Special Session 22: Large Time Behavior in Parabolic PDEs

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The large time behavior, in a broad sense, of solutions of various types of parabolic problems will be discussed. Global as well as blow-up solutions will be examined. To a large extent, the large time behavior is determined by special solutions such as traveling fronts or entire solutions. These will be studied in some of the lectures.

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Spreading speeds for KPP-type equations in general domains

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This talk is about nonlinear propagation phenomena in general unbounded domains of $\mathbb{R}^N$, for reaction-diffusion equations with Kolmogorov-Petrovsky-Piskunov (KPP) type nonlinearities. We are concerned with general domains and we give various definitions of the spreading speeds at large times for solutions with compactly supported initial data. The dependency of the spreading speeds on the geometry of the domain will be explained. Some a priori bounds can be obtained for large classes of domains. The case of exterior domains will also be discussed in detail. Lastly, some very thin domains for which the spreading speeds are infinite will be exhibited.

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Time dependent Ornstein-Uhlenbeck operators and invariant measures

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We consider the operators $K(t)$ defined by

$$K(t)\varphi(x) = \frac{1}{2}Tr(Q(t)D^2\varphi(x))$$

$$+\langle A(t)x + f(t), D\varphi(x)\rangle, \ \ t \in R, \ x \in R^N,$$

where the matrices $Q(t), A(t)$ and the vector $f(t)$ are given data, $Q(t)$ is symmetric and nonnegative. As in the case of constant in time data, the usual L^p spaces with respect to the Lebesgue measure are not the most appropriate setting for such operators. It is much better to work in weighted Lebesgue spaces, with time dependent Gaussian weight with respect to the space variables x. Under suitable assumptions, we define a family of Gaussian measures μ_t in R^N, such that

$$\int_{R^N} (P_{s,t}\varphi)(x)\mu_s(dx) = \int_{R^N} \varphi(x)\mu_t(dx), \ \ s < t$$

for a large class of functions φ. Here $P_{s,t}$ is the backward evolution operator associated to the operators $K(t)$, i.e. $u := P_{s,t}\varphi$ is the solution to $u_s + K(s)u = 0$, $s < t$; $u(t,\cdot) = \varphi$. Then we study the asymptotic behavior of $P_{s,t}$ through the associated evolution semigroup $\mathcal{P}_\tau$ defined by $\mathcal{P}_\tau u(t,x) = P_{t,t+\tau}u(t+\tau,\cdot)(x)$, using the important property that the measure $\mu(dt,dx) := dt \times \mu_t(dx)$ is invariant for such semigroup.

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Multiple blowup on different places at prescribed time

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We are concerned with a Cauchy problem for a semilinear heat equation $u_t = \Delta u + u^p$ with supercritical p in the sense of Sobolev embedding. The author obtained a weak solution blowing up at the origin at multiple blowup time $T_1, T_2, \cdots, T_k$ with $T_i \ll T_{i+1}$ for $1 \le i \le k-1$. In this talk, we consider the existence of a weak solution which blows up on different places at prescribed two time, that is, the first blowup occurs at the origin at $t = T_1$ and the second blowup does at some set D with $0 \notin D$ at $t = T_2$. In the previous result, the difference of blowup time was taken sufficiently large. The blowup time T_1 and T_2 are arbitrarily given in this talk.

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Entire solutions with two fronts of reaction-diffusion equations

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We consider a reaction-diffusion equation which has two stable constant equilibria $0,1$ and one unstable equilibrium $a \in (0,1)$. The typical example is an Allen-Cahn equation. Assume that $u = \phi(x+ct)$ $(c > 0)$, $\psi_1(x+c_1t)$ $(c_1 < 0)$ and $\psi_2(x+c_2t)$ $(c_2 > 0)$ are monotone increasing traveling front solutions connecting $u = 0$ to $u = 1$, $u = 0$ to $u = a$ and $u = a$ to $u = 1$ respectively. We call by an entire solution a classical solution which is defined for all $(x,t) \in \mathbf{R}^2$. In this talk, we present an entire solution which behaves for $t \approx -\infty$ as two fronts $\psi_1(x+c_1t)$ and $\psi_2(x+c_2t)$ on the left and right x-axes respectively while it converges to $\phi(x+ct)$ as $t \to \infty$. Moreover, if $c > -c_1$, we will show the existence of an entire solution which behaves as $\psi_1(-x+c_1t)$ in $x \in (-\infty,(c_1+c)t/2]$ and $\phi(x+ct)$ in $x \in [(c_1+c)t/2,\infty)$ for $t \approx -\infty$.

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On some free boundary problems with moving contact lines and prescribed contact angle

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We investigate certain operator-valued symbols that arise from elliptic or parabolic equations on wedge domains, and from free boundary problems with moving contact lines.

Moving contact lines occur in many situations. They occur in such processes as the coating of solid surfaces by a viscous fluid, spin coating of micro chips, the displacement of one fluid by another fluid along a solid boundary, the spreading of drops on solid surfaces, the motion of a fluid (or a fluid-liquid system) in a container, where the free surface is in contact with the wall of the container.

It is shown that the associated symbols lead to well-posed evolution problems. The tools involve recent results on maximal regularity for non-commuting operators.

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On the asymptotics of gradient blow-up

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We consider semilinear parabolic problems, of the form $u_t - \Delta u = g(u,\nabla u)$ with Dirichlet boundary conditions, which exhibit so-called gradient blow-up (or grow-up) phenomena. Namely, the solutions remain bounded in amplitude, but their gradient may become unbounded in finite (or infinite) time. We survey known and new results concerning the (time and space) asymptotics of such solutions.

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Non-parabolic asymptotic limits of solutions of the heat equation on $\mathbb{R}^N$

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Let $u(t,x)$ be a classical solution of the heat equation on $\mathbb{R}^N$, and set $v(t,x) = t^\mu u(t,xt^\beta)$, where $0 < \mu < N$ and $\beta > 0$. We consider the set $\omega^{\mu,\beta}$ of all $f \in C_0(\mathbb{R}^N)$ for which there exists a sequence $t_n \to \infty$ such that $v(t_n,x)$ converges in $C_0(\mathbb{R}^N)$ to f. We show there exists a solution of the heat equation such that $\omega^{\mu,\beta}$ is highly nontrivial for all rational μ and all $\beta \geq 1/2$. Since this includes values of $\beta > 1/2$, the asymptotic behavior of this solution is not completely described by the standard parabolic rescalings.

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Grow-up and convergence of solutions for a parabolic equation

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We study the behavior of solutions of the Cauchy problem for a parabolic equation with supercritical nonlinearity. It is shown that the grow-up to infinity and convergence to zero occurs depending on initial data. We determine the exact rate by comparison technique based on formal asymptotic analysis.

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