Special Session 16: Dynamical Systems Associated with Nonlinear Phenomena with Energy Dissipation

Toyohiko Aiki, Gifu University, Japan
Nobuyuki Kenmochi, Chiba University, Japan

The main topics of this session is dynamical systems associated with various nonlinear phenomena, such as reaction-diffusion problems, phase transition problems including diverse effects (for instance, hysteresis, memory, irreversibility, non-local constraint). The session provides many opportunities reporting recent results on their new mathematical modelings and related optimal control problems, and discussing new approaches proposed in the presentations of our session. We are interested in real-world problems as well as their abstract treatment based on the theory of nonlinear operators and nonlinear evolution equations in abstract spaces.

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A free boundary problem for elastic materials

Toyohiko Aiki
Gifu University, Japan
aiki@cc.gifu-u.ac.jp

In this talk we consider a free boundary problem for elastic material, which is occupied one-dimensional domain. The typical example of the materials is a spring. Unknown functions of the problem are the location $x = \ell(t)$ of the end of the material and the displacement $u = u(t,x)$. It satisfies that $u_{tt} + \gamma u_{xxxx} - \mu u_{txx} - \kappa u_{xx} = f$, in $Q_\ell(T) = \{(t,x) | 0 < x < \ell(t), 0 < t < T\}$ for $T > 0$, where γ, μ and κ are positive constants and f is a external force. In case $\gamma = \mu = 0$ by the equation the dynamics of the elastic material is represented for linear strain. Also, the equation can be found in shape memory alloy problems. The free boundary condition is as follows: $u(t,\ell(t)) = \ell(t) - \ell_0$, $u_{xx}(t,\ell(t)) = 0$,and $\ell''(t) = g(t) - \kappa u_x(t,\ell(t))$, where g is a given force. The aim of this talk is to propose the free boundary problem and to establish the local existence result.

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On solutions of control system of subdifferential type depending on a parameter

Tolstonogov A. Alexander
Institute for System Dynamics and Control Theory,Siberian Branch, Russian Academy of Sciences, Russia
aatol@icc.ru

In a separable Hilbert space a control system with evolution operators and with mixed non-convex control constraint is considered. Evolution operators are subdifferentials of a proper convex and lower semicontinuous function depending on time. The system depends on a parameter which appears in the initial conditions, the evolution operators, and the control constraint. The solutions of the system are "state-control" pairs. We devote our main attention to the existence of continuous with respect to parameter selectors taking values in the solution sets of the control system. We also consider approximation of continuous with respect to parameter selectors of the control system with convexified constraint on control by similar selectors of the original system (selector relaxation theorem). In addition, we study certain topological properties of the solution sets. An examples of systems monitored by p.d.e's which illustrate the applicability of our abstract results is presented.

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The entropy approach for phase transitions with thermal memory:existence results and long-time behaviour of solutions

Elena Bonetti
University of Pavia, Italy
elena.bonetti@unipv.it

We consider a singular-integrodifferential PDE's system describing phase transitions in terms of nonlinear evolution equations for microscopic motions and for the entropy. The model is derived from a free energy functional (possibly non-convex with respect to the phase parameter) and a pseudo-potential of dissipation. The unknowns are the absolute temperature and a phase parameter. The resulting integro-differential system is strongly nonlinear due to the presence of some singularities in the equation for the temperature (both in the evolution and diffusive terms) and multivalued operators. After recovering existence of solutions for a related initial and boundary value problem, we investigate the long-time behaviour of these

solutions studying the associated ω-limit set. Uniqueness is proved only for a simplified version of the problem.

Some of the above results have been obtained in collaboration with P. Colli, M. Frémond, and E. Rocca.

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On an abstract doubly nonlinear equation with memory

Gianni Gilardi
University of Pavia, Italy
gianni.gilardi@unipv.it

The abstract doubly nonlinear equation $A(u') + B(u) + k * B(u) = f$ is considered, where A and B are maximal monotone operators in Banach spaces and the memory kernel k is a real function. The above equation without memory is well known. In a joint work in progress with U. Stefanelli (Pavia, Italy), sufficient conditions on the ingredients (Banach spaces, operators, memory kernel, source term, initial datum) are given in order that the Cauchy problem $u(0) = u^0$ makes sense and has a solution. The method we use is time discretization. The aim of the talks is to give an outline of such a result.

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A Class of Quasi-Variational Inequalities for Hysteresis

Masahiro Kubo
Nagoya Institute of Technology, Japan
kubo.masahiro@nitech.ac.jp

We study a system of parabolic variational inequalities describing hysteresis phenomina as arise in phase transitions and flows in porous media. We pose convex constraints on the two unknown functions, the temperature and the order parameter (resp. the pressure and the saturation) in phase transitions (resp. in flows in porous media). The constraint on the order parameter depends on the temperature. This refers to a hysteretic relation between these two quantities. Thus, the problem yields a special class of quasi-variational inequalities. We show the existence of a global solution and study its behavior as the coefficient of the interfacial energy vanishes.

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Nonconvex optimization problems for semilinear second order evolution equations

Shin-ichi Nakagiri
Kobe University, Japan
nakagiri@cs.kobe-u.ac.jp

In this paper we study the nonconvex optimization problems for the control systems described by semilinear second order evolution equations in Hilbert space of the form $y'' + A_2(t)y' + A_1(t)y = f(t,y,y') + B_3v_3$ with initial values $y(0) = y_0 + B_1v_1$, $y'(0) = y_1 + B_2v_2$. Here $A_1(t), A_2(t)$ are time varying operators on Hilbert spaces, $f(t,y,y')$ is a nonlinear function, y_0, y_1 are given initial values, v_1, v_2, v_3 are control variables, and B_1, B_2, B_3 are controllers. Under appropriate conditions on operators, initial values and the nonlinear term, we establish the wellposedness result and the Fréchet differentiability of solutions with respect to $v = (v_1, v_2, v_3)$. The attached cost $J(v)$ is given by the nonconvex cost $J(v) = F(v, y(v;T)) + \int_0^T G(t, v, y(v;t))dt$, where T is the final time and $y(v;t)$ is the solution of control system corresponding v. For the nonconvex cost optimization problems, we prove the existence of optimal controls and establish the necessary optimality conditions for optimal controls.

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Large time behaviour of solutions of Nonlinear ODE describing hysteresis

Takanobu Okazaki
Chiba University, Japan
okazaki@faculty.chiba-u.jp

We consider a nonlinear ordinary differential system which describes hysteresis input-output relations. The main part of this system is governed by subdifferential operator and it is used to present various hysteresis effects. In real phenomena, many hysteresis branches are observed. We are interested in verifying our system to express such branches. Our main objective of this talk is to investigate the precise behaviour of orbits of solutions of our system. In fact, our question is investigate how the orbits behave, depending on various parameters of equations and data. Under some conditions, we shall give some theoretical results and some numerical simulations on the large time-behaviour of orbits.

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Cahn-Hilliard system for microstructure evolution in elastic solids

Irena Pawłow
Systems Research Institute, Polish Academy of Sciences, Poland
Irena.Pawlow@ibspan.waw.pl

The paper is concerned with the unique solvability of the Cahn-Hilliard system describing phase separation process in elastic solids. The process is driven by thermomechanical effects which lead to the microstructure evolution connected with pattern formation.

The system under consideration has the form of a fourth order parabolic equation with nonmonotone nonlinearity, coupled with a nonstationary elasticity system.

For such parabolic - hyperbolic system we present recent existence and uniqueness results, obtained in [1], [2].
References:
[1] I. Pawłow, W. Zajączkowski, Classical solvability of 1-D Cahn-Hilliard equation coupled with elasticity, Mathematical Methods in the Applied Sciences, in print.
[2] I. Pawłow, W. Zajączkowski, The three-dimensional Cahn-Hilliard system in elastic solids, in preparation.

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Nonlocal phase field models

Elisabetta Rocca
University of Milan, Italy
elisabetta.rocca@mat.unimi.it
Pavel Krejčí and Jürgen Sprekels

Let us consider a substance, contained in a domain Ω and undergoing phase transitions. Denote by χ the order parameter characterizing the physical state of the material. In order to take into account long-range interactions between particles, we deal with an integro-differential model for volume preserving non-isothermal phase transitions. The physical relevance of nonlocal interaction phenomena in phase separation and phase transition models was already described in the pioneering paper of Van der Waals, dating back to 1893, however only in more recent years both isothermal and non-isothermal models containing nonlocal terms have been analyzed in a more systematic way by some authors (cf., e.g., P. Fife, G. Giacomin and J.L. Lebowitz, P. Bates, H. Gajewski and many others).

The difference between local and non-local models consists in a different choice of particle interaction potential in the free energy functional. The nonlocal contribution to the free energy has typically the form $\int_\Omega k(x,y)\,|\chi(x)-\chi(y)|^2\,dy$ with a given symmetric kernel $k(x,y)$, its classical local Ginzburg-Landau counterpart has the form $(\nu/2)|\nabla\chi(x)|^2$ with a positive parameter ν, and can be obtained as a formal limit as $m \to \infty$ of the nonlocal one with the choice $k(x,y) = m^{N+2}K(|m(x-y)|^2)$, where K is a non-negative function with support in $[0,1]$.

The passage from a non-local to local potential changes dramatically the properties of the model. For example, the maximum principle is lost in the limit and in general it is not possible to guarantee without additional hypotheses that the absolute temperature remains positive during the process.

We study from the analytical viewpoint some generalizations of such models in which the temperature is included as a state variable of the system.

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Stability of the solution to the Falk model system of shape memory alloys

Shuji Yoshikawa
Ube National College of Technology, Japan
yosikawa@math.kyoto-u.ac.jp
Takashi Suzuki

This is a joint work with Takashi Suzuki (Osaka University). We consider the initial-boundary value problem for the Falk model system of shape memory alloys. We show the dynamical stability to the stationary problem of this system.

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