Special Session 25: Dynamical Approach to Pattern-formation Equations, and Related Topics

M. A. Efendiev, Technical University of Munich, Germany

Recently much attention has focused on dynamical aspects of pattern-formation equations arising in biology, chemistry, geology, hydrodynamics and e.t.c. The goal of this special session is present new results and ideas concerning to the qualitative and quantitative aspects of pattern-formation equations as well as related topics.We hope bring together both experts and young researchers (not necessarily ONLY experts on pattern-formation equations) from different fields and exchange ideas which can be useful in the study dynamical aspects of equations formulated below. The topics of session includes: evolution of patterns and their spatial and temporal complexity, variational and topological approach, large-time behaviour, higher-order models, travelling waves, bifurcation of patterns, open problems.

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Localised pattern of a compressed elastic strip: centre manifold and homoclinic solutions

Boris Buffoni
Ecole Polytechnique Federale - Lausanne, Switzerland
Boris.Buffoni@epfl.ch
S. Rey

The localised configurations of an elastic band under traction was studied by A. Mielke by reducing a nonlinear PDE to a "spatial" finite-dimensional dynamical system on a centre manifold (A. Mielke, Hamiltonian and Lagrangian flows on centre manifolds, Lecture Notes 1489, Springer, 1991). The material being homogeneous, these deformations are not due to localised imperfections. Another way of getting localised configurations is to minimise the energy functional (B. Buffoni and A. Mielke, Calc. Var. 13, 2001, pp 469-489).

In this talk, the centre-manifold approach is described and shown to work when the infinite elastic band is compressed along its unbounded direction. There is a continuous family of homogeneous configurations that is parameterised by the compression rate $\beta < 1$ ($\beta = 1$ when there is no compression). It is assumed that, for some critical value β_0, the compression force as a function of β has a strict local extremum and that the linearised equation around the corresponding homogeneous configuration is strongly elliptic. Under these conditions, there are nearby localised deformations that are asymptotically homogeneous (B. Buffoni and S. Rey, Localized thickening of a compressed elastic band, to appear in the Journal of Elasticity). The main difference with traction lies in the fact that the linearised problem is now strongly indefinite. For an introduction to localised pattern in long elastic structures, see the review paper by G. W. Hunt and Cie, Cellular Buckling in Long Structures, Nonlinear Dyn. 21, 2000, pp. 3-29.

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Dynamics of a nonautonomous coagulation system

Fernando P. Da costa
Universidade Aberta, Portugal
fcosta@univ-ab.pt
Rafael Sasportes

We consider a constant coefficient coagulation equation with Becker-Döring type interactions and power law input of monomers $J_1(t) = \alpha t^{\omega}$, with $\alpha > 0$ and $\omega > -\frac{1}{2}$, namely

$$\begin{cases} \dot{c}_1 &= J_1(t) - c_1^2 - c_1 \sum_{j=1}^{\infty} c_j \\ \dot{c}_j &= c_1 c_{j-1} - c_1 c_j, \qquad j \geq 2. \end{cases}$$

For this infinite dimensional system we prove solutions converge to similarity profiles as t and j converge to infinity in a similarity way, namely with either j/ς or $(j-\varsigma)/\sqrt{\varsigma}$ constants, where $\varsigma = \varsigma(t)$ is a function of t only. This work generalizes to the non-autonomous case a recent result of da Costa, van Roessel and Wattis [1] and provides a rigorous derivation, as well as a simplification, of formal results obtained by Wattis [2]. The main part of the approach is the analysis of a bidimensional nonautonomous system obtained through an appropriate change of variables; this is achieved by the use of differential inequalities and qualitative theory methods. The results about rate of convergence of solutions of the bidimensional system thus obtained are fed into an integral formula representation for the solutions of the infinite dimensional system, which is then estimated by an adaptation of methods used in [1].

References:

[1] F.P. da Costa, H. van Roessel, J.A.D. Wattis, Long-Time Behaviour and Self-Similarity in a Coagulation

Equation With Input of Monomers, Markov Process. Related Fields, **12** (to appear in 2006).
[2] J.A.D. Wattis, Similarity Solutions of a Becker-Döring System With Time-Dependent Monomer Input, J. Phys. A: Math. Gen., **37** (2004) 7823–7841.

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On a nonlocal viscose phase separation model

Mohammad hassan Farshbaf shaker
Weierstrass Institute for Applied Analysis and Stochastics, Germany
shaker@wias-berlin.de

We derive a nonlocal model for isothermal phase separation process. We use ideas of previous nonlocal models derived by Gajewski and Zacharias for image segmentation problems. Our model contains some additional viscose term. We present preliminary mathematical results concerning existence, uniqueness, boudedness and longtime behaviour.

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Generalized waves and their qualitative properties

Francois Hamel
Université Aix-Marseille III, France
francois.hamel@univ.u-3mrs.fr
H. Berestycki and H. Matano

In this talk, I will present various generalizations of the usual notions of waves, fronts and propagation speed in a very general setting. These new notions involve uniform limits, with respect to the geodesic distance, to a family of hypersurfaces which are parametrized by time. General intrinsic properties, some monotonicity properties and some uniqueness results for almost planar fronts have been obtained with H. Berestycki. In a second part (with H. Berestycki and H. Matano), we will see how to use these notions to describe the propagation of an almost-planar front around an obstacle for bistable reaction-diffusion equations.

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Waves in dendrites

Gabriel Lord
Heriot Watt University, Scotland
g.j.lord@hw.ac.uk
S. Coombes and Y. Timofeeva

Dendrites form the major components of neurons and many are equipped with excitable channels located in spines.

We develop a mathematical model of dendrites based on the analytically tractable Spike-Diffuse-Spike model of dendritic tissue. The active membrane dynamics of spines are taken as an integrate-and-fire process and the spines are connected to a passive dendritic cable at a discrete set of points. Computationally we obtain a quasi–analytic solution which can be used to study the neural response to complicated spatio-temporal patterns of synaptic input. This approach is extended to a branched dendritic tree.

We examine the robustness of the wave propagation to both space-time white and correlated noise that arise in the cable (eg electrical coupling) and spine-heads (eg stochastic gating) and show a filtering result consistent with experiments of Rose and Fortune.

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Dimension reduction methods and cell-cell communication

Johannes Mueller
Technical University of Munich, Germany
johannes.mueller@gsf.de
Christina Kuttler

The rizosphere denotes the ecosystem at the root of plants. The growth and health of the plant depends heavily on the community of microorganisms (fungi, bacteria etc.) at its root. These microorganisms communicate and interact with each other and with the plant. A quite complicated network evolves. This project aims at further inside in these interaction by means of mathematical modeling.

In a first step, we consider a specific and common mechanism of communication: quorum sensing. The nice thing about quorum sensing is the fact that a lot of mechanisms, especially the regulatory network at the molecular level, are quite well known. These informations lead to a model for the dynamics within the cells as well as for the interaction between cells.

This model consists of a linear parabolic partial differential equation with mixed boundary condition describing the diffusion of signalling substances in the medium. This PDE is coupled (via boundary conditions) with nonlinear ODE's describing the pathway within the cells. It has been possible for the stationary case to use the cell radii as small parameters, and to apply dimension reduction methods. This coupled nonlinear PDE-ODE system is reduced

to a finite set of algebraic equations. The resulting system can then be used to obtain informations of interest for applications.

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Discretizations of chemotaxis-growth system and dimension estimate of their attractors

Etsushi Nakaguchi
Osaka University, Japan
nakaguti@ist.osaka-u.ac.jp

We study semi-/full-discrete approximation of the chemotaxis-growth system due to M.Mimura and T.Tsujikawa [Physica A 230 (1996)]. We establish dimension estimate of attractors for the original and the approximate systems. Our results show that the dimensions of attractors for approximate systems as well as for the original system are estimated in the polynomial order of the chemotactic parameter. This is a joint work with Professor M.Efendiev and Professor W.L.Wendland.

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Degenerate Hopf instability in oscillatory reaction-diffusion equations

Toshiyuki Ogawa
Osaka University, Japan
ogawa@sigmath.es.osaka-u.ac.jp

We study the diffusive coupling of oscillatory media with global inhibition on a circle. It turns out that the degenerate critical point of 0,+1 and -1-modes is the "organizing center" of the behavior of the system. We study the normal form about this degenerate critical point by using a standard bifurcation theory. And this explain the various kinds of spatio-temporal oscillation observed in an electro chemical experiment on a ring electrode.

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Stability of Lipid Bilayers: A Continuum Cartoon

Mark A. Peletier
TU Eindhoven, Netherlands
m.a.peletier@tue.nl
Matthias Röger

Lipid bilayers demonstrate an intriguing combination of properties: at a microscopic scale they consist of individual molecules without covalent bonding, but at larger scales they show behaviour reminiscent of elastic solids. Energy-based models, going back to Helfrich, are often used to describe this macroscopic behaviour; in these models the bilayer is represented as a smooth surface, and the energy functional penalizes the curvature of this surface.

In this talk I describe our work to understand, mathematically, the connection between a microscopic, diffusion-based model of the lipids on one hand and the macroscopic Helfrich-type surface model on the other. This is too complex a task to do in full generality: instead we consider a simplified continuum lipid model that is derived from Density Functional Theory, in which volume exclusion effects are represented by a mean field. For this simplified model we can investigate the connection in detail, and rigorously, resulting in an expansion of the microscopic energy in terms of the layer thickness. The terms of this expansion represent macroscopic quantities, such as the preferred thickness, the penalty to breakage, and the bending stiffness. This gives an interesting insight into the relative importances of these quantities.

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Existence of attracting solutions in non-autonomous delay FDEs

Ana M. Sanz
Universidad de Valladolid , Spain
anasan@wmatem.eis.uva.es
Sylvia Novo and Rafael Obaya

We present some results on the existence of recurrent attracting solutions for non-autonomous recurrent delay differential systems. Our approach relies on the theory of non-autonomous dynamical systems (namely, skew-product semiflows), topological dynamics as well as comparison results for cooperative systems. Under some monotonicity and convexity/concavity conditions on the initial system, and the existence of two ordered minimal sets in the induced skew-product semiflow, we guarantee the existence of an attracting solution for the initial system. This abstract result finds applications in many biology, population dynamics or neural networks problems.

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Spikes in biological systems

Matthias Winter
Brunel University, England
matthias.winter@brunel.ac.uk

The existence and stability of multi-spike solutions for

various biological systems will be considered. Some rigorous results will be presented. These will be illustrated by numerical simulations.

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Describing a class of global attractors via symbol sequences

Matthias Wolfrum
Weierstrass Institute for Applied Analysis and Stochastics, Germany
wolfrum@wias-berlin.de
J. Härterich

We study a singularly perturbed scalar reaction-diffusion equation on a bounded interval with a spatially inhomogeneous bistable nonlinearity. For certain nonlinearities, which are piecewise constant in space, it is possible to characterize all stationary solutions by means of symbol sequences, indicating the behavior in each subinterval. Determining also Morse-indices and zero numbers of the equilibria in terms of the symbol sequences, we are able to give a criterion for heteroclinic connections and a description of the associated global attractor.

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Exponential Attractors for Lotka-Volterra competitive system with cross diffusion

Atsushi Yagi
Osaka University, Japan
yagi@ap.eng.osaka-u.ac.jp

This talk is concerned with the Lotka-Volterra competitive system including self and cross diffusions which was presented by Shigesada et al. in 1979. We report that, under some conditions on cross diffusions, one can construct a dynamical system from the system and the dynamical system possesses exponential attractors. In the proof of construction of exponential attractors, we shall utilize the compact smoothing property of semigroup which was introduced by Efendiev, Miranville and Zelik.

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