Special Session 12: New trends in electromagnetism and micromagnetism

Alouges Francois, Université Paris-Sud Orsay, France
Frank Jochmann, TU-Berlin, Germany
Hong Ming Yin, Washington State University, USA

This aim of this special session is twofold. On one hand, we would like to give a few examples of new techniques in electromagnetism, and electromagnetic scattering. On the other hand, we also want to provide an overview of recent problems met in the somehow smaller domain of micromagnetism (the physical model for ferromagnetic materials). In particular, micromagnetics has received recently a considerable interest from the mathematic community. We emphasize that we wish to present new applications in terms of modelization, numerical techniques and analysis.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Finite element schemes for Landau-Lifshitz equations

François Alouges
Université Paris-Sud , France
francois.alouges@math.u-psud.fr

The discretization of Landau-Lifshitz equations modelling ferromagnetic materials faces the difficulty of dealing with the preservation of the pointwise magnitude of the magnetization vector. When finite elements are moreover considered, one must also provide a weak formulation and a time marching scheme that still satisfy the constraint in some sense. After the approach by Prohl, with penalization, we have developed another strategywhich consists in taking the constraint into account at each discretization point. In the talk, we will give an overview of recent results in that direction. Both analytical and practical viewpoints will be considered.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On the localization of three-dimensional inclusions of small volume from numerical measurements

Mark Asch
LAMFA, Universite de Picardie Jules Verne, France
mark.asch@u-picardie.fr
M. Asch and S.M. Mefire

We present numerical and practical aspects of the localization of three-dimensional inclusions of small volume. The modelling is based on the time- harmonic Maxwell equations with measurements of perturbations in the electric field due to the presence of multiple electromagnetic inclusions of small volume in a bounded domain. A theoretical framework based on a variational approach and asymptotic expansions for the boundary measurements has been recently developed by H. Ammari, M.S. Vogelius and D. Volkov. From a numerical point of view, we evaluate these boundary measurements by computing the electric field and then localize by a MUSIC (MUltiple SIgnal Classification) algorithm. We present numerical results that validate the asymptotic formula as a function of the material properties. We also study the effects of varying frequency and material properties on the localization.

$\longrightarrow \infty \diamond \infty \longleftarrow$

L^2- well-posedness for mixed 3d div-curl systems on bounded regions

Giles Auchmuty
National Science Foundation, USA
gauchmut@nsf.gov

This talk will describe necessary and sufficient conditions for the existence of finite energy solutions of div-curl systems on bounded 3d regions subject to mixed tangential and normal boundary conditions. Under natural assumptions on the data, and provided the region is simply connected, the well-posed problem for this system may be described and has a natural physical interpretation. When the region is not simply connected there remain some open problems about the number of extra conditiona that must be imposed to have a well-posed problem.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Landau-Lifschitz-Gilbert equation with applied electric current

Gaél Bonithon
Université Bordeaux 1, France
Gael.Bonithon@math.u-bordeaux1.fr

In this talk, we study a model on electric current effect on ferromagnetic materials. The magnetic moment variations are described by the Landau-Lifschitz-Gilbert equation adding a transport term, that models the current action. We first prove existence of weak solutions. To achieve this target, we modify the Alouges and Soyeur method (relaxation of the constraint $|u| \equiv 1$) using a truncature function for the transport term. We also prove existence and uniqueness of strong solutions. Uniqueness is equivalent to the conservation of the pointwise norm by the model, that is obtained by studying the well posedness of a quasilinear transport equation derived from the current term of the model.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Asymptotics from Maxwell-Bloch equations to Schrödinger-Boltzmann equations

Dumas Eric
Universite Grenoble1, France
edumas@ujf-grenoble.fr
François Castella

Maxwell-Bloch equations model the nonlinear interaction of an electromagnetic field (solution to Maxwell equations) with matter, such as an atomic vapor, described by a density matrix (solution to Bloch equations). Coupling is due to polarization of the matter. When dissipation is taken into account, transition probabilities between energy levels are supposed to decay rapidly in time, whereas the populations of the energy levels satisfy a Boltzmann equation. We prove such asymptotics in a high-frequency limit, where the fields satisfy a nonlinear Schrödinger equation.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Multi-tracks reading heads modelling by coupling boundary elements and finite differences approaches

Ioana Firastrau
Transilvania University of Brasov, Romania
firastrau@unitbv.ro
L.D. Buda-Prejbeanu, J.-Ch. Toussaint and J.-P. Nozieres

An original approach was developed to model multi-tracks shielded magneto-resistive reading heads for high density tape drives. The shields and the media are of micron size while the sensor has sizes comparable with the characteristic length scales of the magnetic materials which are of the order of nanometer. Because of this large difference of scales between the different parts of the head, the boundary element method (BEM) seems the most adapted to analyze them. Therefore, solving by BEM the Poisson equation for the scalar magnetic potential only the surfaces-interfaces are discretised, saving thus computation time and memory resources. Instead, because of the interactions involved, the response of the magnetic sensor has to be described by using a micromagnetic model based on the finite differences approximation. This macroscopic-microscopic coupling technique allows to fully describe the behavior of the magnetic sensor in its environment. The direct implementation of the full coupling of the micromagnetic and the magnetostatic equations allows to deal with systems having various shapes but it is found to be too time consuming. The efficiency of this technique can be significantly improved by taking an initial micromagnetic configuration close to the equilibrium one, derived by a perfect magnetic imaging method.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Relaxation approximation of some nonlinear Maxwell initial-boundary value problem

Carbou Gilles
MAB Université Bordeaux 1, France
carbou@math.u-bordeaux1.fr
B. Hanouzet

We consider two nonlinear Maxwell systems describing electromagnetic waves propagation in nonlinear material. In the Kerr model the response of the medium is instantaneous and the electric displacement D is related to the electric field by the relation $D = E(1+|E|^2)$. The Kerr-Debye model contains some delay term and is a relaxation approximation of the Kerr model in which $D = E(1+\chi)$ where χ satisfies

$$\partial_t \chi + \frac{1}{\varepsilon}\chi = \frac{1}{\varepsilon}|E|^2.$$

In this talk we prove the convergence of the Kerr Debye model to the Kerr model when the relaxation coefficient ε tends to zero, for the initial-boundary value problem with the ingoing wave condition on the boundary.

The one dimensional case was studied in a previous work. Here we consider the 3 dimensional case in which the boundary is characteristic for both models, with a two-dimensional kernel for Kerr problem and a three-dimensional kernel for Kerr-Debye model. We must then take into account the nonlinear conservation equations for div D and div B.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A mathematical model for nonlinear polarizable media

Frank Jochmann
TU-Berlin, Germany

frankjmn@aol.com

This talk is concerned with the anharmonic oscillator model for nonlinear dielectric media and a Maxwell-Bloch system. The main topics are the existence and uniqueness of solutions in the case of non-Lipschitz continuous restoring forces and some qualitative properties of the solutions if there is no damping term in the equation governing the polarization.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Computing electrostatic charge densities at rounded corners : an improved Peek's formula

Samir Kaddouri
ENSTA, France
samir.kaddouri@ensta.fr
Ciarlet Patrick, Jr

We are interested in computing the charge density at the rounded tip of an electrode of small curvature. As a model, we focus on solving the electrostatic problem for the electric potential. For this problem, Peek's empirical formula describes the relation between the electric field at the surface of the tip and its curvature radius. However, it can be used only for electrodes with either a purely cylindrical, or a purely spherical, geometrical shape. Our aim is to justify rigorously his formula, and to extend it to other geometries. With the help of multiscaled asymptotic expansions, we establish an explicit formula for geometries that coincide with a cone at infinity. We illustrate our mathematical results by some numerical experiments, set in two dimensional, or three dimensional axisymmetric, domains.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Controllability for ferromagnetism systems: the nano wires

Stéphane Labbé
Laboratoire de Mathématique, Universit éParis 11, France
stephane.labbe@math.u-psud.fr
G. Carbou and E. Trélat

The control of the magnetization in ferromagnetic materials is an important challenge (nano electronic, magnetic recording, etc). The main question is to know whether or not it is possible to optimize an external magnetic field evolution in order to lead a particule from a given steady state of the magnetization to another given steady state minimizing the time of transfer and also the energy. In this presentation we propose a first step in the control of the magnetization in ferromagnetic material studying the controllability of the magnetization in nanowires. In such objects, small structures appear called Bloch walls, who could be used to carry informations if not damaged by a too intense external field. G. Carbou and S. Labbé have demonstrated recently that such structure, modulo translations and rotations, are asymptotically stable. Using this stability result, G. Carbou, S. Labbé and E. Trélat prove the controllalibity of walls in nanowires and the robustness of these controls. The method is implemented, and simulations are provided.

$\longrightarrow \infty \diamond \infty \longleftarrow$

An efficient Indirect Integral Equation for Large Scale

David Levadoux
ONERA, France
david.levadoux@onera.fr
F. Alouges

We present an integral equation approach dedicated to the solution of harmonic scattering problems of electromagnetism. We focus on the standard perfect electrical conductor problem. The method relies on the factorisation of one of the Calder??n projectors by an operator approximating the exterior admittance (Dirichlet to Neumann) operator of the scattering obstacle. We show how the pseudo-differential calculus allows us to construct such approximations and that this yields integral equations without internal resonances and being well-conditioned at all frequencies. An implementation technique is elaborated, where again reasonings from pseudo-differential calculus play an important rôle. Some numerical examples are presented which appear to confirm that the new integral equation leads to linear systems which are much better conditioned than the classical ("direct") integral equations and hence have much better behaviour when solved with iterative techniques. Finally, we open new perspectives in showing how one can expect to extend the new approach to the broad class of transmission problems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

The equations of ferromagnetism in domains with spacers

Kévin Santugini repiquet
UNIGE, Switzerland
Kevin.Santugini@math.unige.ch

In recent years, ferromagnetic bodies with weakly magnetic spacers (in particular multilayers) have become a strong focus of reasearch. Ferromagnetic multilayers may present interesting properties deemed impossible to achieve with homogenous configurations: giant magnetoresistance for example. When dealing with such geometries, meshing is always problematic since the spacers are usually thinner than the mesh size. To avoid meshing the spacers, it is natural to analyse the asymptotical influence spacers have on the magnetic behavior of ferromagnetic bodies. We can then derive an efficient model to compute the magnetic behavior of such ferromagnetic bodies. In this presentation, we will show how one can recover a macroscopic model of the behavior of ferromagnetic multilayers using homogenization techniques.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Quasi-stationnary limit for Maxwell-Landau-Lifshitz system in inhomogeneous media

Jean Starynkevitch
Mathematiques Appliquees de Bordeaux, France
jean.starynkevitch@math.u-bordeaux1.fr

We give a result about the strong quasi-stationnary limit in the Maxwell-Landau-Lifshitz system describing smooth ferromagnetic media without exchange energy. Our proof is based on a resolvent estimates for Maxwell equations.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On Mathematical Models of Microwave and Induction Heating

Hong-ming Yin
Washington State University, USA
hyin@wsu.edu

In this talk I shall discuss the mathematical models of microwave and induction heating. The focus will be on characterization of how the electric conductivity affects the electromagnetic fields and the temperature.

$\longrightarrow \infty \diamond \infty \longleftarrow$