Special Session 11: Nonautonomous Dynamical Systems

Russell Johnson, Universita' di Firenze, Italy
Rafael Obaya, Universidad de Valladolid, Spain

It is a fact that a non-autonomous differential equation with bounded coefficients gives rise to a topological flow via a construction of Bebutov type. Thus one can use dynamical and ergodic methods to study the qualitative behavior of the solutions of such equations. Of course the same is true for discrete equations. The systematic development of this insight has produced a body of methods and results which carries the collective label "nonautonomous dynamical systems". The goal of this section is to put in evidence some recent developments in this rapidly developing field.

⟶ ∞◇∞ ⟵

Periodic Solutions of Singular Hamiltonian Elliptic Systems

Flaviano Battelli
Dipartimento di Scienze matematiche, Università Politecnica delle Marche, Ancona (Italy), Italy
fbat@dipmat.univpm.it
Michal Feckan

The existence will be proved of periodic solutions on a given submanifold of $\mathbb{R}^n$ to certain singularly perturbed Hamiltonian systems having symmetry properties. Our result applies to some singular systems arising in the study of Hamiltonian systems with a strong restoring force. These systems are known in literature as "penalized problems" and have been recently been studied by several investigators instead of the reduced D'Alembert equation.

⟶ ∞◇∞ ⟵

On nonautonomous shadowing

Arno Berger
University of Canterbury, New Zealand
arno.berger@canterbury.ac.nz

The versatile technique of shadowing provides an important tool (both theoretical and computational) for the analysis of dynamical systems, and it lies at the heart of hyperbolic dynamics. In this talk we discuss aspects of shadowing in a nonautonomous setting. Under reasonable conditions, a nonautonomous shadowing lemma can be established, but the availability of a good reference system turns out to be crucial. We apply the technique to the problem of determining the asymptotic distribution of orbits in a variety of nonautonomous (both deterministic and stochastic) systems.

⟶ ∞◇∞ ⟵

Quasi-periodic Schroedinger equations and SNA

Kristian Bjerklov
University of Toronto, Canada
bjerklov@math.utoronto.ca

We present some results concerning the Strange Nonchaotic Atractors which appear in the projective flow of quasi-periodic Schroedinger equations.

⟶ ∞◇∞ ⟵

Pullback attractors for asymptotically compact non-autonomous dynamical systems

Tomas Caraballo
Universidad de Sevilla, Spain
caraball@us.es
G. Lukaszewicz and J. Real

We introduce the concept of pullback asymptotically compact non-autonomous dynamical system as an extension of the similar concept in the autonomous framework, and prove a result ensuring the existence of a pullback attractor for a non-autonomous dynamical system under the general assumptions of pullback asymptotic compactness and the existence of a pullback absorbing family of sets. Finally, we illustrate the theory with a 2D Navier-Stokes model in an unbounded domain.

⟶ ∞◇∞ ⟵

Global Attractors of Nonautonomous Difference Equations

David Cheban
State University of Moldova, Moldova
cheban@usm.md
C. Mammana and E. Michetti

The talk is devoted to the study of global attractors of nonautonomous difference equations. We give the tests for the existence of a compact global attractor of quasi-linear difference equations. The obtained results are applied while studying a special triangular map $T : \mathbb{R}^2_+ \to \mathbb{R}^2_+$ describing capital accumulation and population dynamics of the model studied in Brianzoni S., Mammana C. and Michetti E. (2005).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Chain recurrence, growth rates and ergodic limits

Fritz Colonius
University of Augsburg, Germany
fritz.colonius@math.uni-augsburg.de

Averages of functionals along trajectories are studied by evaluating the averages along chains. This yields results for the possible limit points and, in particular, for ergodic limits. Applications to Lyapunov exponents and to concepts of rotation numbers of linear Hamiltonian .flows and of general linear .flows are given. This talk is based on joint work with Roberta Fabbri and Russell Johnson

$\longrightarrow \infty \diamond \infty \longleftarrow$

Spectral properties for the one dimensional quasi-periodic Schrödinger operator

Roberta Fabbri
Universita' di Firenze, Italy
fabbri@dsi.unifi.it

The spectral theory of the one-dimensional Schrödinger operator with a quasi-periodic potential can be fruitfully studied considering the corresponding differential system. In fact the presence of an exponential dichotomy for the system is equivalent to the statement that the energy E belongs to the resolvent of the operator. Starting from results obtained for the question of the positivity of the Lyapunov exponent of the system and the study of the spectrum for the continuos case , we want to consider the discrete case $(Hu)_n = -(u_{n+1} + u_{n-1}) + V(\theta + n\omega)u_n$ acting on $l^2(\mathbb{Z})$ with two frequencies. Observe that in the discrete case we talk of integrated density of states which coincides, modulo π, with the rotation number of the system

$\longrightarrow \infty \diamond \infty \longleftarrow$

A dynamical approach to *p*-Laplace equations

Matteo Franca
Université Politecnica delle Marche, Italy
franca@dipmat.univpm.it

We describe a dynamical approach to study radial solutions for the following quasi-linear elliptic equation:

$$\Delta_p u(x) + f(u(x), |x|) = 0$$

where $x \in \mathbb{R}^n$, $\Delta_p u = div(|Du|^{p-2}Du)$, $p > 1$ denotes the p-Laplace operator, f is continuous in both the variables and locally Lipschitz continuous in the u variable. Our methods prove to be useful in particular when f is spatially dependent and variational methods are not easily applied. We will focus on some recent results concerning the case in which f is negative for u small, and it is positive and subcritical for u large.

We will prove some existence results for ground states and singular ground states, that are new even for the classical case $p = 2$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Bifurcation theory for nonautonomous systems

Russell Johnson
Universita' di Firenze, Italy
russellallan.johnson@unifi.it

We review recent developments regarding stability breakdown in nonautonomous systems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On the fractalization of invariant curves

Angel Jorba
Universitat de Barcelona, Spain
angel@maia.ub.es
Joan Carles Tatjer

In this talk we will present some results dealing with the destruction of attracting invariant curves in quasi-periodically forced 1-D dynamical systems. More concretely, we will focus on the situation in which the length of the curve goes to infinity when a parameter approaches some critical value. We will first discuss some connections between this behaviour and the lack of reducibility of the curve. It will be shown in some cases the curve keeps being a smooth curve as long as it is attracting, although the numerical simulations seem to show that it is not longer a regular curve, but a strange nonchaotic attractor (SNA). We will discuss how this phenomenon brings into question the real existence of some of the SNAs observed numerically in the literature.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Stable and unstable manifolds for quasilinear parabolic systems with fully nonlinear boundary conditions

Yuri Latushkin
University of Missouri-Columbia, USA
yuri@math.missouri.edu
Jan Prüss and Roland Schnaubelt

We investigate quasilinear systems of parabolic partial differential equations with fully nonlinear boundary conditions on bounded or exterior domains in the setting of Sobolev–Slobodetskii spaces. We establish local well-posedness and study the time and space regularity of the solutions. Our main results concern the asymptotic behavior of the solutions in the vicinity of a hyperbolic equilibrium. In particular, the local stable and unstable manifolds are constructed.

⟶ ∞◇∞ ⟵

An extension of the Sacker-Sell spectrum theory

Weishi Liu
University of Kansas
wliu@math.ku.edu

The concepts of exponential dichotomy and normal hyperbolicity are closely related. The well-known Sacker-Sell spectrum theory can be viewed as a link between these two fundamental concepts. However, from this viewpoint of linkage, the Sacker-Sell spectrum theory does not provide a perfect counterpart to normal hyperbolicity. In this talk, we discuss an improvement to the Sacker-Sell spectrum theory for skew-product flows. In particular, if the skew-product flow is obtained as the linearization of a flow over an invariant manifold Y, then our results relate directly to normal hyperbolicity of Y.

⟶ ∞◇∞ ⟵

Exponential stability in non-autonomous delayed equations with applications to neural networks

Sylvia Novo
Universidad de Valladolid , Spain
sylnov@wmatem.eis.uva.es
Rafael Obaya and Ana M. Sanz

We consider the skew-product semiflow induced by a family of finite-delay functional differential equations and we characterize the exponential stability of its minimal subsets. In the case of non-autonomous systems modelling delayed cellular neural networks, the existence of a global exponentially attracting solution is deduced from the uniform asymptotical stability of the null solution of an associated non-autonomous linear system.

⟶ ∞◇∞ ⟵

Some generic results in non-autonomous bifurcation theory

Carmen Nunez
University of Valladolid, Spain
carnun@wmatem.eis.uva.es
Rafael Obaya

The dependence on a parameter of certain families of transformations and flows associated to difference equations and ordinary differential equations with almost periodic coefficients is analyzed. Conditions ensuring the existence of bifurcation points giving rise to the occurrence of almost automorphic (not necessarily almost periodic) dynamics are described. The generic character at those bifurcation points of some properties of ergodic and topological nature is shown.

⟶ ∞◇∞ ⟵

On some stability properties of abstract skew-product semiflows.

Rafael Obaya
University of Valladolid, Spain
rafoba@wmatem.eis.uva.es
Sylvia Novo and Ana Sanz

We consider a continuous skew-product semiflow whose flow on the base is minimal and distal. If the omega limit set of a relatively compact trajectory is uniformly stable then it admits a flow extension which is minimal and distal. This known result was proved by R. Sacker and G. Sell for almost-periodic differential equations and by Y. Yi and and W. Shen in the previous formulation. In this talk we analyze some versions of this theorem assuming weaker hypotheses of the flow on the base. We discuss some implications of these theorems in the context of functional differential equations and monotone dynamical systems.

⟶ ∞◇∞ ⟵

Generalized Attractor-Repeller Pairs, Diagonalizability and Integral Separation

Kenneth J. Palmer
National Taiwan University, Taiwan

palmer@math.ntu.edu.tw
Stefan Siegmund

We introduce the notion of a generalized attractor-repeller pair for x'=A(t)x which in fact is an attractor-repeller pair for the induced (nonlinear) flow on the projective space and use it to characterize diagonalizability. We show that integral separation implies the existence of generalized attractor-repeller pairs and use topological dynamics, more precisely the hull of A(t), to prove a converse theorem. The relation to exponential separation is also discussed.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Stability in a Class of Nonautonomous Linear Delay Differential Equations

Mihály Pituk
University of Veszprém, Hungary
pitukm@almos.vein.hu

In this talk, we shall consider a system of nonautonomous linear delay differential equations. We shall show that under appropriate smallness conditions the stability properties of the delay equation are equivalent to the same properties of an associated system of linear ordinary differential equations.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Reduction principle in the theory of stability of differential equations

Andrejs Reinfelds
Institute of Mathematics, Latvia
reinf@latnet.lv
Klara Janglajew

Consider the following difference equations in Banach space $\mathbf{X}\times\mathbf{Y}$

$$\begin{cases} x(t+1) = A(t)x(t)+f(t,x(t),y(t)), \\ y(t+1) = B(t)y(t)+g(t,x(t),y(t)) \end{cases} \tag{1}$$

where map $A(t)\colon \mathbf{X}\to\mathbf{X}$ is invertible, the maps f and g are ε-uniform Lipschitzian with respect to second and third variable and vanishes at the origin. Let $X(t,s)$, $t,s\in\mathbb{Z}$ and $Y(t,s)$. $t,s\in\mathbb{Z}$, $t\geq s$ be the Cauchy evolution map of the corresponding linear equations

$$\begin{cases} x(t+1) = A(t)x(t) \\ y(t+1) = B(t)y(t), \end{cases}$$

respectively. Assume that operators satisfy the estimates

$$\nu = \max\left\{\sup_{t\in\mathbb{Z}}\left(\sum_{s=-\infty}^{t-1}|Y(t,s+1)||X(s,t)|\right),\right.$$

$$\left.\sup_{\tau\in\mathbb{Z}}\left(\sum_{s=t}^{+\infty}|X(t,s+1)||Y(s,t)|\right)\right\}$$

and

$$\mu = \sup_{t\in\mathbb{Z}}\left(\sum_{s=t}^{+\infty}|Y(t,s)|\right).$$

Theorem: Let $4\varepsilon\nu<1$ and $2\varepsilon\mu<1+\sqrt{1-4\varepsilon\nu}$. Then there exists a continuous map $u\colon \mathbb{Z}\times\mathbf{X}\to\mathbf{Y}$, which is uniform Lipschitzian with respect to the second variable, such that the trivial solution of difference equation

$$x(t+1) = A(t)x(t)+f(t,x(t),u(t,x(t)))$$

is stable, asymptotically stable or nonstable if and only if the trivial solution of difference equation (1) is stable, asymptotically stable or nonstable.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On periodic solutions of forced coupled second order differential equations on manifolds

Marco Spadini
Dipartimento di Matematica Applicata , Italy
marco.spadini@unifi.it

We investigate the forced oscillations of a constrained system of second order differential equations of a particular form. The results described unify several well-known facts about the structure of the set of forced oscillations of perturbed second-order ODE's on manifolds.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Inverse problem for the Sturm-Liouville operator

Luca Zampogni
Università di Firenze, Italy
zampogni@math.unifi.it
Russell Johnson

We use methods of nonautonomous differential equations together with basis techniques of the classical theory of algebraic curves to solve an inverse problem for $(p\varphi')'+q\varphi=-\lambda y\varphi$.

$\longrightarrow \infty \diamond \infty \longleftarrow$