of existence of solutions of nonlinear complementarity problems.

Our results are related to the study of complementarity problems with integral operators.

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Necessary conditions of asymptotic stability for a class of unilateral dynamical systems

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We develop a mathematical tool that can be used to state necessary conditions of asymptotic stability of isolated stationary solutions of a class of unilateral dynamical systems. Instability criteria will also be discussed.

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A Stability Result for Differential Variational Inequalities

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In this contribution we treat Differential Variational Inequalities (DVIs) of the form

$$\mathrm{DVI}(F,Z) \quad 0 \in \dot{x}(t) + F(x(t)) + N_Z(x(t)),\, t \in [0,T]$$

for some $T > 0$. Here $N_Z(x)$ denotes the normal cone (in the sense of convex analysis) to a closed convex set Z in a Hilbert space H at $x \in Z$ and F is a given multi map on H.

We consider perturbations F^n of F and also perturbations Z^n of Z. and assume that the convex closed sets Z^n Mosco-converge to Z. First we analyse stability for normal cones and the dual tangent cones. Then under appropriate conditions for the convergence $F^n \to F$ we can prove the following stability result for DVIs in a separable Hilbert space:

Let x^n be a solution to the $\mathrm{DVI}(F^n,Z^n)$ with $x^n(0) = x_{0,n}$, where $Z^n \ni x_{0,n} \xrightarrow{s} x_0$. Assume that for $T > 0, x^n$ converges to x in $W^{1,2}(0,T;H)$ for $n \to \infty$. Then x is a solution to the $\mathrm{DVI}(F,Z)$ with $x(0) = x_0$. An analogous result is also obtained for slow solutions (solutions of minimal norm) of DVIs. With special variants of DVIs, we derive analogous stability results for complementarity differential systems and projected dynamical systems.

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A Characterization of Lyapunov Pairs

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Let A be a maximal monotone operator in a real Hilbert space H, with domain $D(A)$, and let $f : \overline{D(A)} \to H$ be a Lipschitz continuous mapping defined on the closure $\overline{D(A)}$ of $D(A)$ in H. Given $x \in \overline{D(A)}$, the Cauchy problem

$$\begin{cases} y'(t) + Ay(t) \ni f(y(t)) & \text{for } t \geq 0, \\ y(0) = x \end{cases} \tag{1}$$

has a unique mild solution $y = y_x$. Recall that the lower semicontinuous functions $V, g : \overline{D(A)} \to (-\infty, +\infty]$ form a Lyapunov pair for problem (1) if

$$V(y_x(t)) + \int_0^t g(y_x(s))ds \leq V(x),$$

$$\forall x \in \overline{D(A)},\ \forall t \geq 0.$$

For $g = 0$, the classical definition of a Lyapunov function is recovered. We establish a characterization of a Lyapunov pair (V, g), directly in terms of the given data A, f, V, g, by using a suitable contingent derivative. Precisely, we require the linearity of the operator A or we assume that the semigroup $\{S(t) :\ t \geq 0\}$ generated by $-A$ is compact. The proof relies on an abstract invariance result for an evolution equation that involves the infinitesimal generator of a C_0 semigroup on a Banach space.

This exposition follows the paper: O. Carja and D. Motreanu, Flow-invariance and Lyapunov pairs, *Dynamics of Continuous, Discrete and Impulsive Systems*, to appear.

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Eigenvalue problems for nonlinear elliptic equations with unilateral constraints

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In this paper we study eigenvalue problems for hemivariational and variational inequalities driven by the p-Laplacian differential operator. Using topological methods (based on multivalued versions of the Leray-Schauder alterrnative principle) and variational methods (based on the nonsmooth critical point theory), we prove existence and multiplicity results for the eigenvalue problems we examine.

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Contribution to the Mathematical Modeling of Multipoint, Non-smooth Impact/Contact Dynamics of Human Gait

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This paper suggests a generalized approach to the modeling of human and humanoid motion. Instead of the usual inductive approach that starts from the analysis of different situations of real motion (like bipedal gait and running, playing tennis, soccer, or volleyball, gymnastics on the floor or by using some gymnastic apparatus, etc.) and tries to make a generalization, a deductive approach is suggested where a completely general problem is considered. Once the general model is formulated, different real situations as being special cases can be derived. Such approach needs a serious effort in formulating the general model rigorously intending to be a satisfactory accurate mathematical description of human motion. This paper is an attempt in this direction. The general methodology is explained and demonstrated by synthesis of the spatial 38 d.o.f. model of a human. Special attention is paid towards the contact dynamics modeling, namely impact phenomenon during walk allowing a rigorous way to predict the reaction force. In that sense, the frictional impact/contact is formulated as a Linear Complementarity Problem (LCP) formulation which is solved using the Lemke algorithm. The validity of the modeling approach is supported by experimental measurements as well as by applying the general model to few real examples of human motion: first, simple human walk with different forward speed and second, a problem of climbing the fixed obstacle and jumping. Plenty of graphic presentations, illustrating a similarity between the experimental results and results of numerical simulations of the model derived, are shown in the paper.

Keywords: Biped locomotion, deductive modeling, non-smooth dynamics, impact/contact phenomena, friction, LCP formulation.

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BV solutions of differential inclusions associated with prox-tegular sets

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Jean Fenel Edmond

In this talk I will present some results on BV solutions of nonconvex sweeping processes recently obtained in joint work with Jean Fenel Edmond. The moving set associated to the dynamics will be assumed to be prox-regular. Existence and uniqueness will be discussed. The results are strong enough to cover the usual concept of absolutely continuous solution when appropriate assumptions are done.

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