Special Session 14: Hamiltonian systems

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At this very moment, the main tentative topics are:
* Applications to celestial mechanics, space science, plasma physics, accelerators, etc
* Structure of the phase space
* K.A.M. theory: invariant tori, invariant manifolds
* Splitting of separatrices
* Arnold diffusion: Geometry and estimates
* Bifurcations
* Numerical and symbolic tools for Hamiltonian systems
* Detection and measure of the non-integrability
* Passage through resonance
* Adiabatic invariants
* Stability
* Variational methods
* Transport in conservative systems
* Hamiltonian PDEs

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Nonlinear oscillations in Hamiltonian PDEs

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We describe new existence results of periodic and quasi-periodic solutions of Hamiltonian PDEs. Both infinite dimensional bifurcation phenomena and small divisors difficulty arise.

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Periodic solutions of Birkhoff-Lewis type for the nonlinear wave equation

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We prove existence of periodic solutions of the nonlinear wave equation with minimal period going to infinity. These solutions accumulate at the origin which is an elliptic equilibrium of the associated infinite dimensional hamiltonian system. The proof is achieved by a Nash-Moser Implicit Function Theorem.

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Detecting global instability in Hamiltonian systems by means of geometrical methods

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We describe the geometric features of a mechanism for detecting global instability in a priori-unstable nearly integrable Hamiltonian systems.

The mechanism presented is based on decomposing the motion in two types of different dynamics, one called inner that takes place inside a normally hyperbolic invariant manifold, where a lot of regular objects (i.e., invariant tori) live, and another one called outer (or scattering map), that takes into account the asymptotic motions to the normally hyperbolic invariant manifold. The combination of both types of dynamics gives rise to chaotic dynamics and instability.

This mechanism has been applied to several problems. For instance, it was first applied to the existence of orbits of unbounded energy in generic geodesic flows on a torus with a periodic time-dependent potential.

More recently, it has been applied to overcome the large gaps problem in Arnold diffusion, and also to show that certain mechanical systems, including a geodesic flow in any dimension plus a quasi-periodic perturbation by a potential, have orbits of unbounded energy.

The proof is based on geometric considerations of invariant manifolds and their intersections. The main tools

include the scattering map of normally hyperbolic invariant manifolds, as well as standard perturbation theories (averaging, KAM and Melnikov techniques).

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Collisionless symmetric minimizers for the n-body Lagrangian functional

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Periodic and quasi-periodic solutions of the n-body problem are critical points for the Lagrangian action functional. It is possible to prove the existence of families of such orbits by restricting the functional to suitable spaces of symmetric Sobolev loops. Algebraic and combinatorial properties can be computed, in terms of finite group characters, in order to prove topological features of the corresponding minimizers (being collisionless, coercive, ...). We will illustrate some simple examples in dimension 3 and 2.

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Diffusion with optimal time in the large gap problem

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We present a topological mechanism for diffusion in the large gap problem for a Hamiltonian system on $\mathbb{R}^n \times \mathbb{T}^n \times \mathbb{R} \times \mathbb{T} \times \mathbb{T}$, given by $H_0(I) + \sum_{i=1}^n \pm \left(\frac{1}{2}p_i^2 + V_i(q_i)\right) + \varepsilon h(p_1, \ldots, p_n, q_1, \ldots, q_n, I, \phi, t; \varepsilon)$, where each V_i has a unique non-degenerate global maximum, and $\partial^2 H_0/\partial I^2 > \delta$ for some $\delta > 0$.

We show that if h satisfies some explicit non-degeneracy conditions, which are C^2-open and C^∞-dense, then there exist trajectories for which $|I(T) - I(0)| \geq O(1)$ with $T \leq O((1/\varepsilon)\ln(1/\varepsilon))$. There are known upper bounds for $|I(T) - I(0)|$ which show that this time T is optimal. The proof is based on the theory of normally hyperbolic manifolds and on the method of correctly aligned windows.

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Effect of Plasma Sputtering on Dust Grain Dynamics in Planetary Magnetospheres

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Plasma sputtering can significantly reduce the size of charged dust grains orbiting within the magnetosphere of Saturn in only a few years. With both mass and charge varying in time, the resulting equations of motion are non-hamiltonian. Except for the small influence of solar radiation pressure, this systems is axisymmetric and the question arises as to the existence of global invariants in the absence of a hamiltonian. For larger grains, where gravity dominates we show that a formal hamiltonian may be constructed by treating the velocities as canonical momenta. For smaller grains, where the planetary magnetic field dominates, a hamiltonian description is apparently not possible. Nevertheless, an exact invariant is derivable from the axisymmetric equations of motion. Implications for the history and structure of Saturn's E ring and for observations by the Cassini orbiter CDA and UVIS experiments will be discussed.

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Dynamics near a homoclinic orbit to a saddle-center of Hamiltonian system

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Consider a family of real analytic two degrees of freedom Hamiltonian systems with a saddle-center equilibrium p, $H_\mu(p) = 0$, $\mu \in R^2$. Suppose for $\mu = 0$ the corresponding Hamiltonian system has a homoclinic loop Γ to p. Evidently, for $\mu \neq 0$ the loop is already destroyed, but some multi-round loops can exist. The present result concerns the existence of homoclinic orbits of the roundness $2^k \cdot 3^m$ ($k, m \in Z^+$, $k + m \geq 1$) in nonresonance case. We also prove that in the nonresonance case in some neighbourhood of every such multi-round homoclinic orbit there exist four countable families of one-round periodic orbits, accumulating at the homoclinic orbit.

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periodic solutions for regularizing NLS equations in d dimensions

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We consider the d-dimensional nonlinear Schrödinger equation with Dirichlet boundary conditions and with a regularizing non-linearity, we prove the existence of small periodic solutions which at leading order are wave packets. The main difficuly in proving the existence of such solutions is to solve a " small divisor problem" which we do by using a renormalization theory approach.

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On the dynamics of a multiple pendulum. Non-integrability, topological properties.

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The planar motion of a multiple pendulum is studied. A multiple pendulum is a system of mass points with constraints given by second degree polynomial equations. Special attention is given to the triple pendulum case. The double pendulum is an integrable system. When the number of particles is more then 3 the system is rather complicated. And for the triple pendulum non-integrability is shown. For this case the motion may be chaotic.

To study the dynamics and the topological properties of the system both analytical and numerical methods were employed. This problem is similar to the problem of chaotic behavior of geodesics on the torus with some special metric.

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On the stability of linear potential gyroscopic systems

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The problem of the gyroscopic stability of the system with degenerate matrix of the gyroscopic forces is considered. Sufficient conditions of the stability for these systems are obtained.

Let us consider the small oscillations of the dynamical system in the region of an equilibrium position, described by the differential equation with n degrees of freedom:

$$M\ddot{z} + G\dot{z} + Kz = 0, \qquad z \in \mathbb{R}^n \qquad (1)$$

The matrix of kinetic energy $M = M^T$ is positive definite, $K = K^T$ is the matrix of potential energy, $G^T = -G$ - is the matrix of gyroscopic forces, $rankG = 2k$.
Theorem. The sufficient conditions of the stability of the trivial solution of equation (1) is that matrices A and B are positive definite, where

$$\begin{aligned} A &= K - GM^{-1}G/4 \\ B &= (GM^{-1}G + \gamma^2 M)/4 - K \end{aligned}$$

$\gamma^2 = -\lambda^2$, λ is the nonzero root of the characteristic equation

$$det(G - \lambda M) = 0$$

Notice. The theorem is true for the case, when G reads:

$$G = \gamma S^T I_k S,\ I_k = diag(I, \dots, I, 0), I = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

S is the non-degenerate $(n \times n)$ matrix. When $n = 3$ any non-trivial skew-symmetric matrix G has this form for $k = 1$. The application of this theorem is considered for the problem of two mutually gravitating bodies.

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Scattering maps of a normally hyperbolic invariant manifold: geometric properties and examples

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We consider the scattering map for manifold Λ, which is invariant and normally hyperbolic under a map f defined on a manifold M, and whose stable and unstable manifolds intersect on a homoclinic manifold Γ. This scattering amp is a useful tool to detect heteroclinic orbits to Λ.

We show that in case that f is symplectic (resp. exact symplectic) and that the invariant manifold M is symplectic, the scattering map is also symplectic (resp. exact symplectic). We use this geometric information to obtain efficient calculations of the scattering map. This perturbation theory generalize and extends several results that were obtained before using Melnikov theory. Similar results are true for Hamiltonian flows. Several examples are presented.

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On the total disconnectedness of the quotient Aubry set

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In Mather's studies on the existence of Arnold diffusion, it turns out that it might be useful to understand certain metric aspects of the quotient Aubry set. In particular, it seems to be interesting to know whether this set has a "small" dimension.

We prove that under suitable hypotheses on the Lagrangian, the associated quotient Aubry set, corresponding to a certain cohomology class, is totally disconnected, i.e., every connected component consists of a single point. Moreover, the regularity conditions we get for the Lagrangian are optimal, as the counterexamples provided by Mather in "Examples of Aubry set" show.

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Entropy penalized weak KAM theory

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We present an entropy penalization problem and we discuss its relations with approximate solutions of Hamilton-Jacobi equations, the convergence of associated discrete schemes, as well as several applications, such as: a generalization of the Hopf-Cole transformation which converts non-linear Hamilton-Jacobi equations into linear evolution equations, the study of fixed point problems, the approximation of certain linear evolution equations and the construction of entropy penalized Mather measures.

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