Special Session 26: Nonlinear Parabolic and Elliptic PDEs and Applications

Ratnasingham Shivaji, Mississippi State University, USA
Peter Takac, Universitat Rostock, Germany

Ground-state Positivity, Negativiy, and Compactness for a Schrödinger Operator in $\mathbb{R}^N$

Béné]dicte Alziary
Université de Toulouse 1, France
alziary@univ-tlse1.fr
Jacqueline Fleckinger and Peter Takáč

Positivity, negativity, and the asymptotic behavior at infinity of a weak solution u to the Schrödinger equation, $-\Delta u + q(x)u - \lambda u = f(x)$ in $L^2(\mathbb{R}^N)$ is investigated. Let φ denote the positive eigenfunction associated with the simple eiganvalue Λ of the Schrödinger operator $-\Delta + q(x)\bullet$ in $L^2(\mathbb{R}^N)$. Assume that the potential q is strictly positive and has a superquadratic growth as $|x| \to \infty$. Then it is shown that, for every spectral parameter λ, u/φ belongs to $L^\infty(\mathbb{R}^N)$ provided f/φ is also bounded. More, for $-\infty < \lambda < \Lambda$, the solution u is φ-positive (i.e $u \geq c\varphi$ with $c \equiv const > 0$) and the solution u is φ-negative (i.e $u \leq -c\varphi$ with $c \equiv const > 0$) for $\Lambda < \lambda < \Lambda + \delta$, where $\delta > 0$ is a number depending on f.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Indefinite superlinear boundary value problems

Thomas Bartsch
University of Giessen, Germany
Thomas.Bartsch@math.uni-giessen.de
N. Ackermann, P. Kaplicky and P. Quittner

We consider the dynamics of the semiflow associated to the parabolic problem $u_t - \Delta u = f(x,u)$ on a bounded domain with Dirichlet boundary conditions and superlinear, indefinite nonlinearity f. In addition to new existence results for equilibria we obtain some information on connecting orbits. The use of heat flow methods in order to find solutions of the associated elliptic problem is particularly suitable in the case of indefinite nonlinearities because the heat flow is order preserving unlike the negative gradient flow used in the classical variational approach. One consequence is that we can prove the existence of nodal equilibria.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Multiple positive solutions for a class of singular problems

Maya Chhetri
UNC Greensboro, USA
maya@uncg.edu
Stephen Robinson

We will discuss the existence of multiple positive solutions for a class of singular boundary value problem. We employ the method of sub and super solutions to establish the result.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Asymmetric eigenvalue problems for the p-laplacian with Neumann boundary conditions

Mabel Cuesta
Universite du Littoral ULCO, France
mcuesta@ulb.ac.be
M. Arias, J. Campos and J-P. Gossez

We investigate the Neumann problem:

$$-\Delta_p u = \lambda[m(x)(u^+)^{p-1} - n(x)(u^-)^{p-1}] \text{ in } \Omega,$$

$$\frac{\partial u}{\partial \nu} = 0 \text{ on } \partial\Omega. \tag{1}$$

The relevant functional $\int_\Omega |\nabla u|^p$ restricted to the manifold

$$\begin{aligned} M_{m,n} &:= \{u \in W^{1,p}(\Omega) : B_{m,n}(u) \\ &:= \int_\Omega [m(u^+)^p + n(u^-)^p] = 1\} \end{aligned} \tag{2}$$

presents a difficulty in connexion with the (PS) condition. It turns out that the (PS) condition remains satisfied at all levels when $\int_\Omega m \neq 0$ and $\int_\Omega n \neq 0$, but it is not satisfied anymore at level 0 when $\int_\Omega m = 0$ or $\int_\Omega n = 0$. In this latter case, which we will call the singular case, we do not know whether the (PS) condition still holds at all positive. However one can show that the Palais-Smale condition of Cerami holds at all positive levels. Another difficulty arises when dealing with problem (1), which is now connected with the geometry of the functional. It turns out that in the singular case, at least one of the two natural candidates for local minimum fails to belong to the manifold $M_{m,n}$. To bypass this difficulty we will consider a minimax procedure defined from a family of paths having free endpoints .

$\longrightarrow \infty \diamond \infty \longleftarrow$

An Improved Poincaré Inequality for the p-Laplacian

Pavel Drábek
University of West Bohemia, Czech Rep
pdrabek@kma.zcu.cz

An improved Poincaré inequality is investigated for the energy functional

$$\mathcal{E}_{\lambda_1}(u) = \frac{1}{p}\int_\Omega |\nabla u|^p \,\mathrm{d}x - \frac{\lambda_1}{p}\int_\Omega |u|^p \,\mathrm{d}x \qquad (1)$$

on $W_0^{1,p}(\Omega)$, $1 < p < \infty$, where Ω is a bounded domain in $\mathbf{R^N}$, $-\lambda_1 \in \mathbf{R}$ is the first eigenvalue of the Dirichlet p-Laplacian Δ_p on $W_0^{1,p}(\Omega)$, $\lambda_1 > 0$. Analysis is focused on the case $1 < p < 2$. The "qudratization" of (1) within an arbitrarily small cone around the axis spanned by φ_1, where φ_1 stands for the first eigenfunction of $-\Delta_p$ associated with λ_1, is the main tool in our approach. We point out the striking difference between the cases $1 < p < 2$ and $2 \le p < \infty$. This is the joint work with Peter Takáč from the University of Rostock.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Fundamental Negativity and Application to Some Parabolic Problem

Jacqueline Fleckinger
UNIV TOULOUSE 1, France
jfleck@univ-tlse1.fr

It was shown by Clément-Peletier that there exists $\delta(f) > 0$ such that if $\lambda_1 < a < \lambda_1 + \delta$, the equation $-\Delta u - au = f > 0$ defined on a smooth bounded domain with Dirichlet boundary conditions has a non positive solution (for $f \in L^p$); here λ_1 is the first eigenvalue of the Dirichlet Laplacian.

With Alziary and Takaç we have improved this result to an equation of Schrödinger type defined on the whole space $\mathbb{R}^N$ with a potential $0 < q(x) \to +\infty$ growing fast enough (superquadratic growth) .

We use this result to study the sign of the solution of the associated parabolic problem

$$\partial_t u + (-\Delta + q)u - au = f,\, in]0;\infty[\times \mathbb{R}^N;$$

$$u(0,x) = u^0(x),\, x \in \mathbb{R}^N.$$

This extends an earlier result obtained with J.I.Diaz concerning a bounded domain Ω.

$\longrightarrow \infty \diamond \infty \longleftarrow$

quasilinear elliptic equations with singular and critical nonlinearities

Jacques Giacomoni
Laboratoire MIP-Ceremath (Institut de Maths de Toulouse, France
jgiacomo@univ-tlse1.fr
Adimurthi

we investigate the following problem:

$$(P_\lambda)\begin{cases} -\Delta u = \lambda(f(x,u)) & \text{in } \Omega \\ u|_\partial\Omega = 0 \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^n$, $f(x,u)$ is singular at $u = 0$ and has with a critical growth at ∞. We study the question of existence and multiplicity of solutions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Bifurcations in elliptic quasilinear problems

Petr Girg
KMA-FAV, Zapadoceska univerzita v Plzni, Czech Rep
pgirg@kma.zcu.cz

This talk is concerned with general bifurcation results of Dancer's type (bifurcation of continua of positive and negative solutions) for quasilinear elliptic problems. Prototype of problems we study is the following one:

$$-\Delta_p u = \lambda |u|^{p-2}u + h(x,u(x);\lambda) \quad \text{in } \Omega;$$
$$u = 0 \quad \text{on } \partial\Omega,$$

Here, Ω denotes a bounded domain in $\mathbb{R}^N$ ($N \ge 1$), Δ_p stands for the Dirichlet p-Laplacian defined by $\Delta_p u := \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ for $1 < p < \infty$, λ ($\lambda \in \mathbb{R}$) serves as the bifurcation parameter, and $h\colon \Omega\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ is a Carathéodory function. We consider bifurcations from zero and from infinity. In the case of one dimensional problem, we generalize some of the results for bifurcations at higher eigenvalues. Namely, we will present some new results on the so called Fredholm alternative for the p-Laplacian at higher eigenvalues. Main results were obtained jointly with P. Takáč, P. Drábek and J. Benedikt and are scattered in several papers and manuscripts.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On the p-laplacian on $\mathbb{R}^N$

Jean-pierre Gossez
Universite Libre de Bruxelles, Belgium
gossez@ulb.ac.be

We will consider the p-laplacian on $\mathbb{R}^N$ and deal successively with its first eigenvalue, its spectrum, its Fučik spectrum and the antimaximum principle. Some well-known results relative to the case of the Dirichlet problem on a bounded domain extend but some do not. This is a report on several joint works with J. Fleckinger, F. de Thélin and L. Leadi.

⟶ ∞◇∞ ⟵

Ratios of eigenvalues of p-Laplacians and other consequences of some elementary inequalities

Evans Harrell
Georgia Institute of Technology, USA
harrell@math.gatech.edu
J. Fleckinger and F. de Thélin

Universal upper bounds are derived for the ratios $\frac{\lambda_2}{\lambda_1}$ of the first two eigenvalues of the p-Laplacian on bounded domains, with Dirichlet boundary conditions. These are consequences of some elementary inequalities, which have other consequences for estimates of eigenvalues and eigenfunctions, and for Hardy-type inequalities. Time perrmitting, some of those will be reviewed.

⟶ ∞◇∞ ⟵

Multiple positive solutions to nonlinear singular elliptic problems

Jesus Hernandez Alonso
Universidad Autonoma de Madrid, Spain
jesus.hernandez@uam.es

We provide an overview of some recent work concerning existence of multiple positive solutions to some semilinear singular elliptic problems. This includes not only radial solutions obtained by phase plane arguments but also results where either variational or asymptotic bifurcation arguments have been used. Sshaped curves and free boundary(i.e., with ”dead cores”) non-negative solutions are also considered.

⟶ ∞◇∞ ⟵

A Reaction-Diffusion System from Climate Modeling

Georg Hetzer
Auburn University, USA
hetzege@auburn.edu

Energy balance climate models describe the evolution of a long-term mean of temperature by employing the relevant balance equations for the heat fluxes involved. Typically, the horizontal heat flux is parameterized by a diffusion operator. The transition between ice-ages and interglacials strongly affects the earth's vegetation, which in turn leads to a change in the albedo. So-called daisy world models have been employed for investigating this bio-feedback. A simple model will be described which leads to a non-autonomous reaction-diffusion system. Some results concerning the asymptotic behavior will be presented.

⟶ ∞◇∞ ⟵

Turing Patterns on Spheres

Jon Jacobsen
Harvey Mudd College, USA
jacobsen@math.hmc.edu
Julijana Gjorgjieva

We consider Turing patterns for reaction-diffusion equations on the surface of growing spheres. In particular, we are interested in the effect of dynamic growth on the pattern formation. We consider isotropic growth of the sphere, with otherwise arbitrary temporal dynamics. In the case of exponentially growing domains we perform a linear stability analysis and compare the results with numerical simulations.

⟶ ∞◇∞ ⟵

Structure of the set of large radial solutions of polyharmonic equations with superlinear growth

Monica Lazzo
University of Bari, Italy
lazzo@dm.uniba.it
Paul Schmidt and Ildefonso Diaz

We study polyharmonic equations with superlinear, p-homogeneous nonlinearities and seek ”large” radial solutions on a ball, that is, radially symmetric classical solutions that are unbounded. Our main results concern the structure of the set of all large radial solutions, in dependence on the shape and growth of the nonlinearity, the order of the equation, and the space dimension. For example, in the fourth-order case with a positive, even nonlinearity, the set of all large radial solutions is homeomorphic to the unit circle if the equation is subcritical, and homeomorphic to a line if the equation is critical or supercritical.

⟶ ∞◇∞ ⟵

Generalizations of Logarithmic Sobolev inequalities

Jochen Merker
University of Rostock, Germany
jochen.merker@uni-rostock.de

We consider L^p-logarithmic Sobolev inequalities such as

$$\int \frac{|u|^p}{\|u\|_p^p} \log\left(\frac{|u|^p}{\|u\|_p^p}\right) dx \leq \frac{n}{p} \log\left(C_{n,p}^p \frac{\|\nabla u\|_p^p}{\|u\|_p^p}\right)$$

and their relation to hypercontractivity of nonlinear semigroups such as the semigroup generated by

$$u_t = \Delta_p u^{1/(p-1)} \quad .$$

In particular, we discuss generalizations involving Gagliardo-Nirenberg inequalities and fractional derivatives, and we study optimal constants. This is a joint work with Peter Takac, University of Rostock.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A dynamical systems framwork for symmetries in PDE's

Pablo Padilla
University of Mexico (UNAM), Mexico
pablo@mym.iimas.unam.mx

The problem of determining a priori the symmetries that solutions of a certain PDE will exhibit is of both theoretical and practical importance.

From the abstract point of view, it was addressed by S. Lie and later on by E. Cartan, who was interested in the precise mathematical formulation of the notion of covariance in general relativity.

It arises naturally in some geometric questions and A.D. Alexandrov introduced the method of moving planes in order to show that the only compact (closed and with no boundary) surface enclosing a prescribed volume with zero mean curvature had to be a standard sphere. From the practical point of view, knowing the symmetries of solutions allows us to simplify the problem and often to obtain explicit solutions.

In this talk we propose a dynamical system framework to obtain symmetries of solutions to certain PDE's with variational structure. I comparison with previous results and approaches is also given.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Entire solutions of singular elliptic inequalities on complete manifolds

Marco Rigoli
Universita degli studi di milano, Italy
rigoli@mat.unimi.it
P. Pucci

We present some qualitative properties for solutions of singular quasi elliptic differential inequalities on complete Riemannian manifolds such as the validity of the weak maximum principle at infinity and non existence results We also discuss a few direct geometrical applications.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Asymptotic behavior of large radial solutions of polyharmonic equations with superlinear growth

Paul Schmidt
Auburn University, USA
pgs@auburn.edu
Monica Lazzo and Ildefonso Diaz

We study the blow-up behavior of ”large” radial solutions of polyharmonic equations with superlinear, p-homogeneous nonlinearities, that is, radially symmetric classical solutions on a ball that are unbounded. We show that, independent of the shape and growth of the nonlinearity, the order of the equation, and the space dimension, there are exactly three different modes of blow-up, two of which involve either small-amplitude or large-amplitude oscillations.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Standing Pulse solution of a reaction-diffusion equation of logistic growth

Junping Shi
College of William and Mary, USA
shij@math.wm.edu
Chongchun Zeng

The dynamical behavior of a reaction-advection-diffusion equation can be partially described by its convergent direction, where a one-dimensional equation for the average profile of the filament is $u_t = Du_{xx} + xu_x + f(u)$. For the logistic growth function $f(u) = u(1-u)$, we prove the existence and uniqueness of a standing pulse solution on the real line when $D > 1$, and we also show the standing pulse is globally asymptotically stable. A ”traveling wave” with varied velocity is also discussed.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Multiple positive solutions for classes of elliptic systems with combined nonlinear effects

Ratnasingham Shivaji
Mississippi State University, USA
shivaji@ra.msstate.edu
Jaffar Ali and Mythily Ramaswamy

We study the existence of multiple positive solutions for a class of elliptic systems where the nonlinearities involved belong to a class of postive functions that have a combined sublinear effect at infinity.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A biharmonic problem on a domain with a reentrant corner

Guido Sweers
Delft University of Technology & Universität zu Köln, Germany
g.h.sweers@ewi.tudelft.nl
S.A. Nazarov

Fourth order hinged plate type problems are usually solved via a system of two second order equations. For smooth domains such an approach can be justified. However, when the domain has a reentrant corner the bilaplace problem with Navier boundary conditions may have two different types of solutions, namely u_1 with $u_1, \Delta u_1 \in H^1$ and $u_2 \in H^2 \cap H^1$. We will compare these two solutions. A striking difference is that in general only the first solution, obtained by decoupling into a system, preserves positivity, that is, a positive source implies that the solution is positive. The other solution, generically sign-changing for positive source terms when the reentrant is larger than $\frac{3}{2}\pi$, is more relevant in the context of the hinged plate.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Abstract concentration compactness and some applications

Ian Schindler
Université de Toulouse 1, France
ian.schindler@gmail.com
K. Tintarev

We present the concentration compactness principle formalized by K. Tintarev and the speaker and survey some applications.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A Variational Approach to the Fredholm Alternative for the p-Laplacian

Peter Takac
University of Rostock, Germany
peter.takac@uni-rostock.de

We are concerned with the existence of a weak solution $u \in W_0^{1,p}(\Omega)$ to the degenerate quasilinear Dirichlet boundary value problem

$$-\Delta_p u = \lambda |u|^{p-2} u + f(x) \ \text{ in } \Omega; \quad u = 0 \ \text{ on } \partial\Omega. \qquad \text{(P)}$$

It is assumed that $1 < p < \infty$, $p \neq 2$, Ω is a bounded domain in $\mathbb{R}^N$, and $f \in L^\infty(\Omega)$ is a given function. Eigenvalue λ_1 being simple, let φ_1 denote the eigenfunction associated with it. We investigate the validity of the Palais--Smale condition for the corresponding energy functional with $\lambda = \lambda_1$ (or λ near λ_1) using a quadratic approximation of this functional about φ_1. We establish several “positive” and “negative” results which provide new information about the geometry of this energy functional. We obtain at least *three* distinct solutions if either $p < 2$ and $\lambda_1 - \delta \le \lambda < \lambda_1$, or else $p > 2$ and $\lambda_1 < \lambda \le \lambda_1 + \delta$ (with $\delta > 0$ small enough). The proofs use a minimax principle. This is joint work with Pavel Drábek from the University of West Bohemia, Pilsen, Czech Republic.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On a climate model with a dynamic nonlinear diffusive boundary condition

Lourdes Tello
Universidad Politecnica de Madrid, Spain
l.tello@upm.es
J.I. Diaz

In this work we present some new results on the mathematical treatment of a two dimensional climate model (latitude – deep) which models the coupling of the mean surface temperature of the Earth with the ocean temperature. The model was proposed by Watts and Morantine in 1990 and contains several mathematically new aspects: for instance, the boundary condition at the top layer is given by a dynamic diffusive energy balance. This balance incorporates a second order diffusion operator, the derivative of the temperature with respect the time and a multivalued term (the coalbedo function). Some related models arise in very different contexts as the Cahn-Hilliard equation with dynamic boundary conditions or some adsorption-diffusion phenomena in Chemical Engineering, among others.

Besides recalling some results on the existence of a bounded weak solution and some positive and negative answers for the uniqueness of solution, we consider the associate stationary problem and prove some convergence results when times goes to infinity.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Concentration compactness in mountain pass problems and other applications

Kyril Tintarev
Uppsala University, Sweden
tintarev@math.uu.se

The concentration compactness principle, relative to a given group of unitary operators ("dislocations"), and formalized by I. Schindler and the speaker as a lemma of functional analysis, provides a decomposition of an arbitrary bounded sequence into a sum of asymptotically orthogonal dislocated weak limits plus a remainder that weakly vanishes under any sequence of dislocations. We survey the following recent applications to semilinear elliptic problems: 1.Unconstrained mountain pass problems with penalty at infinity 2.Reversal of the P.-L.Lion's condition of penalty at infinity 3.Existence of solutions to semilinear elliptic problems in half-space 4.Existence of solutions to problems with oscillatory critical nonlinearity 5.Existence of minimizers in the Hardy-Sobolev inequality

$\longrightarrow \infty \diamond \infty \longleftarrow$

Fučik spectrum for Schrödinger equations and applications

Zhitao Zhang
Academy of Mathematics & Systems Science, Peoples Rep of China
zzt@math.ac.cn
T. Bartsch and Zhiqiang Wang

We investigate Fučik spectrum for Schrödinger equations $-\Delta u + V(x)u = \alpha u^+ + \beta u^-,\ x \in \mathbb{R}^N$. We construct the first nontrivial curve in the spectrum by minimax methods, and show some properties of the curve, for example, we show that the eigenfunctions corresponding to eigenvalues (α, β) in the first Fucik curve are foliated Schwarz symmetric if $V(x) = V(|x|), \forall x \in \mathbb{R}^N$. Finally we establish some existence results of multiple solutions for jumping nonlinearity problems.

$\longrightarrow \infty \diamond \infty \longleftarrow$