Special Session 20: Nonlinear Dispersive Waves

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The session will discuss the theory and applications of nonlinear dispersive equations. Applications in view include lasers, water-waves and plasma physics. Numerical simulations will also be featured.

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On The Local Well-Posedness for Some Systems of Coupled KdV Equations

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Using the theory developed by Kenig, Ponce, and Vega, we prove that the Hirota-Satsuma system is locally well-posed in Sobolev spaces $H^s(\mathbb{R}) \times H^s(\mathbb{R})$ for $3/4 < s \leq 1$. Using the results obtained by Christ, Colliander, and Tao, we show ill-posedness for the Hirota-Satsuma system in $H^s(\mathbb{R}) \times H^{s'}(\mathbb{R})$ for $-1 \leq s < -3/4$ and $s' \in \mathbb{R}$. We introduce some Bourgain-type spaces $X^a_{s,b}$ for $a \neq 0$, $s,b \in \mathbb{R}$, and we obtain local well-posedness for the Gear-Grimshaw system in $H^s(\mathbb{R}) \times H^s(\mathbb{R})$ for $s > -3/4$, by establishing new mixed-bilinear estimates involving the two Bourgain-type spaces $X^{-\alpha_-}_{s,b}$ and $X^{-\alpha_+}_{s,b}$ adapted to $\partial_t + \alpha_- \partial_x^3$ and $\partial_t + \alpha_+ \partial_x^3$ respectively, where $|\alpha_+| = |\alpha_-| \neq 0$.

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A unified theory for nonlinear steady travelling waves in constant, but arbitrary, depth

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A non-linear coupled-mode system of horizontal equations is derived with the aid of Luke's (JFM, 1967) variational principle, which models the evolution of nonlinear water waves in intermediate depth and over a general bathymetry. The vertical structure of the wave field is exactly represented by means of a local-mode series expansion of the wave potential. This series contains the usual propagating and evanescent modes, plus two additional modes, the free-surface mode and the sloping-bottom mode, enabling to consistently treat the non-vertical end-conditions at the free-surface and the bottom boundaries. The system fully accounts for the effects of non-linearity and dispersion. The main features of this approach are the following: (i) various standard models of water-wave propagation are recovered by appropriate simplifications of the coupled-mode system., and (ii) a small number of modes (up to 5 or 7) are enough for the precise numerical solution, provided that the two new modes (the free-surface and the sloping-bottom ones) are included in the local-mode series. In the present work, the consistent coupled-mode system is applied to numerical investigation of families of steady travelling wave solutions in constant depth, corresponding to a wide range of water depths, ranging from intermediate to shallow wave conditions.

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Recent Progress on BBM type Equations

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Some new results on BBM type equations and their systems analogs are stated. Potential applications are also disussed.

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Some hints on shallow water : sedimentation and avalanches

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Recent results around shallow water equations will be presented with special consideration on sedimentation and avalanches.

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Influence of Topography on Water Waves

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The motion of the free surface of a layer of perfect, incompressible and irrotationnal fluid under the only influence of gravity can be described using the water waves equations. Numerous models have tried to approximate the solutions of these equations (should they exist in some cases), mostly in the simple case of a flat bottom. Our motivation here is to consider the water waves problem in the so-called long waves regime for uneven bottoms, and for two different bottom topography scales. New asymptotic models are formally derived, and the obtained approximation is rigourously justified. To finish, we see how we can refind the KdV approximation from the previous models.

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Spatially Periodic Problems in Nonlinear Dispersive Theory

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A general class of nonlinear dispersive wave equations of the form

$$u_t - Lu_x + f(u)_x = 0, \quad x \in \mathbb{R}, t \geq 0$$

was derived to describe long-crested waves with small amplitude propagating in one direction. Here, f is a nonlinear function, typically, a polynomial and L is the dispersion operator defined through its Fourier symbol α, say. Thus, L and α are related by the relation

$$\widehat{Lv}(\xi,t) = \alpha(2\pi\xi)\widehat{v}(\xi,t)$$

where

$$\widehat{v}(\xi,t) = \int_{-\infty}^{\infty} v(x,t)e^{-2\pi i\xi x}\,dx$$

for all wavenumbers ξ. The symbol α is taken to be a real, even continuous function vanishing at the origin and becoming unbounded as $\xi \to \pm\infty$. Theory on existence of periodic traveling wave solutions, the analog of the classical cnoidal waves, and their stability is established. Progress is also made on the question of the large wavelength limit of these periodic wave trains.

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Models for crossing laser beams

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New intermediate models for crossing laser beams are introduced. We obtain some uniform error estimates with respect to the angles of incidence. We perform some numerical simulations in real physical situations.

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From wave equations system to Zakharov system: a limit process in laser plasma interactions

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In this talk, we investigate a new system of wave-type equations describing laser plasma interactions. We will first present some numerical computations. Then, we will explain how one can obtain the Zakharov system used in [1] by performing a limit process in the wave system.
Reference
[1] On a quasilinear Zakharov system describing laser-plasma interactions". Diff and Int. Eqs., vol 17, (2004), 297-330.

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Numerical Solution of Boussinesq Systems

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Recent progress on the numerical solution of Boussinesq systems of water wave theory will be reviewed. The results include error estimates for Galerkin-finite element methods for various Boussinesq systems in one and two space dimensions, and numerical computations on the generation, interactions and stability of solitary wave solutions of these systems.

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Asymptotic and numerical study of water waves

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We will comment on various asymptotical models for water-waves and describe an algorithm for the numerical resolution of free surface water waves.

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Global dispersive solutions for the Gross-Pitaevskii equation

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We study the Gross-Pitaevskii equation with non-zero constant boundary value at infinity in two dimensions. The perturbation from the constant background satisfies an equation similar to the nonlinear Schrödinger equation (NLS) but with quadratic terms and the linear part behaving similar to the wave equation near zero frequency. We prove unique existence of global solutions which approach to given small solutions of the linearized equation as time goes to infinity. This type of result is not yet available for the NLS with the same nonlinearity. The key idea is a certain transform of the unknown function, which effectively inserts derivatives in the quadratic terms, and the main task is a bilinear space-time non-stationary phase estimate for them.

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Global existence for damped nonlinear Schrödinger equations

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We study the Cauchy problem for the damped nonlinear Schrödinger equations. We prove that for any initial data in H^1, the solution exists globally in time, provided that the damping coefficient is sufficiently large.

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On semirelativistic Hartree equations

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We study semirelativistic evolution equations with nonlocal nonlinearity of Hartree type. Local and global well-posedness of the Cauchy problem is discussed in the usual Sobolev spaces. Moreover, we discribe how the solutions behave when mass tends to zero or infinity. It is shown that the solutions are well approximated by the corresponding solutions of the massless wave equations in the vanishing mass limit and by those of Schrödinger equations with heavy mass when mass tends to infinity.

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Weakly transverse Boussinesq systems

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We derive new systems of Boussinesq type in the scaling of KP for long weakly nonlinear gravity surface waves. Contrary to the KP equation, they do not suffer a (unphysical) zero mass constraint)and they hav e the correct convergence rate to the full Euler system.

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Hamiltonian long-wave expansions for water waves over a rough bottom

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This concerns the problem of nonlinear wave motion of the free surface of a body of fluid with a varying bottom. The object is to describe the character of wave propagation in a long-wave asymptotic regime for two and three dimensional flows. We consider bottom topography which is periodic in horizontal directions on a short length-scale, with the amplitude of variation being of the same order as the fluid depth. The bottom may in addition exhibit slow variations at the same length-scale as the order of the wavelength of the surface waves. For two dimensional flows, we also consider the case of a random rough bottom. We obtain effective Boussinesq equations and in the appropriate unidirectional limit, KdV or KP type equations.

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The Korteweg-de Vries equation in a quarter plane and a bounded domain

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The talk will focus on the initial- and boundary-value problems (IBVP) of the Korteweg-de Vries (KdV) equation posed in a quarter plane and on a bounded interval with nonhomogeneous boundary conditions. The problems arise naturally in certain circumstances when the KdV equation is used as a model for waves and a numerical scheme is needed. It will be shown that the IBVP is locally and globally well-posed in certain Banach spaces. Then, these well-posedness results will be applied to obtain the exact theory of convergence of the two-point boundary value problem to the quarter-plane boundary value problem, which provides a justification for the use of the two-point boundary value problem in numerical studies of the quarter plane problem.

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Time decay of solution for the KdV equation with multiplicative time-dependent noise

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We consider the time decay property of pathwise solution for the KdV equation on a circle with multiplicative time-dependent noise. We show that a multiplicative time-dependent noise gives rise to the time decay of solution. This phenomenon is often called the stabilization by noise.

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Korteweg-de Vries-Type Equations and their Properties Related to the Fractional Airy Transform

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One of the powerful methods of investigating Cauchy problems for nonlinear evolution equations consists in reducing them to integral equations. A key role in this reduction is played by the fundamental solution of the linear Cauchy problem. Fractional derivatives of the fundamental solution are often used for obtaining various functional estimates. It is shown that calculation of some of these fractional derivatives may lead to establishing new properties of solutions of both linear and nonlinear problems. Korteweg-de Vries-type equations are chosen as main examples. Derivation of the weak rotation approximation for the Ostrovsky equation is related to the issue in question.

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Moving poles of the the two soliton solution

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We study the time evolution of the poles in the complex plane of the two soliton solution of KdV. In particular, we are interested in the behavior of the poles during the interaction of the two solitons, and specifically whether or not poles associated with one soliton can switch to the other soliton. As a consequence of our analysis, we are able to define precisely the interaction time and characterize it in terms of the behavior of the poles. Moreover, by considering the two soliton solution u(t, z), for $z = x + ib, b \neq 0$ fixed, we provide new examples of finite time blowup of complex valued solutions of KdV.

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