Special Session 28: Delay Differential Equations

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The section presents lectures about the dynamics generated by delay differential equations. The emphasis is on nonlinear autonomous equations, on global problems, and on state-dependent delays. Applications are also considered.

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Center manifold for some partial functional differential equations

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In this work, we prove the existence of a center manifold for some partial functional differential equations, whose linear part is not necessarily densely defined but satisfies the Hille-Yosida condition. When the unstable space is reduced to zero, we also show the attractiveness of the center manifold. We prove that the flow on the center manifold is completely determined by an ordinary differential equation in a finite dimensional space. Finally, we prove that the stability of the equilibrium is completely determined by the stability of the flow on the center manifold in some critical cases.

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Controlling oscillations with time delays: From oscillators to networks

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The presence of time delays in the control action can have beneficial consequences for shaping the behavior of dynamical systems. The delay typically originates in the feedback function for a single system, or in the couplings of interconnected systems. This talk will present a systematic investigation of these delay effects in the context of oscillators near a supercritical Andronov-Hopf bifurcation. Particular attention will be devoted to the interplay between the delays and the network topology in determining the collective dynamics.

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A local Hopf Bifurcation Theorem for differential equations with state dependent delays

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We present a new version of a local Hopf Bifurcation Theorem which is applicable to delay differential equations with state dependent delays of the form

$$x'(t) = f\Big(\alpha, x\big(t - r(x_t)\big)\Big), \quad t \in \mathbb{R}, \quad \alpha \in J \subset \mathbb{R}.$$

Here $J \subset \mathbb{R}$ is an interval and

$$f : J \times \mathbb{R}^n \to \mathbb{R}^n,$$

is 2 times continuously differentiable. The segment x_t, $t \in \mathbb{R}$, is an element of the space $C^1([-h,0]|\mathbb{R}^n)$ of continuously differentiable real - valued functions $\phi : [-h,0] \to \mathbb{R}^n$, $h > 0$.
The smoothness - hypotheses which are needed for the proof of Hopf Bifurcation are different from the usual ones.

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How lasers generate new delay differential equations problems

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Semiconductor lasers used in most of our every-day applications are highly sensitive to optical feedback. Because of the fast time scale of the laser (1 picosecond), the effect of the feedback is considerably delayed (1 nanosecond). Specific laser systems are investigated in laboratories, are simulated numerically using delay differential equation models, and are leading to new mathematical questions. We illustrate this by two recently studied laser problems. The first one is a laser subject to polarization-rotated feedback (Gavrielides, et al, Proc SPIE 6115, to appear (2006)). The laser exhibits square-wave polarization intensities with rapidly decaying oscillations after each jump. These observations renew the interest on the stability of square-wave oscillations. The second problem concerns the synchronization of two coupled lasers

(W??nsche et al, Phys. Rev. Letters 94, 163901 (2005)). A delayed phase equation is derived showing a stair-like bifurcation diagram. For both problems, experiments, numerical simulations, and analytical results will be presented.

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Dynamics of a simple gene regulatory switch

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We discuss a dynamics of a simple regulatory switch, that arises in multiple organizms. Our model consists of a system of time-periodic, coupled delay- differential equations. The periodic forcing as additive and the system is monotone, which allows us to show that all bounde solutions converge to a unique periodic orbit with the same period as the forcing function.

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Quasilinear neutral differential difference equations

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We consider a class of quasilinear neutral differential difference equations which are equivalent to dynamical systems in a product space generated by an ordinary differential equation and a shift map. The latter representation of the initial value problem allows to study existence and uniqueness for rather general spaces of initial data and to identify those data which produce regular solutions of the neutral equation. The approach covers a class of hyperbolic partial diferential equations from population dynamics with strong non-linearities as well as non-autonomous equations.

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On the problem of linearization for functional differential equations with state-dependent delays

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In this talk we discuss sufficient, and for some cases, necessary and sufficient conditions to guarantee exponential stability of a constant or periodic steady-state solutions of several classes of functional differential equations with state-dependent delays. As an application of our results, we formulate a necessary and sufficient condition for the exponential stability of the trivial solution of a threshold-type delay system.

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Existence of traveling waves connecting equilibrium point and periodic solution for a class of time delayed and non-local reaction-diffusion equations

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Many physical and biological systems described by reaction-diffusion equations have oscillatory supporting media. Thus the corresponding reaction equations possess periodic solutions and the orbits connecting an equilibrium point and a periodic solution. We are then interested in whether there will exist traveling waves solutions joining an equilibrium point and a periodic solution with the addition of the diffusion to the system. In this talk we develop a singular perturbation technique to show the existence of this type of traveling wave solutions of large wave speed for reaction-diffusion equations with time delayed and non-local response. Unlike the classical singular perturbation method, our approach is based on a transformation of the differential equations to integral equations in a Banach space and the rigorous analysis of the property for a corresponding linear operator. Our approach eventually reduces the singular perturbation problem to a regular perturbation problem. Application of our result will also be presented.

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Center Manifold Theory for Functional Differential Equations of Mixed Type

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We study the behaviour of solutions to nonlinear autonomous functional differential equations of mixed type in the neighbourhood of an equilibrium. We show that all solutions that remain sufficiently close to an equilibrium can be captured on a finite dimensional invariant center manifold, that inherits the smoothness of the nonlinearity. In addition, we provide a Hopf bifurcation theorem for such equations. We illustrate the application range of our results by discussing an economic life-cycle model that gives rise to functional differential equations of mixed type.

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Smooth center manifolds for differential equations with state-dependent delay

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For a class of delay differential equations, representing equations with state-dependent delay, C^k-smooth local center manifolds are constructed at stationary points. The construction works for arbitrary integer $k \geq 1$.

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On the approximation of attractors for infinite delay differential equations. A logistic model

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The aim of this talk is to present a study of the relationship between attractors for infinite delay differential equations (IDDE) and related attractors for finite delay. Indeed, using truncation it is proved an upper semicontinuity result from the truncated attractors to the infinite one, focusing on a logistic model with infinite delay as example.

$$\frac{\mathrm{d}x}{\mathrm{d}t}(t) = rx(t)\Big(1 - K^{-1} \int_{-\infty}^{0} w(s)P(x(s+t))\mathrm{d}s\Big),\ x_0 = \psi$$

References:
[1] T. Caraballo, P. Marín-Rubio, and J. Valero, Autonomous and non-autonomous attractors for differential equations with delays, *J. Diff. Eqns.* 208 (2005), no. 1, 9-41.
[2] T. Caraballo, P. Marín-Rubio, J. Valero, Attractors for differential equations with unbounded delays. Submitted.
[3] G. Hines, Upper semicontinuity of the attractor with respect to parameter dependent delays. J. Differential Equations **123** (1995), 56–92.
[4] P. Kloeden, Upper semi continuity of attractors of retarded delay differential equations in the delay. ANZIAM J. To appear.

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Differntial delay equations with state dependent time lags

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John Mallet-Paret

We shall discuss results obtained in collaboration with John Mallet-Paret over the past few years concerning the equation $ax'(t) = f(x(t), x(t-r))$, where $a > 0$, f is a given function and $r = r(x(t))$. we are interested in the asymptotic behaviour of solutions as a approaches 0. we shall discuss the important example (**) $ax'(t) = -x(t) - kx(t-r)$, where $a > 0$, $k > 1$ and $r = 1 + cx(t)$ for some $c > 0$.

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Dynamics generated by delayed unimodal positive feedback

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The dynamics of scalar differential equations with delayed unimodal positive feedback may vary from the simplest (convergence of all solutions to an equilibrium) to the most complicated case (chaotic behaviour). This class involves some illustrious examples, like the Mackey-Glass and the Nicholson blowflies equation. Conditions are given to ensure that the dynamical system is eventually monotone. We present some results regarding global attractivity, existence of periodic solutions and heteroclinic orbits.

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Event collisions in systems with delayed switches

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If the equations of motion of a dynamical system contain a switch depending on the state from a fixed time ago the system tends to oscillate. For example, if an unstable linear system is controlled by a delayed two-state switch (relay) we can expect a periodic motion back and forth between both states of the switch. Conventional wisdom states that the dynamics near periodic motions of this type can be described by smooth finite-dimensional return maps even though the dimension of the phase space is infinite-dimensional and the dependence on initial conditions is, in general, not continuous. This is true under certain genericity conditions. However, violations of these conditions can occur robustly when varying a parameter of the system, for example, the delay of the switch. We investigate what happens near these violations, also called event collisions, in the simplest but most common cases. The return map near a "colliding" periodic orbit is continuous and piecewise smooth. We demonstrate that it is also possible that the dimension of the image of the return map increases and study which dynamical behavior can occur near collisions.

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Delay Equations with Rapidly Oscillating Stable Periodic Solutions

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Delay equations

$$\dot{x} = \mu(-x + f(x(t-1)))$$

are considered where f is piecewise constant and models either positive or negative feedback. It is proved analytically that there exist delay equations admitting rapidly oscillating stable periodic solutions. Previous results were obtained with the aid of computers, but only for particular piecewise constant feedback functions. The present proofs work for a whole class of piecewise constant feedback functions. In the case of negative feedback functions, given an odd integer n, we give sufficient conditions on the two parameters describing the shape of f and on the stiffnes parameter μ such that there exist stable periodic solutions with n zeros per unit time interval. Numerical experiments indicate that these conditions also might be necessary.

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Soft landing and state-dependent delay

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Automatic soft landing is modeled by a differential equation with state-dependent delay. It is shown that in the model soft landing occurs for an open set of initial data, which is determined by means of a smooth invariant manifold.

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