Special Session 23: New Developments in Nonlinear Partial Differential Equations and Control Theory

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This session will focus on qualitative behavior of solutions to nonlinear evolution equations that are motivated by strong applications in mechanics and physics. Particular examples include: nonlinear wave equations, nonlinear Shrodinger equations, nonlinear plate equations, Navier Stokes and Euler equations, Gunzburg Landau equations etc.

In addition to classical issues of wellposedness of nonlinear problems, of particular interest to the session are questions related to asymptotic (in time) behavior of solutions, stability/instability, formation and propagation of singularities, regularity of solutions.

Both controlled and uncontrolled , forward and inverse dynamics will be considered. In fact, new developments in control theory of nonlinear PDE's bring forward several open questions as well as techniques that could be used in order to forge a desirable (from the point of view of applications) outcome of the dynamics. These include problems such as stabilization, controllability, formation of attractors /inertial manifolds, optimization, identification and reconstruction of the data such as initial conditions or sources in the equations. However, these techniques depend heavily on a deep understanding of a free- uncontrolled dynamics. Thus, the aim of the session is to bring together both constituencies, so people working on different aspects of mathematical properties of solutions to nonlinear evolutionary dynamics could exchange expertize and benefit from it. Particular emphasis will be given to new techniques and methodologies developed for classes of problems under consideration.

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Controllability Properties of Nonlinear Rotation-Free Thermoelastic Systems

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In this talk, we provide results of local and global controllability (exact and global) for 2-D nonlinear thermoelastic systems, in the absence of rotational inertia. We will present results pertaining to the presence of both Lipschitz and non-Lipschitz nonlinearities. The plate component may be taken to satisfy either the clamped or higher order (and physically relevant) free boundary conditions. In the accompanying analysis, critical use is made of sharp observability estimates which obtain for the linearization of the thermoelastic plate. Moreover, key use is made of the analyticity of the underlying linear semigroup.

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Modelling the Flutter Instability Problem of Aeroelasticity

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The central problem of AeroElasticity is an endemic instability flutter that occurs at high enough airspeed in subsonic flight. Currently almost all the work is computational. In contrast we retain the full continuum models and formulate it as a boundary value problem for the Euler Full Potential Equation or the simpler quasilinear Transonic Small Disturbance Potential Equation. The flutter speed is a Hopf Bifurcation point for a non-linear convolution/evolution equation in a Hilbert Space, and is determined by the linearized equations. The latter formulates as Neumann type problem in Lp-spaces, $1 < p < 2$, and the solution involves the Possio Singular Integral Equation.

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Long-time behaviour of a coupled wave/plate PDE model

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This talk will deal with long-time behaviour of a system of coupled second-order evolution equations with nonlinear dampings. More precisely, the system comprises a semilinear wave equation on some smooth domain $\Omega \subset \mathbb{R}^3$, and Berger's plate model on a portion of the boundary of Ω, say Γ_0; the coupling is strong as it takes place (through velocities traces) on Γ_0. The talk will focus on the issues of (i) existence of a global attractor for the dynamical system defined by the overall PDE problem, and of (ii) dimension and smoothness of the attractor. Most results presented in the talk are jointly obtained with Igor Chueshov (Kharkov University, Ukraine) and Irena Lasiecka (University of Virginia).

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Weak and Regular Solutions for a Non-linear Shell Problem with Small Finite Deflections

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W.T. Koiter classifies shell problems according to the order of magnitude of the deflection. The Small Finite Deflections, characterized by small displacement gradients and by rotations whose squares do not exceed the middle surface strain in order of magnitude leads a geometrically nonlinear model discussed. In this prsentation we will consider a version of this model which is based on the intrinsic shell modeling techniques introduced by Michel Delfour and Jean-Paul Zolésio. Mathematically it takes the form of a set of coupled partial differential equations where the shell equation is a nonlinear, strongly coupled with hyperbolic equations. We shall prove the existence of weak solutions as well as regular solutions of this system.

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Solutions to the hyperbolic-parabolic system modeling fluid-structure interaction in blood flow

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We will focus on a free-boundary problem for a quasilinear system of PDEs modeling blood flow in compliant arteries. The system of PDEs is of hyperbolic-parabolic type. Hyperbolicity captures wave propagation in elastic arteries, and parabolicity describes the flow of a viscous incompressible fluid modeling blood flow. The system is obtained from the incompressible Navier-Stokes equations coupled with the equations for a linearly elastic Koiter shell by using homogenization theory. The resulting, effective model, captures the main features of blood flow in elastic arteries. More precisely, a comparison between the numerical simulations of the effective equations and the experimental results performed at the Texas Heart Institute in Houston showed excellent agreement. We will discuss the existence of a solution to the effective free-boundary problem and the difficult challenges and new insights that it brings towards the understanding of the fluid-structure interaction in blood flow.

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Hadamard Wellposedness of a Two-Dimensional Boussinesq Equation with Applications to Structural Acoustic Problems

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We consider a Boussinesq type equation defined on a smooth and bounded domain $\Omega \in R^2$. It is shown that the model admits "finite energy" solutions that are Hadamard well-posed. In particular, it is shown that nonlinear restorative forces acting upon the plate prevent a finite-time blow up weak solutions.

The Boussinesq equation is then considered in a larger context as a component of a structural acoustic model that describes oscillations of a pressure (acoustic wave equation) in an acoustic chamber. By applying suitable control mechanism on a flexible wall of the chamber, the goal of the work is to show that the pressure (noise in an acoustic environment) can be effectively controlled. The presentation highlights the effects of a particular restorative force acting on the plate and interacting with an acoustic medium that is responsible for both wellposedness and control of the overall nonlinear model.

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Modern Boundary Conditions

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Angelo Favini, Ciprian Gal, Gisele Ruiz Gldstein & Silvia Romanelli

Consider a parabolic (or hyperbolic) equation of the form

$$D^j u(t) = Au(t)$$

where $D = d/dt$, $j = 1$ or 2, A is an elliptic differential operator in spatial variables x of order $2n$, possibly nonlinear, acting in a domain R in Euclidean space. Recently much attention has been devoted to associated nonclassical boundary conditions of the form

$$D^j u + Bu = 0 \text{ on G, the boundary of R, j} = 1,2$$

$$Au + Cu = 0 on G,$$

or others; B and C are operators of order less than $2n$. These dynamic, Wentzell, and acoustic boundary conditions will be briefly surveyed, from the perspective of physical interpretation and wellposed problems. Examples discussed will include acoustic wave equations with both acoustic and nonlinear Wentzell boundary conditions.

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Linear and Nonlinear Kinetic Boundary Conditions for the Wave Equation

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Ciprian Gal and Jerome A. Goldstein

Of concern is the wave equation with kinetic boundary conditions. Kinetic boundary conditions arise from incorporating the effects of kinetic energy and potential energy on the boundary as well as inside the region; a derivation via classical methods of the calculus of variations and the physical interpretation of the kinetic boundary conditions will be given. All of the standard boundary conditions (Dirichlet, Neumann, and Robin) will be obtained as special cases of the kinetic boundary conditions. Connections between kinetic, acoustic and general Wentzell boundary conditions will be discussed (cf. [1]).

We shall also consider the wave equation with kinetic boundary conditions which incorporate the effect of friction. This will lead to an initial boundary value problem of the form

$$\rho(x) u_{tt} = \nabla \cdot (T(x) \nabla u) \text{ in } \Omega$$

$$\rho(x) u_{tt} + \frac{T^2(x)}{\rho(x)} \frac{\partial u}{\partial n} + \frac{T(x)}{\rho(x)} c(x) u \in -\beta(u_t) \text{ on } \partial\Omega$$

where $T(x)$ represents the tension in the region, $\rho(x)$ is the density of the material, $c(x)$ is a function which depends on the potential energy, and β is a maximal monotone graph which is nondecreasing. We shall show that the above problem is well-posed in a certain space. The results in this section are joint with Ciprian Gal and Jerome A. Goldstein [2].

REFERENCES:
1. G.R. Goldstein, *Derivation and physical interpretation of general boundary conditions*, Advances in Differential Equations, to appear.
2. C. Gal, G.R. Goldstein, and J.A. Goldstein, *Wave equations with nonlinear dynamic boundary conditions*, in preparation.

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On a proportional and derivative robust optimal feedback for linear quadratic control problems

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linear-quadratic control prolem the derivation of the optimal feedback is well known and goes back to the method of invariant embeding of R. Bellman. This embeding is done with respect to time and gives rise to a Riccati equation satisfied by the operator : state to adjoint state. It has been showed that this invariant embeding can be viewed as a continuous version of a Gauss *LU* block factorization of the optimality system (state - adjoint state). In this paper we want to investigate backwards what a *QR* factorization would mean in terms of feedback. It turns out that it gives an expression of the optimal feedback not only in terms of the present state but also of its derivative. So we assume that not only the present state is observed but also its derivative. In the case the observed state and its derivative are compatible through the state equation, the formula for the optimal control turns out to be the same as the classical one. If they are not compatible then an additional Riccati equation has to be solved and the optimal control is given as an affine function of the measured state and its derivative. The interpretation of the optimality of this control is that at each time the state equation will be satisfied only in the least square sense. This property implies some robustness.

The method is presented for the control of parabolic and hyperbolic infinite dimensionnal systems but could be applied in other situations. Some numerical experiments will be given.

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Semilinear hyperbolic equations with a localized dissipation in an exterior domain

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I will give a global existence result of solutions together with decay estimates of the energy for some semilinear hyperbolic equations with a localized dissipation in an exterior domain.

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Long time behavior of solutions to nonlinear strongly damped wave equations

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The problems of global non-existence of solutions to initial boundary value problems for semi-linear and quasi-linear strongly damped wave equations as well as the problem of existence of global attractor of the semigroups generated by initial boundary value problem for a class of systems of semi-linear strongly damped wave equations will be discussed.

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Global and almost global existence for nonlinear wave equations in an exterior domain

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In this talk I wish to present a result concerning a mixed problem for the nonlinear wave equation in an exterior domain. The main issue is to establish a uniform decay estimate for the solution, provided the obstacle satisfies a suitable geometric condition under which we have the local energy decay property. Then we are able to consider global behavior of the perturbed solution by the effect of the nonlinearity.

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Stability of a nonlinear intrinisic shell model

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We consider a nonlinear shell modelled with the intrinsic methodology introduced by Michel Delfour and Jean-Paul Zolésio. The nonlinearity arises from allowing small finite deflections of the shell and is related to the nonlinearity found in the full Von Kármán plate model. Hadamard well-posedness of this shell model has previously been demonstrated by John Cagnol *et al.*. In this work we consider stability of the model under an internal damping.

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Degenerate Ornstein-Uhlenbeck operators and invariant measures

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We consider the operator $\mathcal{L}$ defined by

$$\begin{aligned}\mathcal{L}u(x) &= \frac{1}{2}\sum_{i,j=1}^{d} q_{ij}D_{ij}u(x) + \sum_{i,j=1}^{d} b_{ij}x_jD_iu \\ &= \frac{1}{2}\mathrm{Tr}(QD^2u(x)) + \langle Bx, Du(x)\rangle,\end{aligned}$$

where B and Q are real $d\times d$-matrices, Q is symmetric and nonnegative. This operator is hypoelliptic provided that the symmetric matrices Q_t defined by

$$Q_t := \int_0^t e^{sB}Qe^{sB^*}ds, \qquad t>0, \tag{1}$$

have nonzero determinant for some (equivalently, for all) $t>0$. Since it has unbounded coefficients, the usual L^p spaces with respect to the Lebesgue measure are not the best functional settings for $\mathcal{L}$. It is more natural and fruitful to use an invariant measure, that is a weighted Lebesgue measure $\mu(dx) = \rho(x)dx$ such that

$$\int_{\mathbb{R}^d} \mathcal{L}u\,\mu(dx) = 0$$

for all $u\in C_0^\infty(\mathbb{R}^d)$. It is well known that an invariant measure exists iff all the eigenvalues of B have negative real part, it is unique (up to constants), and it is the Gaussian measure $\mu := \mathcal{N}_{0,Q_\infty}$ with mean 0 and covariance matrix Q_∞ defined by (2) for $t=+\infty$.

The realization L of $\mathcal{L}$ in $L^2(\mathbb{R}^d,\mu)$ is dissipative and it is the infinitesimal generator of the (strongly continuous, contraction) Ornstein-Uhlenbeck semigroup in $L^2(\mathbb{R}^d,\mu)$. If $\mathcal{L}$ is uniformly elliptic, that is if $\det Q \neq 0$, the characterization of the domain of L as the space $H^2(\mathbb{R}^d,\mu)$ is already well known. If $\mathcal{L}$ is degenerate elliptic, then $D(L)$ is not contained in $H^2(\mathbb{R}^d,\mu)$, but the elements of $D(L)$ enjoy different regularity and summability properties with respect to different variables, a typical behavior of hypoelliptic operators. In the first part of this talk I will describe some maximal regularity properties of the elements of $D(L)$.

The second part of the talk is devoted to perturbations of degenerate Ornstein-Uhlenbeck operators. This is a rather delicate subject, because hypoellipticity and the regularization properties may be destroyed by perturbations. In fact, I will consider only a particular class of operators, such as

$$\mathcal{K}u(x,y) = \frac{1}{2}\,\Delta_x u -$$

$$\langle My + D_yU(y) + x, D_xu\rangle + \langle x, D_yu\rangle,$$

in $\mathbb{R}^{2n} = \mathbb{R}^n_x \times \mathbb{R}^n_y$, where M is a symmetric positive definite matrix, and $U \in C^1(\mathbb{R}^n, \mathbb{R})$ is a possibly unbounded function (these operators are motivated by stochastic perturbations of motions). In this case the invariant measure is $\mu(dx, dy) = \rho(x, y)dx\,dy$, where

$$\rho(x,y) = e^{-(\langle My,y\rangle + |x|^2)}e^{-2U(y)}.$$

If $U \equiv 0$, $\mathcal{K}$ is just one of the operators $\mathcal{L}$ in (1). Under suitable assumptions on U we can show that realizations of $\mathcal{K}$ in $L^1(\mathbb{R}^{2n}, \mu)$ and in $L^2(\mathbb{R}^{2n}, \mu)$ are m-dissipative, so that they generate contraction semigroups. However, a nice description of the domains of such realizations as in the case $U \equiv 0$ is not available for the moment.

References:
G. Da Prato, A. Lunardi: *On a class of degenerate elliptic operators in L^1 spaces with respect to invariant measures*, preprint.
G. Da Prato, A. Lunardi: *Maximal dissipativity of class of elliptic degenerate operators in weighted L^2 spaces*, to appear in Discrete Contin. Dynam. Systems.
B. Farkas, A. Lunardi: *Maximal regularity for Kolmogorov operators in L^2 spaces with respect to invariant measures*, preprint.

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Global attractor for nonlinrer wave equations with some nonlinear dissipations in exterior domains

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We discuss the existence and absorbing properties of global attractor for the problem

$$u_{tt} - \Delta u + \rho(x, u_t) + \lambda u + g(x,u) = f(x) \text{ in } \Omega \times (0, \infty),$$

$$u(x,0) = u_0(x), u_t(x,0) = u_1(x) \text{ and } u|_{\partial\Omega} = 0,$$

where Ω is an exterior domain in R^N and $\lambda > 0$.

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Large-time behavior of solutions for the damped wave equation

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We consider the Cauchy problem for the semilinear damped wave equation

$$(D) \qquad \begin{cases} u_{tt} - \Delta u + u_t = f(u), & (t,x) \in \mathbf{R}_+ \times \mathbf{R}^N \\ (u, u_t)(0,x) = (u_0, u_1)(x), & x \in \mathbf{R}^N, \end{cases}$$

related to the semilinear heat equation

$$(H) \qquad \begin{cases} \phi_t - \Delta\phi = f(\phi), & (t,x) \in \mathbf{R}_+ \times \mathbf{R}^N \\ \phi(0,x) = \phi_0(x), & x \in \mathbf{R}^N, \end{cases}$$

where $f(u) = \pm|u|^{\rho-1}u, \pm|u|^{\rho}$ etc. with $\rho > 1$.

The damped wave equation has been recognized to approach to the corresponding heat equation as $t \to \infty$. First, we give the precise L^p-L^q estimate on the difference of solutions to the Cauchy problems for the linear damped wave equation and linear heat equation, by using the explicit formulas of solutions. These estimates suggest that the solution to (D) behave samely as that to (H) as $t \to \infty$. In fact, we will survey the followings in the talk.

(I) When the semilinear term is sourcing, that is, $f(u) = +|u|^{\rho-1}u, \pm|u|^{\rho}$ etc., the solution globally exists for small data if $\rho > \rho_c(N) = 1 + \frac{2}{N}$ (Fujita exponent), and blows up within a finite time for suitable data if $\rho \le \rho_c(N)$.

(II) When the semilinear term becomes absorbing, that is, $f(u) = -|u|^{\rho-1}u$, the solution globally exists for large data, and the asymptotic profile is the Gauss kernel $G(t,x)$ if $\rho > \rho_c(N)$. If $\rho < \rho_c(N)$, then the solution decays with same rate as the similarity solution $w_a(t,x) = t^{-\frac{1}{\rho-1}}g(\frac{|x|}{\sqrt{t}})$ to (H).

Especially, we show our recent results in case of (II) for $1 \le N \le 4$. In the proof, the weighted L^2 enrgy method, developed by Todorova and Yordanov, J. Differential Equations 174(2001), and the explicit formula of solutions play an important role.

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Control of Elastic Systems with Restricted Nonlinearities

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Previous articles by the present author have dealt with control in static equilibrium of linear elastic systems by means of a variety of control actuator systems, including distributed “rod type” actuators with a particular orientation distribution, volume actuators employing pneumatic or hydraulic actuation, etc. In all of these the standard equations of linear elasticity have served as the starting point for the development. Computations quickly reveal the limitations of this linear framework. On the other hand a very general nonlinear approach to elasticity is to complex to allow any meaningful control theory to develop. Here we seek a middle ground based on the eigenspaces of the tensor defining the elastic energy. Within these spaces selected nonlinearities can be superimposed on the linear structure, retaining such properties as translation invariance, invariance under infinitesimal rotation, etc. A number of examples will be presented.

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Degree Theory and Proper Fredholm Maps: Quasilinear Elliptic Systems

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We consider classes of quasilinear elliptic systems with nonlinear boundary conditions on bounded domains. We develop and apply a topological degree theory for proper Fredholm maps that applies to these systems, reminiscent of the Leray- Schauder degree. Applications are made directly to compressible and incompressible nonlinear elasticity and Navier-Stokes equations with global continuation and existence results. The theory applies particularly to mixed-order elliptic type.

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Large deviations for stochastic Navier-Stokes equations: A PDE approach.

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We will present recent results on Hamilton-Jacobi-Bellman equations associated with optimal control of stochastic Navier-Stokes equations. In particular we will discuss the method of half-relaxed limits for such equations and its applications to establish large deviation principle at single times for solutions of stochastic Navier-Stokes equations with small noise intensities.

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Stabilizing Steady State Solutions of the 3D Navier-Stokes Equations and Other Dissipative Models

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In this talk we show that by stabilizing a steady state solution to the Galerkin approximation of the Navier-Stokes equations, using certain linear feedback control, one is in fact stabilizing a nearby steady state solution to the fully three-dimensional Navier-Stokes equations. In other words, we overcome the spell over problem. Similar results hold also in the context of Nonlinear Galerkin method, which are based on the theory of Approximate Inertial Manifolds. It is worth mentioning that all our conditions are explicit and verified by the computed approximate Galerkin solution and that no a priori assumptions are made on the unknown exact solution of the Navier-Stokes equations. Our techniques are very general and can be easily applied to other dissipative systems.

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Weighted L^2-Estimates for Dissipative Wave Equations with Variable Coefficients

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We establish weighted L^2-estimates for the wave equation with a damping term with potential $a(x)$ in R^n. Fourier analysis remains a powerful tool when the potential $a = a(t)$ is a function of time and has been used by many authors. When $a = a(x)$ the Fourier technique becomes cumbersome. The alternative is a multiplier method. In general multiplier techniques yield weaker decay estimates than Fourier techniques whenever the latter can be applied. In what is presented we strengthen the multiplier method and get a sharp result. Applications include $a(x) \sim a_0|x|^{-\alpha}$ for large $|x|$, where $\alpha \in [0,1)$. We show that the energy $\|u_t\|^2_{L_2} + \|\nabla u\|^2_{L_2}$ decays like $t^{-(n-\alpha)/(2-\alpha)-1+\delta}$ for any $\delta > 0$. The approach can be adapted to more general hyperbolic equations with damping.

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L^p–L^q decay estimates for solutions of wave equations with time-dependent dissipation and applications

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Aim of the talk is to give an overview on recent results on asymptotic properties of solutions to the Cauchy problem for wave equations with time-dependent dissipation. Two different scenarios are sketched. On the one hand, in the case of noneffective dissipation solutions are related to corresponding solutions of the free wave equation modified by the energy decay rate. Contrary to that, for effective dissipation terms a relation to a corresponding parabolic problem occurs. In both cases, L^p–L^q decay estimates will presented together with a discussion of their sharpness. Furthermore, for effective dissipation applications to semilinear problems will be discussed.

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Pointwise Carleman estimates at the H^1-level with no lower order terms for non-conservative Schródinger equations on a Riemannian manifold: control theoretic implications

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We present pointwise Carleman estimates at the H^1-energy level without lower order terms for general Schrödinger equations containing H^1-terms and defined on an n-dimensional Riemannian manifold. As a consequence, we obtain: global uniqueness, observability and stabilization results. The setting includes non-conservative Schrödinger equations with variable coefficient (in space) principal part defined on a Euclidean domain. The paper generalizes prior work of Lasiecka-Triggiani-Zhang (2004) with constant coefficient principal part on a Euclidean setting.

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When does a Schrödinger heat equation permit positive solutions

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We introduce some new and near optimal classes of time dependent functions whose defining properties take into account of oscillations around singularities. We study regularity properties of solutions to the heat equation with coefficients in these classes which are much more singular than those allowed under the current theory. An application to the Navier-Stokes equations is given. In particular, we derive a new type of a priori estimate for solutions of Navier-Stokes equations. The novelty is that the gap between the a priori estimate and a sufficient condition for smoothness is logarithmic.

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Null and Approximate Controllability of Stochastic Semilinear Parabolic Equations

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In this talk, I shall present some null and approximate controllability results for backward and forward stochastic semilinear parabolic equations. Due to the lack of compactness and exact controllability, the usual fixed point and/or global inverse function techniques for the controllability of deterministic semilinear partial differential equations have to be delicately refined.

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