Special Session 1: Mathematical Aspects of Wave Propagation

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This Special Session is devoted to different aspects of the modern Wave Propagation theory. The topics included are wave propagation in different media, oscillations of mechanical systems, random waves and oscillations, nonlinear optics, nonlinear waves, optical solitons, dispersion-managed solitons, quasi-linear pulses, solitary waves, solitons in plasmas, etc.

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Modeling light trapping in nonlinear periodic structures

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An area of intense research is that of photonics, where light propagation features are controlled by clever engineering of periodic optical structures. For example, the fiber bragg grating where an additional intensity dependent nonlinear index of refraction allows soliton like propagation with tunable velocities. In this work we consider nonlinear periodic geometries. We show that the additional transverse dimension allows for a richer dynamics of light trapping, bending and switching, provided stable gap soliton-like bullets exist. We first show the behavior of soliton-like bullets in a nonlinear periodic waveguide. We then follow by medeling the interaction of these solitons with defects in the photonic structure. Finally, a reduced finite dimensional model to study the dynamics of trapped energy carried by incident 2D gap solitons (GS) into localized defects in Bragg resonant Kerr nonlinear (photonic) gratings is presented.

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Degeneration of creeping waves on an anisotropic impedance surface

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Creeping waves propagate along the convex surface of an obstacle and give the filed in deep shadow. Dielectric coverings on the obstacle can be described in many cases by Leontovich impedance boundary conditions. If the material of the covering is anisotropic this condition becomes matrix. Creeping waves on an anisotropic impedance surface were studied, for example, in [1] with the use of "impedance stretching" technique (see also [2]). It can be seen from these results that a specific case of anisotropic impedance appears, namely when the "equivalent" impedances of the two types of creeping waves coincide. This happens together with the degeneration of the matrix, $\begin{pmatrix} \det\mathbf{Z} & -Z_{sa} \\ Z_{sa} & 1 \end{pmatrix}$ where $\mathbf{Z} = \begin{pmatrix} Z_{aa} & Z_{sa} \\ Z_{sa} & Z_{ss} \end{pmatrix}$ is the impedance matrix written in coordinates (s,a), s is the arc-length of the geodesics followed by creeping waves, a is transverse surface coordinate. Two subcases are possible. The simpler one is when $Z_{sa} = 0$. This case is similar to the case of isotropic impedance equal to one which is studied in [3] (see also [2]). The other case with $Z_{sa} \neq 0$ appears more difficult. The usual form asymptotic decomposition occurs inapplicable. Modifications required are in introducing additional exponential factor proportional to $k^{1/6}$ (k is wave number, large parameter) and in carrying out the asymptotic expansion in inverse powers of $k^{1/6}$, but not $k^{1/3}$ as usually. The recurrent procedure is to be described and the principal order expression for the creeping wave in that degenerated case will be presented. This expression allows the following effects to be noticed:

- Additional quick (like $k^{1/6}s$) dependence of the wave field appears.
- Such parameters of the surface as torsion are presented in this quick dependence (to compare, in the general case torsion is presented only in the correction to the amplitude and in the case of impedance equal to one it appears in the amplitude factor, but without large parameter).
- In the leading order creeping wave of both types have coincident polarization, difference appears only in the first order correction, i.e. at the order $k^{-1/6}$.
- Some additional multipliers appear.

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[1] I. Andronov, D. Bouche "Theoretical analysis of creeping waves", *Annales des Telecomm.*, 1993, **vol. 49**, No 3–4. pp. 193-210.

[2] F. Molinet, I. Andronov, D. Bouche *"Asymptotic and Hybrid Methods in Electromagnitics"*, IEE Electromagnetic Waves Series 51, 2005.
[3] D. Bouche "La methode des courants asymptotiques", These, Universite Bordeaux-1, 1992.

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Optimal design of an elastic string with respect to its optical length

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We discuss some problems of the optimal design (optimal mass distribution $\rho(x)$) for an elastic string with the fixed end points and given tension distribution $p(x)$. The criterion is the *time of control* associated with the *optical length*. The functional $T = 2\int_0^l \sqrt{\rho(x)/p(x)}dx$ is known as the time of exact controllability of the string and the total mass is given by $M = \int_0^l \rho(x)dx$. We discuss the following **Problem**: Given the length of the string, the tension function , and its total mass find a distribution of the density $\rho(x)$ such that the time of control T is a maximal / minimal possible. We prove the existence of $T_{\max}$ and construct the corresponding design explicitly. We further prove that $\inf T = 0$ and construct one of the possible minimizing sequences. We consider some examples of the string (control of which was recently studied). For a rotating string, the previous results may not be applied, but another approach shows that $\sup T = \infty$. An elastic string with the tension $p(t)$ (a function of time, not coordinate) is considered as well.

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Stochastic Perturbation of power law optical solitons

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The soliton perturbation theory is used to study and analyze the stochastic perturbation of optical solitons, with power law nonlinearity, in addition to deterministic perturbation terms, that is governed by the nonlinear Schrodinger's equation. The Langevin equations are derived and analysed. The deterministic perturbation terms that are considered here are due to filters and nonlinear damping.

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Energy of a General Linear Wave Equation Driven by Fractional-in-Time Noise

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This talk considers a general linear stochastic wave equation driven by fractional-in-time noise with Hurst parameter, $H \in (0,1)$. The equation is solved and its energy is studied. Series expansions for both the energy and the expected energy are given. Asymptotic results for the expected energy for large and small times are found. These results shed light on the inteplay between the fractional-in-time noise and the wave operator. It is also shown that the expected energy is continuous as a function of H at $H = 1/2$. Finally, analytic continuation is used to provide an alternate analysis for $H < 1/2$.

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Asymptotic models for diffraction by thin wires

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This presentation deals with the asymptotic analysis of the outgoing solution to the Hemholtz equation (denoted by u^ε) outside perfectly conducting thin wires . In particular, consider the segment $I_\alpha = \{\mathbf{x}(x,y,z) \in \mathbb{R}^3\ ,\ x = y = 0\ ,\ |z| \leqslant \alpha\}$ and a family of regular open sets (thin wires) $\omega_\alpha^\varepsilon \subset \mathbb{R}^3$ containing I_α such that $\omega_\alpha^\varepsilon$ contracts into I_α when $\varepsilon \to 0$. In the case $\alpha = +\infty$, we establish the existence and uniqueness of the outgoing solution to the Helmholtz equation and study the asymptotic behavior of u^ε as $\varepsilon \to 0$ using Fourier transform. In the case α is finite, using prolate spheroidal coordinates, the same asymptotic analysis is done. In particular, these results are valid in the neighbourhood of the tips.

In the context of the scattering by thin wires, these analytic results could be the starting point for the design of new volumic numerical methods that would provide accurate results without meshing the 3D geometry of wires. These methods would provide a satisfying alternative to the heuristic Holland model.

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Oscillatory motion of solitons in two-dimesional waveguides

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It is well known that the one-particle type spatial soliton exists by maintaining a balance between propagational dispersive as the linear contribution and propagational refraction from nonlinear effects. When such a soliton propagates in a medium with a spatially varying, inhomogeneous index of refraction, the soliton may follow a non-straight line path. In this paper, we present evidence of the oscillatory motion of solitons in a triangular index profile. In both the Kerr and power law media, we have shown that the soliton behaves as a classical particle that is trapped in a potential well. Here, we calculate the spatial acceleration, and corresponding spatial period. Using the 620 nm wavelength input laser, soliton periods on the order of magnitude of 16 cycles per meter of travel distance occur for stable filaments or self-channel solitons having intensities of 3x1016 watts/m2. These behaviors are developed through the higher nonlinear Schrödinger's equation (HNLSE). Also, our results are compared the swing effect for a Gaussian index profile. These results are critical for optical switching, automatic multiplexing and the all-optical computing

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Classical Wave Propagation, Quantum Physics, and Modern Mathematical Asymptotics: the Holy Trinity of Classical Wave Theory

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Seismo-acoustic wave propagation, imaging, and inversion modeling and computation are complicated by the complex, layered environments that extend over very large domains in the interior of the earth. Over the past decades, these problems have been attacked by direct approximations on the wave field (e.g., perturbation theory, asymptotic ray theory, spectral analysis, Gaussian beams), derivations of approximate wave equations (e.g., scaling analysis, approximation theory), and computational partial differential equation (pde) methods (e.g., finite differences, finite elements, spectral methods). Rather than focus on these more traditional approaches to direct and inverse, classical wave propagation modeling and computation, this talk will examine an approach based on the application of what is loosely referred to as phase space and path integral methods. These methods were developed primarily in the quantum physics and theoretical pde communities, and include constructions such as Feynman's path integral formulation of non-relativistic quantum mechanics, and the theories of pseudodifferential and Fourier integral operators, for example. The principal aims of this approach are (1) to exploit well-posed, one-way (marching) methods in these inherently two-way(global) problems, (2) to exploit the correspondences between classical wave propagation, quantum physics, and microlocal analysis (modern mathematical asymptotics), and (3) to extend Fourier methods to analyze inhomogeneous environments.

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Stochastic Volterra equations in Hilbert space

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We consider the following stochastic Volterra equation in a Hilbert space H

$$X(t) = X_0 + \int_0^t a(t-\tau)AX(\tau)d\tau + \int_0^t \Psi(\tau)dW(\tau), \quad (1)$$

where $t \geq 0$, A is a closed unbounded linear operator in H with a dense domain $D(A)$ and $a \in L^1_{\mathrm{loc}}(\mathbb{R}_+)$. The equation (1) is driven by a Wiener process W (genuine or cylinrical one). The process Ψ is an appropriate stochastic process.

A deterministic version of the equation (1) arises in several applications as model problems. Moreover, the techniques like perturbation or coordinate transformation allow to transfer results for such model problems to parabolic integro-differential equations on smooth domains.

If the kernel function a is completely positive and the operator A generates a C_0-semigroup, we can provide sufficient conditions for mild solutions to be also strong solutions to the eq. (1), see [1].

Let us note that if

$$a(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}, \quad \alpha > 0, \quad (2)$$

the eq. (1) is an integral form of the equation

$$D^\alpha u(t) = Au(t) + f(t), \quad t \in (0,T],$$

where D^α, $\alpha > 0$, is a fractional derivative and A is as above.

Note that the kernel function a is completely positive only for $\alpha \in (0,1]$, but for $\alpha > 1$ is not. So, the above mentioned results can not be directly used (see, [2]). The talk will present the recent results concerning the equations (1) with the kernel (2).

References:
[1] A.Karczewska, C.Lizama, Strong solutions to stochastic Volterra equations, submitted.
[2] A.Karczewska, C.Lizama, Stochastic fractional Volterra equations in Hilbert space, submitted.

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Convectons

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Recent simulations [1,2] of binary fluid convection with a negative separation ratio reveal the presence of multiple numerically stable spatially localized steady states we have called 'convectons'. These states consist of a finite number of convection rolls embedded in a non-convecting background and are present at supercritical Rayleigh numbers. The convecton length decreases with decreasing Rayleigh number; below a critical Rayleigh number the convectons are replaced by relaxation oscillations in which the steady state is gradually eroded until no rolls are present (the slow phase), whereupon a new steady state regrows from small amplitude (the fast phase) and the process repeats. Both ^{3}He-^{4}He mixtures [1] and water-ethanol mixtures [2] exhibit this remarkable behavior. Stability requires that the convectons are present in the regime where the conduction state is convectively unstable but absolutely stable. The multiplicity of stable convectons can be attributed to the presence of a 'pinning' region in parameter space, or equivalently to a process called homoclinic snaking [3]. In the pinning region the fronts bounding the convecton are pinned to the underlying roll structure; outside it the fronts depin and allow the convecton to grow at the expense of the small amplitude state (large Rayleigh numbers) or shrink back to the small amplitude state (low Rayleigh numbers). The convectons may exist beyond the onset of absolute instability but the background state is then filled with small amplitude traveling waves. A theoretical understanding of these results will be developed.

References:
[1] O. Batiste and E. Knobloch. Simulations of localized states of stationary convection in ^{3}He-^{4}He mixtures. Phys. Rev. Lett. 95, 244501 (2005).
[2] O. Batiste, E. Knobloch, A. Alonso, I. Mercader. Spatially localized binary fluid convection. Journal of Fluid Mechanics (in press).
[3] J. Burke and E. Knobloch. Localized states in the generalized Swift-Hohenberg equation. Physical Press E (in press).

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Some Aspects of Wave Propagation Using Modified Nonlinear Schrodinger Equation

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We have investigated high power nonlinear optical wave propagation in doped fibers and other materials possessing higher order nonlinearity. Both single and coupled waves have been considered. First we have investigated evolution of two spatially separated laser beams which are propagating in a cubic quintic nonlinear media. Variational formalism has been employed for the derivation of evolution equations of several parameters relevant to laser beams. We have shown that due to mutual nonlinear interaction two parallel circular Gaussian beams become elliptical Gaussian after traveling finite distance. It is revealed that though in the Kerr media no stable composite bound state exists, oscillating stationary bound state exists in quintic media. Dragging and trapping of a weak laser beam by a strong laser beam has been discussed. The role of quintic nonlinearities in induced focusing, formation of composite bound states, soliton dragging and on all optical switching have been highlighted. Next we have investigated the propagation characteristics of a cosh-Gaussian laser beam in a chalcogenide glass. At the end we have investigated the propagation characteristics of a chirped optical pulse in anomalously dispersive media possessing saturating nonlinearity.

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Fractional Laplace Motion

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Fractional Laplace motion (FLM) is a new stochastic process obtained by subordinating fractional Brownian motion (FBM) with parameter 0 ¡ H ¡ 1 to a gamma process. In the special case H = 1/2, FLM densities solve a fractional evolution equation with exponentially weighted derivatives, a variation on the usual fractional derivative. Developed recently to model hydraulic conductivity fields in geophysics, FLM also seems to be an appropriate model for certain financial time series. Its one dimensional distributions are scale mixtures of normal laws, where the stochastic variance has the generalized gamma distribution. These one dimensional distributions are more peaked at the mode than a Gaussian, and their tails are heavier. This talk is an overview of the basic properties of the FLM process. which include covariance structure, densities, moments, stochastic representations, infinite divisibility, stochastic self-similarity, and tail behavior. Time permitting, we shall also discuss the corresponding fractional Laplace noise, which may exhibit long-range dependence.

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Boundary controllability of Maxwell's equations with heterogeneous medium and nonzero conductivity inside a general domain

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In this presentation we consider the question of control of Maxwell's equations in a non-homogeneous medium with positive conductivity by means of boundary surface currents. The existing literature on this topic only deals with zero internal conductivity case, to the best of the author's knowledge. In present work we assume that domain is bounded simply connected star-shaped region in R^3 which is made up of a heterogeneous medium with positive conductivity, with controls being applied over the entire boundary. Using the Hilbert Uniqueness Method of Lions, the exact boundary controllability over a sufficiently long time period is established for this case, provided that both the size and the spatial gradient of the conductivity term is small enough to satisfy certain technical inequality (even if the medium is homogeneous). Also for our proof to work, the functions describing the electric permittivity and the magnetic permeability must satisfy certain technical inequalities, which roughly imply that these functions may not be radially decreasing.

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Diffraction of an electromagnetic wave by a elongated prolate body using Heun bi-confleunt equation

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Following the Geometrical Theory of Diffraction approach of creeping waves introduced by J. B. Keller and the contribution of I. Andronov and D. Bouche (*) on elongated bodies, the authors have developed formulas that can be used in concrete situations. The idea is first to have high frequencies asymptotics expansions of the total electromagnetic field near the body.

In a first step, the most common formulas have been explained close to the lit-shadow boundary and in the deep shadow region on the surface. By using geodesic coordinate system, analytics solutions of the main term of the total electromagnetic field have been derived for an observation point located close to the surface.

The problem is secondly solved by the same way for moderatly elongated bodies. The Fourier transform of the electromagnetic field satisfies the Airy equation as in the first case. Solutions of the amplitude equation requiring the transverse geodesic curvature have been developed.

The case of elongated bodies involve bi-confluent Heun equation. New Fock functions, used to describe the solutions form developed for creeping waves are requiring the transverse geodesic curvature.

Finally, we have applied our theorical developments to a strongly elongated prolate spheroid illuminated by a plane wave propagating in its axial direction. Numerical results for the asymptotic currents along the geodesic followed by creeping waves will be shown and compared to the results obtained by solving the EFIE.

(*) ANDRONOV, I., and BOUCHE, D. : "Asymptotics of creeping waves on a strongly prolate body" Annales des Telecomunications, 1994, 49,(3-4), pp. 205-210.

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Wave Propagation and Energy Transformation in Checkerboard Spatiotemporal Microstructures

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We consider propagation of waves through a rectangular checkerboard-type spatio-temporal material structure (dynamic material) in one spatial dimension and time. Both spatial and temporal periods in this material are assumed to be of the same order of magnitude. The rectangles in a checkerboard are assumed to be filled with materials having equal impedance but different phase speeds. Within certain parameter ranges, we observe numerically the formation of distinct and stable limiting characteristic paths ("limit cycles") that attract neighbouring characteristics after a few time periods. The average speed of propagation along the limit cycles remains the same throughout certain ranges of parameters of the microgeometry making the material microstructure stable. A dynamic material is a thermodynamically open system, as it is involved in a permanent exchange of energy and momentum with the environment. Material assemblages that produce the limit cycles are special in this aspect. Specifically, to make a wave travel through such an assemblage, an external agent may need to supply energy to maintain the material pattern. We see analytically and numerically that the energy may be accumulated in a wave travelling through this pattern and will become infinite due to the mechanism similar to that working in a swing. Moreover, due to the properties of limit cycles, waves can be compressed spatially to produce pulses with extraordinarily high power densities. This feature is impossible with laminates; it is specifically due to the checkerboard property pattern.

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A new method for the determination of the electromagnetic impulse response of a target

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The response of a target to an electromagnetic impulse remains an important problem especially when the impulse becomes very narrow so that its spectrum extends far in the high frequency domain. In such situations, standard methods for solving the problem (finite differences, marching in time procedures applied to the integro-differential equations in the time-domain,...) become impractical. An alternative approach consists in treating the limiting case of an infinitely narrow incident impulse which is mathematically defined by a Dirac distribution. Once the solution to this problem which is called the impulse response, is known, the solution of the narrow impulse problem may be obtained by a convolution product. In this paper, a new approach for determining the impulse response of an arbitrary perfectly conducting smooth convex object is presented. The method takes advantage of a well known theorem which allows to identify separately the terms defined on the support of the Dirac distributions and elsewhere in the time domain Kirchhoff integro-differential equations verified by the diffracted field. As a result, separate equations are obtained for the impulse components and the non-impulse components of the diffracted field. It is shown that the impulse components are identical to the time domain geometrical optics field whereas the non-impulse components verify a system of integro-differential equations which can be solved by a marching in time technique once the initial values of the components and their first order derivatives are known. The important point is that these equations have no singularity related to the incident field. They can therefore be solved with a sampling of the surface depending only on its geometry and not on the shape of the impulse as in the standard techniques. Another important result is that the initial values of the non-uniform components and their first order derivatives are given by the second and third order terms of the Luneburg-Kline expansion. Our method gives therefore the correction term to the ray method in the time domain.

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Form methods for damped wave equations

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The methods of bilinear (or, more generally, sesquilinear) forms has been developed since the 1950s as an effective Hilbert space technique to deal with abstract Cauchy problems of parabolic type, generalizing the Spectral Theorem to the non self-adjoint case (in pretty much the same way as the Lax-Milgram Lemma generalizes the Riesz-Fréchet Theorem), and furthermore allowing for a number of results about qualitative properties of solutions. It is a folk theorem that each problem governed by an analytic semigroup can be discussed by means of form techniques, but it seems that little attention has been paid so far to a special class of abstract Cauchy problems of parabolic type: those second order problems modeling damped wave equations like $\ddot{u}(t) = Au(t) + \rho A\dot{u}(t)$ or $\ddot{u}(t) = Au(t) + \rho A^{\alpha}\dot{u}(t)$. In our talk, we introduce a motivating problem and discuss its (analytical) well-posedness by means of form techniques, generalizing some known results of, among others, Chen-Triggiani and Xiao-Liang to the case where A is not self-adjoint.

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Solvable model for Helmholtz resonator

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The Kirchhoff model gives a convenient Ansatz

$$\Psi(x,\nu,\lambda) = \Psi^N_{out}(x,\nu,\lambda) + A_{out}\, G^N_{out}(x, x_{\Gamma_H}, \lambda)$$

for calculation of the scattered wave in the outer domain of the Helmholtz resonator, in terms of the scattered wave and the Green function of the Neumann Laplacian in the outer domain, with a pole x_Γ at the point-wise opening

connecting the outer domain with the cavity. We suggest an explicit formula for the Kirchhoff coefficient A_{out}, and the corresponding formula for A_{int} in the cavity, based on construction of a solvable model for the Helmholtz resonator with narrow and short connecting channel.
PACS numbers:73.63.Hs,73.23.Ad,85.35.-p,85.35.Be

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Trapped modes in steady flow problems

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We consider the problem of the steady two-dimensional flow of a heavy, ideal fluid, over localized perturbations of a horizontal bottom. We assume irrotational motion and neglect the effects of surface tension. A linearized version of the problem is discussed in terms of a perturbed stream function. We investigate unique solvability for all values of the (unperturbed) stream velocity; in the particular case of a submerged hollow of rectangular shape, we provide sufficient conditions on the hollow's size for the existence of non trivial solutions of the homogenous problem (trapped modes).

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Analysis and Discretization of Semilinear Stochastic Wave Equations with Power Law Nonlinearity and Q-Regular Space-Time Noise

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The one-dimensional wave equations with certain power laws of quasi-nonlinearity perturbed by Q-regular space-time random noise are considered. This model describes the displacement of a noisy nonlinear string. We shall discuss existence and uniqueness of (strong) solutions using energy-type methods based on the construction of Lyapunov-functions. Appropriate truncations and finite-dimensional approximations are presented while using an approach exploiting the explicit knowledge on eigenfunctions of related second order differential operators. Moreover, the probabilities of large fluctuations are estimated and some nonstandard partial-implicit difference methods for their numerical integration are suggested in order to control its energy functional in a dynamically consistent fashion. Parts of this presentation are related to ideas of an ongoing joint work with Boris Belinskiy on the randomly perturbed, nonlinear beam model (4th order SPDE).

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About Some Aspects in Numerical Investigation of Nonlinear Schrödinger Equation

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The investigation of soliton supporting systems is of great importance both for the applications and for the fundamental understanding of the phenomena associated with propagation of solitons. Recently, elaborate models such as Coupled Nonlinear Schrödinger Equations (CNLSE) appeared in the literature. They involve more parameters and possess richer phenomenology but, as a rule, are not fully integrable and require numerical approaches. The non-fully-integrable models possess as a rule three conservation laws: for (wave) "mass", (wave) momentum, and energy and these have to be faithfully represented by the numerical scheme. In this instance, we follow generally [1] but focus on a new implementation of the conservative scheme that makes use of complex variables. This allows us to invert (albeit complex-valued) but five diagonal matrices while the real-valued scheme requires the inversion of nine-diagonal matrices [1]. This gives a significant advantage in the efficiency of the algorithm. To this end, we generalize the computer code developed earlier for real-valued algebraic systems in [2].
As a featuring example we consider 3D unsteady propagation with axial symmetry, which leads to (2+1)D CNLSE. We use the second Douglas scheme (called the stabilizing correction) which is based on splitting of the spatial operator and effective inversion of 1D operators. In order to guarantee the conservation law, the intensity at each step along the propagation direction is renormalized. To make the scheme fully implicit for the nonlinear case, we introduce also internal iteration.

The paper is partially supported by Bulgarian Ministry of Education and Science under Grant VY-MI-106/2005.

References:

[1] W.J.Sonnier, C.I.Christov, Strong coupling of Schrodinger equations: Conservative scheme approach, *Mathematics and Computers in Simulation*, **69**, 514-525 (2005).

[2] C.I.Christov, Gaussian elimination with pivoting for multi–diagonal systems, *Internal Report 4, University of Reading* (1994).

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Fractional stable distributions and their applications to solution of fractional differential equations

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The class of fractional stable distributions (FSD's) introduced recently by the author and his collaborators is discussed. Containing alpha-stable distributions as a subclass, the set of FSD's covers all fundamental solutions of two-term fractional (both in space and time) partial differential equations. The main properties of FSD's are described: characteristic functions, Laplace and Mellin transforms, convergent and asymptotic series representation, integral representation, relations to elementary and special functions, duality interrelations and the others. Formulating some of the properties in terms of random variables yields the algorithms of simulation of random variables with FSD by Monte Carlo method and estimation of their parameters are demonstrated and illustrated by numerical examples. On the base of the FSD's properties, the algorithm of simulation of interarrival times in the fractional Poisson process is derived and applied to modeling of compound Poisson and Kolmogorov-Feller processes of fractional orders. This simulation can be considered as a direct numerical method for solution of a fractional stochastic equation with the Riemann-Liouville fractional derivative with respect to time. The talk is ended by a short review of physical applications of FSD's .

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On the behaviour of the implied volatility under a long-memory stochastic volatility model

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Following the ideas presented in [CR] and in [CCR] we propose, by fractional integration, a stochastic long-memory volatility model. By means of Malliavin Calculus we study the behaviour of the corresponding implied volatility, both for short-dated and for long-dated options.
References:
[CR] F. Comte and E. Renault (1998): Long memory in continuous time stochastic volatility models Mathematical Finance, 8, 291-323.
[CCR] F. Comte, L. Coutin and E. Renault (2003): Affine fractional stochastic volatility models with application to option pricing Discussion paper n. 2003-10, University of Montreal.

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Inverse problems involving smart obstacles

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Direct and inverse acoustic scattering problems involving smart obstacles are proposed and some ideas to study them are suggested. A smart obstacle is an obstacle that when hit by an incoming acoustic wave reacts circulating on its boundary a pressure current, that is a quantity dimensionally given by pressure divided by time, in order to generate a scattered wave that pursues a preassigned goal. In our models the smart obstacle pursues one of the following goals: i) to be undetectable, ii) to appear with a shape and/or acoustic boundary impedance different from its actual ones, iii) to appear with a shape and/or acoustic boundary impedance and in a location in space different from its actual ones. That is, in the first case the smart obstacle tries to be furtive, in the second case it tries to be masked that is it tries to appear as another obstacle that we call the mask and finally in the third case it tries to appear as another obstacle in a location in space different from its actual one. We refer to this last apparent obstacle as the ghost. The direct scattering problem considered is the following; given the incoming acoustic field, the obstacle, its acoustic impedance and its goal formulate an adequate mathematical model for the problems previously considered and find the optimal strategy to pursue the assigned goal within the proposed model. The inverse scattering problem considered is the following: given the knowledge of several far fields generated by the smart obstacle when hit by known incident acoustic fields it reacts with the optimal strategy and the knowledge of the goal pursued by the obstacle find the obstacle (i.e. find the shape, acoustic impedance and spatial location of the obstacle). For simplicity in this paper we limit our attention to the case of the obstacle that tries to be masked when the incoming acoustic field is time harmonic. Moreover in the inverse problem we assume that the acoustic boundary impedance of the obstacle and of the mask are known. In this case the direct scattering problem is translated in a constrained optimization problem and its solution is characterized as the solution of a set of auxiliary equations, that is a boundary value problem for a system of two Helmholtz equations. The inverse scattering problem is translated in a two steps optimization procedure, that is an inverse problem for the system of two Helmholtz equations mentioned above. Finally in a test case the inverse problem is solved numerically starting from synthetic data. Material related to the problems described here is contained in the websites: http: //www.econ.univpm.it/recchioni/w6, http: //www.econ.univpm.it/recchioni/w13.

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