Special Session 19: Qualitative Properties of Evolution Equations

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This Special Session is devoted to qualitative properties of evolution partial differential equations. The topics included are for instance, convergence to steady states, description of permanent regime of dissipative equations, finite time blow-up.

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Energy decay of solutions of a wave equation of p-laplacian type with a nonlinear dissipation

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In this work we consider the initial boundary problem for the nonlinear wave equation of p-Laplacian type with a nonlinear dissipation of the type

$$(P)\begin{cases} (|u'|^{l-2}u')' - \mathrm{div}(|\nabla_x u|^{p-2}\nabla_x u) \\ \qquad + \sigma(t)g(u') = 0 \text{ in } \Omega\times[0,+\infty[, \\ u = 0 \text{ on } \partial\Omega\times[0,+\infty[, \\ u(x,0) = u_0(x),\ \ u'(x,0) = u_1(x) \text{ in } \Omega. \end{cases}$$

where Ω is a bounded domain with a smooth boundary, $p \geq 2$. Assume that the solution exists in the class

$$C(\mathbb{R}_+, W_0^{1,p}(\Omega)) \cap C^1(\mathbb{R}_+, L^l(\Omega)). \tag{1}$$

$\lambda(x), \sigma(t)$ and g satisfies the following hypotheses:
(H1) $\sigma : \mathbb{R}_+ \to \mathbb{R}_+$ is a non increasing function of class C^1 on $\mathbb{R}_+$ satisfying

$$\int_0^{+\infty} \sigma(\tau)\,d\tau = +\infty.$$

(H2) Consider $g : \mathbb{R} \to \mathbb{R}$ a non increasing C^0 function such that

$$g(v)v > 0 \ \text{ for all } v \neq 0.$$

and suppose that there exist $c_i > 0$; $i = 1,2,3,4$ such that

$$c_1|v|^m \leq |g(v)| \leq c_2|v|^{\frac{1}{m}} \text{ if } |v| \leq 1,$$

$$c_3|v|^s \leq |g(v)| \leq c_4|v|^r \text{ for all } |v| \geq 1,$$

where $m \geq 1$, $l-1 \leq s \leq r \leq \frac{n(p-1)+p}{n-p}$.
We define the energy associated to the solution given by (1) by the following formula

$$E(t) = \frac{l-1}{l}\|u'\|_l^l + \frac{1}{p}\|\nabla_x u\|_p^p.$$

Our main result is the following
Theorem : Let $(u_0,u_1) \in W_0^{1,p} \times L^l(\Omega)$ and suppose that $(H1)$ and $(H2)$ hold. Then the solution u(x, t) of the problem (P) satisfies

(1) If $l \geq m+1$, we have, $\forall t > 0$

$$E(t) \leq C(E(0))exp\left(1 - \omega\int_0^t \sigma(\tau)\,d\tau\right).$$

(2) If $l < m+1$, we have, $\forall t > 0$

$$E(t) \leq \left(\frac{C(E(0))}{\int_0^t \sigma(\tau)\,d\tau}\right)^{\frac{p}{(mp-m-1)}}.$$

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Asymptotic solutions to higher-order boussinesq systems

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We consider a class of higher-order Boussinesq systems which was presented in a paper by J.L.Bona, M.Chen and J.C.Saut (Nonlinearity 17 (2004) 925-952). We aim to study the global existence and the decay of solutions to the Boussinesq system.

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Null controllability for degenerate parabolic equations and Carleman estimates

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We consider a one-dimensional heat equation for which the coefficient in the elliptic part degenerates at one end of the spatial domain. We prove a Carleman estimate for this equation using a pseudo-convex weight related to the degeneracy rate of the coefficient. This allows us to prove null controllability for the adjoint equation by localized controls. We extend these results to the semilinear degenerate case. Such degenerate equations arise in probability theory (study of transition probabilities and Feller's semigroups, in fluids problems (for instance after suitable transformations of Prandtl equations), as well as in population genetics (see e.g. Wright-Fischer model).

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Asymptotic behaviour for quasilinear parabolic equations with lower order terms

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We present some recent results about quasilinear parabolic equations of doubly nonlinear structure, e.g., p-Laplacean or porous media operator.

The equations contain also nonlinear terms depending on the spatial gradient of the solution. We consider nonnegative solutions to the Cauchy problem.

We investigate the behaviour of the solutions for large times, namely sup bounds, gradient estimates, finite speed of propagation.

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Asymptotic stability and blow up for semelinear wave equations with dynamic boundary conditions

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In this paper we consider a multi-dimensional wave equation with dynamic boundary conditions, related to the Kelvin-Voigt damping. Global existence of solutions starting in the stable set is proved, also blow up for solutions with initial data in the unstable set is obtained.

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Large time behaviour of solutions of a reaction - diffusion equations under dynamical boundary conditions

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We discuss the behaviour in the large of solutions of a reaction - diffusion equations under dynamical boundary conditions $\sigma\partial_t u + \partial_\nu u = 0$,especially blow up phenomena will be of interest. Moreover, the type of the dynamical boundary condition will be of interest, especially its impact on dissipativity properties of the flow.

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Asymptotic behaviour for 1D radiative and reactive flows.

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We consider the 1D reactive and radiative Navier-Stokes system describing a model of gaseous star. We present results concerning the large-time behaviour depending on the order of the kinetics involved.

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Periodic solutions in Marchuk model with time-dependent immune reactivity

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In the presentation we deal with Marchuk model of immune system. One of the main parameters of the model are the coefficient which describes the state of infected organism and the coefficient which describes the rate of production of antibodies. In the classical model this coefficients are constants. We deal with the case when this coefficients are time-dependent. In particular, we are interested in the case of periodic coefficients which can describe a periodic changes of the parameters of the immune system due to periodic changes of the environment. We examine the asymptotic behaviour of the solutions. We prove, under assumptions, the solutions tends to periodic functions.

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On attractor dimension estimate for 2D shear flow of micropolar fluid with free boundary

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This research is motivated by a problem from lubrication theory. We consider a free boundary problem of a two-dimensional boundary driven micropolar fluid flow. The existence of a unique global in time solution of the problem and the global attractor for the associated semigroup are known.

In this paper we estimate the dimension of the global attractor in terms of the given data and the geometry of the domain of the flow by establishing a new version of the Lieb-Thirring inequality with constants depending explicitly on the geometry of the domain. We obtain also some new estimates for the Navier-Stokes shear flows.

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Asymptotic behavior of linear parabolic problems with the dirichlet and neumann conditions imposed on varying subsets

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We study the asymptotic behavior of the solutions u_n of linear parabolic problems posed in a fixed domain $\Omega \times (0,T)$ with variable operators depending on time. The solution u_n is assumed to satisfy a Dirichlet boundary condition on Γ_n, where Γ_n is an arbitrary sequence of subsets of $\partial\Omega$, and a Neumman boundary condition on the remainder of $\partial\Omega$. We obtain a representation of the limit problem which is stable by homogenization and where it appears a generalized Fourier boundary condition.

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Boundary Stabilization of the damped wave equation with Cauchy-Ventcel dynamic boundary conditions

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This talk is concerned with the study of optimal and uniform decay rates of the wave equation subject to Cauchy Ventcel dynamical boundary conditions.

$$\begin{cases} u_{tt} - \Delta u = 0 \text{ in } \Omega \times]0,\infty[, \\ v_{tt} + \partial_\nu u - \Delta_\Gamma v + g(v_t) = 0 \text{ on } \Gamma_1 \times]0,\infty[, \\ u = v \text{ on } \Gamma \times]0,\infty[, \\ u = 0 \text{ on } \Gamma_0 \times]0,\infty[, \end{cases} \tag{2}$$

where Ω is a bounded domain of $\mathbb{R}^n$ $(n > 2)$ having a C^3 boundary $\partial\Omega = \Gamma$, such that $\Gamma = \Gamma_0 \cup \Gamma_1$.

We prove that the boundary dissipation $g(v_t)$ is strong enough to assure the asymptotic stability of the system. The results presented in the literature deal with localized dissipations acting in a strategic neighbourhood of the boundary (sometimes in the whole domain) in order to stabilize the system. In this work we prove the reciprocal procedure (which remained an open problem), namely: to prove that a frictional dissipation acting on the boundary is strong enough, via transmission process ($u|_\Gamma = v$), to stabilize the whole system.

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On the p-Laplace operator in domains becoming unbounded

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We study the asymptotic behaviour of the Dirichlet problem for operators of the p-Laplacian type in domains becoming unbounded.

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Qualitative properties of solutions of fractional integro-differential equations with special regard to assimptotic behavior

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Consider the abstract integro-differential equation

$$(1) \qquad u'(t) = \int_0^t \frac{(t-s)^{\alpha-2}}{\Gamma(\alpha-1)} Au(s)\,ds + f(t),$$
$$u(0) = u_0 \in X, \qquad 0 \le t \le T,$$

where $1 < \alpha < 2$, $A : D(A) \subset X \to X$ is an abstract, linear and unbounded operator, and X in a complex Banach space. In this paper we focus on the properties of the solution $u : X \to X$ of (1) with special regard to its asymptotic behavior which shows of interest in the context of the numerical methods applied to solve equations which prototype is (1). In fact, we prove that there exists $C > 0$ such that

$$\|u(t)\| \leq \frac{C}{t^{\alpha}}, \qquad t > 0,$$

from which we derive an analogous result for the Euler method applied to solve (1).

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Entropy methods for the large time behavior of reaction-diffusion systems

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Entropy methods have recently been used with success in the obtention of quantitative estimates of the rate of convergence to equilibrium in the context of kinetic equations and nonlinear diffusion equations. We present here an approach based on the entropy for obtaining such rates (with explicit constants) in the context of reaction-diffusion systems appearing in problems of reversible chemistry. This approach is an elegant alternative to the use of linearization. We focus on a particular system of four equations in which no global (in time) uniform bounds was known, in order to illustrate the concept of ”entropy methods with slowly growing a priori estimates”, first introduced by G. Toscani and C. Villani for the treatment of the Boltzmann equation with soft potentials.

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Wellposedness and optimal decay rates for wave equation with nonlinear boundary damping-source interaction

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We establish, subject to some natural additional assumptions imposed on the relation between the source and the damping, both wellposedness and effective optimal decay rates for the solutions of a semilinear model of the wave equation. The theory presented allows to consider both superlinear and sublinear behaviour of the dissipation in the presence of unstructured sources. In addition, blow up of solutions in finite time is also established when two competing sources of polynomial type act on the system.

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On some fractional evolution equations with nonlocal conditions

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In this paper Leray - Schauder principle is used to establish existence results of solutions of some nonlinear fractional evolution equations with nonlocal conditions in Banach spaces. Some properties of solutions of the considered problem are studied. An application is given for a nonlinear fractional partial differential equation with nonlocal initial conditions.
Keywords and phrases: Fractional evolution equation, nonlocal initial conditions, Schauder fixed point theorem.

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Entropy Methods for spatial inhomogeneous coagulation-fragmentation models with diffusion

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Jose A. Carrillo and Laurent Desvillettes

In this talk, we consider a spatial inhomogeneous version of the Aizenman-Bak model of coagulation and fragmentation of polymers [AB] as an example of models which combine diffusion and nonlinear reactions in terms of an entropy (free energy) functional. In particular, we present how to establish a so-called entropy/entropy-disspation estimate, which is a functional inequality controlling the relative entropy with respect to a steady state in terms of the entropy disspation, i.e. the rate of the entropy decay. Once established the entropy/entropy-dissipation estimate entails readily (exponential) decay towards the steady state.

It is a major advantage of the entropy method to be quite robust w.r.t. related models. This is due to the fact that it mainly relies on functional inequalities which have no direct link with the original PDE. As a matter of fact, the presented proofs draw certain ideas from a previous study of entropy methods for systems of reaction-diffusion equations [DF, DF2]. In 1D, we use a-priori-estimates derived from the entropy decay and the corresponding bounds on the entropy dissipation.

[AB] M. Aizenman, T. Bak, Convergence to Equilibrium in a System of Reacting Polymers. Commun. math. Phys. 65, (1979), pp. 203–230.

[DF] L. Desvillettes, K. Fellner, Exponential Decay toward Equilibrium via Entropy Methods for Reaction-Diffusion Equations, to appear in J. Math. Anal. Appl.

[DF2] L. Desvillettes, K. Fellner, Entropy Methods for Reaction-Diffusion Equations: Slowly Growing A-priori Bounds, Preprint

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Complex Degenerate Advection Diffusion Systems

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We are concerned with the global existence and approximation of degenerate reaction advection diffusion systems modeling the photochemical generation and atmospheric dispersion of pollutants.

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Problem of Cauchy for system which describes filtration of natural gas

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In this paper we moulded a system which describes filtration of natural gas. We give a conditions for the coefficients of the system for which the system has nonnegative solutions, conditions for the coefficients of the system for which the system is stable; conditions for the coefficients of the system for which the system is unstable' The proposed model is based on a real example of Bulgaria-the Eocene interval in the Lower Kamchia Depression. It is a small foredeep basin, developed at a front of the East Balkan thrust belt, closely related to the Black sea evolution. The basin sedimentary-fill is represented by a thick, predominantly siliciclastic deposits. As a possible example of homogenous system could be described part of the Avren Fm, which possessed all appropriate values of mentioned above, hydrocarbon components, as well as favorable burial history. Avren Fm is composed of lamely siltstones to fine-grained sandstones with thick, irregular sets of medium- to coarse-grained sandstones (tributary to distributory channels with fan-delta feeding). The formation an average thickness reached to 800-1200m. It might be considered as the main petroleum target at the basin due to source potential and good reservoir properties, respectively.

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Large-time behaviour of solutions of the porous media equation in an exterior domain

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The topic of this contribution is the large-time behaviour of solutions of the exterior-domain Cauchy–Dirichlet problem for the porous media equation with homogeneous boundary data. The radially-symmetric problem in the exterior of the unit ball with, as it were, a space dimension that is any real number, is considered first. The solution of this problem converges to one of two explicit self-similar solutions, dependent upon the dimensionality of the problem. The critical space dimension is two, for which there is no distinction between the self-similar solutions, and the convergence is exceptional. The technique used to determine this behaviour is a comparison principle involving a variable that is a weighted integral of the solution. Attention turns thereafter to the problem in an arbitrary domain in a natural number of space dimensions with no conditions of symmetry. The large-time behaviour is determined by a special invariance principle and the behaviour established for the radially-symmetric situation. The behaviour reveals similarities and contrasts with that known for other problems for the porous media equation. (The work is joint with J. Goncerzewicz.)

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Reaction-diffusion on network-like domains

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We consider the reaction-diffusion equation defined on a sequence of bounded open sets $(\Omega_n)_{n\in N}$ which converge to Ω in the sense of Mosco. We prove that main properties of the attractor, as stability of steady states, are invariant under domain perturbation. We apply these results to dumpbell and other types of domains forming what we call networks.

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Asymptotic behavior of solutions of hyperbolic problems on a cylindrical domain.

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The asymptotic behavior of the hyperbolic evolution problems of order two, on a cylindrical domain $\Omega = \Delta \times \omega$, with coefficients dependent on a parameter is examined. The convergence of the solution of such problems towards a solution of a problem of the same type defined in ω is proved, and the rate of convergence estimates is given. One can see work like a singular perturbation of the hyperbolic problems in some directions.

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Rate of Convergence to a Singular Steady State for a Heat Equation with Strong Absorption

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We study the initial boundary value problem for a heat equation with strong absorption. We first prove that the solution of this problem converges to the unique singular steady state for a class of initial data. This gives an example of dead-core which is developed in infinite time. Furthermore, we also derive the exact convergence rate (the dead-core rate) by a matching process.

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Multi-dimensional bistable reaction-diffusion fronts

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Conical-shaped curved fronts with positive aperture angle are known to exist for homogeneous reaction-diffusion equations $u_t = \Delta u + f(u)$ in $\mathbb{R}^N$ with unbalanced bistable nonlinearities f. Classification, uniqueness and stability results can also be obtained. In this talk, I will report on recent results in the case when the nonlinearity f is balanced (its integral is equal to 0 on the range of u). By approximating the balanced case by unbalanced nonlinearities, we were able to prove the existence of non-planar front-solutions in the balanced case. The level sets behave like hyperbolic cosines in dimension 2, while they are paraboloid like in dimensions 3 and higher, due to curvature effects. The existence of such curved fronts, for any positive speed, provides counter-examples to the parabolic version of a conjecture of De Giorgi.

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A Functional Reaction-Diffusion Problem with Hysteresis

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This talk is based on joint work with J.I. Díaz and Lourdes Tello. Simple heuristic climate models lead to reaction-diffusion equations on the 2-sphere with slow diffusion and memory. Additionally, the reaction part exhibits a jump discontinuity (at the snow line) in case of so-called Budyko-type models. Finally, a Babuška-Duhem hysteresis accounts for a frequent repetition of sudden and fast warming followed by much slower cooling as observed from paleoclimate proxy data. Global existence and the existence of a trajectory attractor will be discussed for the resulting system.

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Solvability of some volterra type integral equations in Hilbert spaces

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We consider an integral equation of Fredholm and Volterra type with spectral parameter depending on time. Conditions of solvability are established when the initial value of the parameter coincides with an eigenvalue of Fredholm operator.

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Blow-up results for some nonlinear delay differential equations

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In this work we study the blow up phenomena for some scalar delay differential equations. In particular, we make connection with the blow up of ordinary differential equations that are related to the delay differential equations.

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On the Fractal Hamilton-Jacobi-KPZ equations

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Nonlinear and nonlocal evolution equations of the form $u_t = Lu \pm |\nabla u|^q$, where L is a pseudodifferential operator representing the infinitesimal generator of a Lévy stochastic process, have been derived as models for growing interfaces in the case when the continuous Brownian diffusion surface transport is augmented by a random hopping mechanism. The goal of this talk is to present recent research on properties of solutions to this equation resulting from the interplay between the strengths of the "diffusive" linear and "hyperbolic" nonlinear terms, posed in the whole space R^N, and supplemented with nonnegative, bounded, and sufficiently regular initial conditions.

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Convergence in some Degenerate Parabolic Equations with Delay

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Busenberg and Huang (JDE 1996) showed that small positive equilibria can undergo stable Hopf bifurcation (giving rise to spatially inhomogenous time-periodic solutions) in the delay reaction-diffusion equation $u_t = u_x x + ku(1 - u(x,t-r))$, with homogeneous Dirichlet boundary conditions. Here, sufficient conditions will be given which ensure global convergence of non-negative solutions to a positive equilibrium for a class of delayed parabolic equations possessing nonlinear degenerate diffusion. In particular, small equilibria cannot be destabilised by delay. This class includes the KPP-type reaction term above as a special case, but with nonlinear diffusion.

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Solitary and Self-similar Solutions of Two-component System of Nonlinear Schrödinger Equations

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Conventionally, to learn wave collapse and optical turbulence, one must study finite-time blow-up solutions of one-component self-focusing nonlinear Schrödinger equations (NLSE). Here we consider simultaneous blow-up solutions of two-component system of self-focusing NLSE. By studying the associated self-similar solutions, we prove two components of solutions blow up at the same time. These self-similar solutions may come from solitary wave solutions with multi-bumps forming abundant geometric patterns which cannot be found in one-component self-focusing NLSE. Our results may provide the first step to investigate optical turbulence in two-component system of NLSE.

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General decay of solutions of a semilinear viscoelastic equation

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In this work, we consider the semilinear problem

$$(P)\begin{cases} u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + |u|^\gamma u = 0, \\ \qquad \text{in } \Omega \times (0,\infty) \\ u(x,t) = 0,\ x \in \partial\Omega\ , t \geq 0 \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ x \in \Omega. \end{cases}$$

where Ω is a bounded domain with a smooth boundary, $\gamma > 0$, with

$$\begin{array}{lllll} 0 & \leq & \gamma \leq \dfrac{2}{n-2}\,, & n \geq 3 & \\ \gamma & \geq & 0, & n = 1, 2, & \end{array} \tag{1}$$

and the relaxation function g satisfies
(G1) $g : \mathbb{R}_+ \to \mathbb{R}_+$ is a differentiable function such that

$$g(0) > 0, \qquad 1 - \int_0^\infty g(s)ds = l > 0.$$

(G2) There exists a positive constant ξ such that

$$g'(t) \leq -\xi(t)g(t),\ t \geq 0.$$

(G3) ξ is a nonincreasing differentiable function satisfying

$$|\frac{\xi'(t)}{\xi(t)}| \leq k, \quad 0 < \xi(t) \leq M, \quad \xi'(t) \leq 0, \quad \forall t > 0$$

We establish a general decay result. More presicely, we establish the follwing:
Theorem *Let* $(u_0, u_1) \in H_0^1(\Omega) \times L^2(\Omega)$ *be given. Assume that* (1) *and*$(G1)$-$(G3)$ *hold.*

Then, for each $t_0 > 0$, *there exist strictly positive constants* K, α *such that the solution of* (P) *satisfies*

$$\mathcal{E}(t) \leq Ke^{-\alpha\int_{t_0}^t \xi(s)ds}, \qquad t \geq t_0.$$

This work generalizes and improves earlier results by Cavalcanti, Messaoudi, Berrimi, and others.

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Kolmogorov equations and option pricing

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The talk will present a survey of the theory of partial differential equations of Kolmogorov type arising in physics and in mathematical finance. These evolutionary equations, which are generally non-uniformly parabolic, are naturally associated to stochastic models with memory. Financial derivatives with dependence on the past provide some typical examples: in particular, Asian options and the modeling of stochastic volatility for the evaluation of derivative securities will be discussed.

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Gaussian estimates for degenerate operators on Lie groups

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We prove some Gaussian lower bounds for the positive solutions of a family of partial differential equations in $\mathbb{R}^n \times \mathbb{R}$, in the form

$$Lu := \sum_{i,j=1}^{m} X_i X_j u + X_0 u - \partial_t u = 0,$$

where every X_j is a first order vector field (i.e. $X_j = \sum_{k=1}^{n} a_{j,k}(x)\partial_{x_k}$). We assume that the X_j's satisfy the Hörmander condition on commutators, so that any (possibly weak) solution u to the equation $Lu = 0$ is a smooth classical solution. We prove a Gaussian lower bound of the positive solutions that are defined on a domain of the form $\mathbb{R}^n \times]0,T[$ and, as a consequence, we get a lower bound for the fundamental solution of L.

Our technique extends the Aronson's method for the uniformly parabolic equations, that relies on the repeated use of an invariant Harnack inequality. The natural framework for the study of our operator L is the non-Euclidean geometry of the Lie groups. In order to adapt the Aronson's method to the Lie group geometry we solve some suitable optimal control problems.

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Extremal equilibria for parabolic equations and applications.

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In this talk, we show that a wide class of parabolic reaction diffusion equations have two extremal ordered equilibia. These are maximal and minimal equilibria, in the ordering sense, which are stable from above and below, respectively, and the global dynamics of the ploblem is confined between them. Therefore they are the "caps" of the global atractor. The same dynamical behavior is obtained when analyzing only positive solutions. In some cases uniquenes of positive solutions can also be obtained, showing that the maximal equilibria is globally asymptotically stable for nonnegative solutions. We will also show how to recover several classical results on the existence of positive solutions of elliptic equations with this dynamical approach.

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The blow-up problem for a semilinear parabolic equation with a potential

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Let Ω be a bounded smooth domain in $\mathbb{R}R^N$. We consider the problem $u_t = \Delta u + V(x)u^p$ in $\Omega \times [0,T)$, with Dirichlet boundary conditions $u = 0$ on $\partial\Omega$ and initial datum $u(x,0) = Mu_0(x)$ where $M \geq 0$, u_0 is positive and compatible with the boundary condition. We give estimates for the blow up time of solutions for large values of M. We find that, for M large, the blow up set concentrates near the points where $u_0^{p-1}V$ attains its maximum.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Asymptotic stability for non-autonomous damped Kirchhoff equations

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G. Autuori and P. Pucci

We study the asymptotic stability for solutions of the nonlinear damped Kirchhoff equation, with homogeneous Dirichlet boundary conditions. More precisely, we consider the problem

$$\begin{cases} u_{tt} - M(\|\nabla u\|_2^2)\Delta u + Q(t,x,u,u_t) = f(x,u), \\ \qquad (t,x) \in \mathbb{R}_0^+ \times \Omega, \\ u(t,x) = 0, \quad (t,x) \in \mathbb{R}_0^+ \times \partial\Omega, \end{cases}$$

where Ω is bounded domain of $\mathbb{R}^n$, $n \geq 3$, $M(s) = a + bs^\gamma$, $s \geq 0$, $a, b \geq 0$, $a + b > 0$ and $\gamma \geq 1$. Roughly speaking the continuous function Q is modelled by $\alpha(t)|u_t|^{q-2}u_t$, while f by $\beta(x)|u|^{p-2}u$, where $\alpha \in C(J)$, $\beta \in C(\overline{\Omega})$, α, $\beta \geq 0$, while the exponents satisfy the main restrictions

$$1 < p < \infty, \qquad 1 < q \leq \max\left\{p, \frac{2n}{n-2}\right\}.$$

(Joint work with G. Autuori and P. Pucci).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Boundary stabilization of solutions of a nonlinear system of Timoshenko type

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In this talk we are concerned with a multidimensional Timoshenko system subjected to boundary conditions of memory type. We establish general rate decay results. The usual exponential and polynomial decay rates are only special cases.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nonlinear-diffusive logistic equations with spatial heterogeneity

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The subject of our investigation is non-negative steady states for diffusive logistic equations with the the nonlinear diffusion of p-Laplacian and spatially heterogeneous reactions. It is shown that the stationary solutions coincide with the carrying capacity function in each region where it is constant if a parameter for diffusion is sufficiently small.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Traveling waves in the Allen-Cahn equations

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This work is concerned with traveling waves in the Allen-Cahn equation. I introduce a method to construct supersolutions and subsolutions and construct traveling waves in multi-dimensional Euclidean space.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Asymptotic analysis and estimates of blow-up time for the radial symmetric semilinear heat equation in the open-spectrum case

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We estimate the blow-up time for the reaction diffusion equation $u_t = \Delta u + \lambda f(u)$, for the radial symmetric case, where f is a positive, increasing and convex function growing fast enough at infinity. Here $\lambda > \lambda^*$, where λ^* is the "extremal" critical value for λ, such that there exists an "extremal" weak but not a classical steady-state solution at $\lambda = \lambda^*$ with $||w(\cdot,\lambda)||_\infty \to \infty$ as $0 < \lambda \to \lambda^*-$. Estimates of the blow-up time are obtained by using comparison methods. Also an asymptotic analysis is applied when $f(s) = e^s$, for $\lambda - \lambda^* \ll 1$, regarding the form of the solution during blow-up and an asymptotic estimate of blow-up time is obtained. Finally some numerical results are also presented.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Attractors for the 3D Navier-Stokes system

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Let $\Omega \subset \mathbb{R}^3$ be a bounded open subset with smooth boundary. For a given $\nu > 0$ we consider the Navier-Stokes system

$$\begin{cases} \dfrac{\partial u}{\partial t} - \nu\triangle u + (u\cdot\nabla)u = -\nabla p + f(t), \\ div\ u = 0, \\ u|_{\partial\Omega} = 0, \\ u(\tau) = u_\tau. \end{cases}$$

Whereas in the two dimensional case the existence of the global attractor is a well known result in both the autonomous and nonautonomous cases, the three dimensional case contains some difficult problems to overcome. On the one hand, it is not known whether the weak solution corresponding to the Cauchy problem is unique or not. On the other, and this is the main difficulty, in the classical results the weak solutions were proved to be continuous in time only with respect to the weak topology of the phase space.

For external forces f in H we define a family of multivalued semiflows G_R from the ball of radius $R \geq R_0$ into itself, where R_0 is a fixed constant depending on the parameters of the problem. We prove for any $R \geq R_0$ the existence of a global attractor $\mathcal{A}_R$ but considering the attracting property in the weak topology of the phase space. Moreover, it is shown that the global attractor does not depend on R, i.e., $\mathcal{A}_R = \mathcal{A}_{R_0}$, for all $R \geq R_0$.

These results are extended also to the nonautonomous case.

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Mild solutions and their long-time behavior for the 2D Boussinesq equation in a disc

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Two-dimensional "good" Boussinesq equation is known to govern small nonlinear oscillations of elastic membranes. Its damped version with a source term is considered in a unit disc with homogeneous boundary conditions corresponding to a simply supported boundary. This model is argued to describe small nonlinear oscillations of a circular membrane under the influence of acoustic pressure. Mild solutions of the mixed problem in question are of interest. They are constructed by the method of eigenfunction expansions developed by the author in his earlier works. Convergence of the corresponding series is proved in anisotropic Sobolev spaces. These function spaces allow to reveal the effect of nonlinear smoothing due to periodicity conditions in the angular coordinate. Long-time behavior of the mild solutions is established.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positivity properties and nonuniqueness in the quenching problem

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We consider the prototype of a singular semilinear parabolic equation,

$$u_t = \Delta u - u^{-q}, \qquad 0 < q < 1. \qquad (\star)$$

The corresponding Dirirchlet problem in a smooth bounded domain with arbitrarily prescribed continuous nonnegative boundary data possesses at least one continuous weak solution. Such a solution can be obtained, for example, as the limit of a decreasing sequence of solutions to suitably regularized problems. On the other hand, however, $(\star)$ has an explicitly known singular stationary solution w. The talk deals with positivity properties of the weak solution constructed via the above limit process. As a result, we obtain a negative answer to the long time open uniqueness question for $(\star)$: Namely, it turns out that this solution, when emanating from the singular steady state w, becomes positive (and thus regular) immediately and therefore must be different from w.

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