Special Session 2: Semigroups, Evolution Equations, and Boundary Conditions

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Of concern are well-posed problems, usually initial-boundary value problems. Recently much attention has focused on boundary conditions involving time derivatives or terms of the highest order in the PDE, such as general Wentzell boundary conditions, dynamic boundary conditions, acoustic boundary conditions, and so on. Recent and continuing advances in the theory, applications, and physical interpretations of these boundary conditions form an ongoing active research area. This briefly summarizes the ideas behind Special Session 02.

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A Degenerate Elliptic-parabolic Problem with Nonlinear Dynamical Boundary Conditions

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We are interested in the following degenerate elliptic-parabolic problem with nonlinear dynamical boundary conditions $P_{\gamma,\beta}(f,g,z_0,w_0)$

$$\begin{cases} z_t - \operatorname{div} a(x,Du) = f, \; z \in \gamma(u), \;\; \text{in } Q_T \\ w_t + a(x,Du)\cdot\eta = g, \; w \in \beta(u), \;\; \text{on } S_T \\ z(0) = z_0 \;\text{ in } \Omega, \; w(0) = w_0 \;\text{ in } \partial\Omega. \end{cases}$$

($Q_T :=]0,T[\times\Omega$, $S_T :=]0,T[\times\partial\Omega$). The nonlinear elliptic operator div $a(x,Du)$ is modeled on the p-Laplacian operator $\Delta_p(u) = \text{div }(|Du|^{p-2}Du)$, with $p > 1$, γ and β are maximal monotone graphs in $\mathbb{R}^2$ such that $0 \in \gamma(0)$ and $0 \in \beta(0)$. Particular instances of this problem appear in various phenomena with changes of phase like multiphase Stefan problem and in the weak formulation of the mathematical model of the so called Hele Shaw problem. Also, the problem with non-homogeneous Neumann boundary condition is included.

Under certain assumptions on γ, β and a, we prove existence and uniqueness of renormalized solutions of problem $P_{\gamma,\beta}(f,g,z_0,w_0)$ for data in L^1, and also that these renormalized solutions are weak solutions if the data are in $L^{p'}$.

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Long-period limit of nonlinear dispersiver waves: the BBM-equation

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The focus of the present study is the standard BBM-equation $\eta_t + \eta_x + \eta\eta_x - \eta_{xxt} = 0$ which models unidirectional propagation of small amplitude long waves in dispersive media. The equation is posed on the entire real line and the interest here is the relationship between two different types of solutions. The problem has been studied with initial data in various Sobolev spaces defined on $\mathbb{R}$ and for periodic initial data, say of period $2l$. The principal new result is an exact theory of convergence of the periodic solutions to the solutions in Sobolev spaces as $l \to \infty$.

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Global Weak Solutions to a Generalized Hyperelastic-Rod Wave Equation

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In this lecture we consider a Generalized Hyperelastic-Rod Wave Equation (or Generalized Camassa-Holm Equation). Physically, it models the motion of nonlinear dispersive waves in compressible hyperelastic rods and the wave motion in shallow water. Geometrically, it describes the exponential curves of the Riemannian structure induced by H^1 on the manifold of the diffeomorphisms on S^1. We prove the existence of a strongly continuous semigroup of global weak solutions by showing the existence, uniqueness, and stability of the vanishing viscosity limit. The results were obtained in collaboration with Professors Helge Holden and Kenneth H. Karlsen.

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On the Gibbs character of the Dirichlet-to-Neumann semigroup

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Let H be a Hilbert space and $\mathcal{T}(H)$ the ideal of the trace class operators on H. A C_0–semigroup $\{S(t)\}_{t\geq 0}$ on H is called *Gibbs semigroup*, if for $t>0$, $S(t)\in\mathcal{T}(H)$. In this note we define the Dirichlet-to-Neumann semigroup $S(t)$ as follow: Let Ω be a smooth bounded domain in $\mathbb{R}^n$ and γ be a C^∞ positive definite matrix-valued function on $\overline{\Omega}$. The solution of

$$(\mathbf{P}) \qquad \begin{cases} \operatorname{div}(\gamma\nabla v)=0 & \text{in } \Omega,\\ v=f & \text{on } \partial\Omega. \end{cases}$$

is called γ-harmonic lifting of $f\in L^2(\partial\Omega)$. The Dirichlet-to-Neumann operator is an unbounded operator Λ_γ on $L^2(\partial\Omega)$ defined as

$$\Lambda_\gamma f=-\partial v/\partial\nu_\gamma=-\nu\cdot\gamma\nabla v\mid_{\partial\Omega},$$

where ν is the outer normal vector at $\bar{x}\in\partial\Omega$ and v is the γ-harmonic lifting of f, with $D(\Lambda_\gamma):=\{f\in H^{1/2}(\partial\Omega)\mid \Lambda_\gamma f\in L^2(\partial\Omega)\}$.

It is well-known that Λ_γ is a self-adjoint and compact resolvent operator on $L^2(\partial\Omega)$, consequently, it generates an analytic and compact C_0–semigroup $S(t)$. We establish the conditions under which $S(t)$ is a Gibbs semigroup. Furthermore, we define an approximating family for this semigroup and we see how the Gibbs character of this semigroup implies the precise information on the convergence of the approximating family toward $S(t)$.

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Quadratic optimal control problems for degenerate differential systems

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We are concerned with the quadratic regulator problem described by

$$\frac{d}{dt}x=Ax+Bu,$$
$$y=Gu,\ x(0)=x_0$$

where the operator A generates a C_0-semigroup on the Hilbert space X, operators B and G are linear and continuous from U to X and from U to Y, U and Y being two Hilbert spaces. The cost functional to be minimized over $L^2(0,T;U)$ is

$$J(x_0;u)=\int_0^T F(x(t),y(t),u(t))dt+$$
$$\left\langle \begin{bmatrix} x(t)\\ y(t)\end{bmatrix}, M\begin{bmatrix} x(t)\\ y(t)\end{bmatrix}\right\rangle$$

where

$$F(x,y,u)=\left\langle \begin{bmatrix} x\\ y\end{bmatrix}, Q\begin{bmatrix} x\\ y\end{bmatrix}\right\rangle+|u|^2,$$

Q,M are symmetric nonnegative continuous linear operators, with $M=[M_{ij}]$, $M_{22}>cI>0$.

Although the optimal control does not exist, it admits a vatiational characterization by means of a convenient two-point system.

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The semigroup of film casting: Elliptic constraints and linear transport

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In this presentation we derive non-standard elliptic estimates for systems of linear momentum equations subject to mixed periodic-Dirichlet boundary conditions. These results allow to study the spectral determinacy and regularity of the linear semigroup associated to linearizations of Yeow's equations of film casting (Y.L. Yeow, JFM 66, 1974). These equations are formed by a mass transport equation coupled to a two-dimensional elliptic constraint. We will address the issues of eventual compactness and differentiability of the underlying semigroup and point out some physical consequences.

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Nonexistence for the Laplace Equation with a Dynamic Boundary Condition of Fractional Type

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We consider the Laplace equation in a half-space with a dynamic nonlinear boundary condition of order between 1 and 2. Namely, the boundary condition is a fractional differential inequality involving derivatives of non-integer order as well as a nonlinear source. Nonexistence results and necessary conditions for local and global existence are established. In particular, we show that the critical exponent depends only on the fractional derivative of lowest order.

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Evans Functions, Jost Functions, and Fredholm Determinants

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The principal results of this talk consist of an intrinsic definition of the Evans function in terms of newly introduced generalized matrix-valued Jost solutions for general first-order matrix-valued differential equations on the real line, and a proof of the fact that the Evans function, a finite-dimensional determinant by construction, coincides with a modified Fredholm determinant associated with a Birman–Schwinger-type integral operator up to a nonvanishing factor.

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Limited Flux Diffusion Equations

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To correct the infinite speed of propagation of the classical diffusion equation Ph. Rosenau proposed the tempered diffusion equation

$$u_t = \nu \operatorname{div} \left(\frac{uDu}{\sqrt{u^2 + \frac{\nu^2}{c^2}|Du|^2}} \right). \tag{1}$$

Equation (1) was derived by Y. Brenier by means of Monge-Kantorovich's mass transport theory and he named it as the *relativistic heat equation*.

We prove existence and uniqueness of entropy solutions for the Cauchy problem for the quasi-linear parabolic equation

$$\frac{\partial u}{\partial t} = \operatorname{div} \mathbf{a}(u, Du), \tag{2}$$

where $\mathbf{a}(z,\xi) = \nabla_\xi f(z,\xi)$ and f being a function with linear growth as $\|\xi\| \to \infty$, satisfying other additional assumptions. In particular, this class includes the relativistic heat equation (1) and the flux limited diffusion equation

$$u_t = \nu \operatorname{div} \left(\frac{uDu}{u + \frac{\nu}{c}|Du|} \right) \tag{3}$$

used in the theory of radiation hydrodynamics.

We study the evolution of the support of entropy solutions of relativistic heat equation. For that purpose, we give comparison principles between sub-solutions (or super-solutions) and entropy solutions of the Cauchy problem and then using suitable sub-solutions and super-solutions, we establish the following result.

"Let C be an open bounded set in $\mathbf{R}^N$. Let $u_0 \in (L^1(\mathbf{R}^N) \cap L^\infty(\mathbf{R}^N))^+$ with support equal to $\overline{C}$. Assume that given any closed set $F \subseteq C$, there is a constant $\alpha_F > 0$ such that $u_0 \geq \alpha_F$ in F. Then, if $u(t)$ is the entropy solution of the Cauchy problem for the equation (1) with u_0 as initial datum, we have that

$$\operatorname{supp}(u(t)) = \overline{C} \oplus \overline{B_{ct}(0)} \quad \text{for all } t \geq 0."$$

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$C^{(n)}$-almost periodic solutions of some evolution equations

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We prove the existence and uniqueness of the so-called $C^{(n)}$-almost periodic solutions to the ordinary differential equation $x'(t) = A(t)x(t) + f(t), \ t \in \mathbb{R}$, where the matrix $A(t) : \mathbb{R} \to \mathcal{M}_k(\mathbb{C})$ is τ-periodic and $f : \mathbb{R} \to \mathbb{C}^k$ is $C^{(n)}$-almost periodic. We also prove the existence and uniqueness of an ultra-weak $C^{(n)}$-almost periodic solution in the case when $A(t) = A$ is independent of t. Finally we prove also the existence and uniqueness of a mild $C^{(n)}$ almost periodic solution of the semilinear hyperbolic equation $x'(t) = Ax(t) + f(t,x)$ considered in a Banach space, assuming $f(t,x)$ is $C^{(n)}$-almost periodic in t for each $x \in X$, satisfies a global lipschitz condition and takes values in an extrapolation space $F_{A_{-1}}$ associated to A.

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Some classes of higher order differential operators on hilbert spaces

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We present some results concerning higher order operators with Wentzell-type boundary conditions obtained in the joint works [1], [2] with A. Favini, G. Ruiz Goldstein and J.A. Goldstein. In the framework of a systematic investigation of possibly degenerate elliptic operators with (dynamic) boundary conditions of Wentzell type (i.e. boundary conditions involving the operator itself), here we focus on selfadjoint properties of uniformly elliptic operators of the type $A = \Delta(a(x)\Delta)$, or $B = (\nabla(a(x)\nabla))^2$, on suitable L^2-spaces, and of degenerate elliptic operators of the type $C = (-1)^{n+1}x^n(1-x)^n D^{(2n)}$ on the Sobolev space $H_0^n(0,1)$, for any $n \in \mathbf{N}, n \geq 1$. Hence the analyticity on the right half plane of all the corresponding generated semigroups is achieved.

Reference

[1] A. Favini, G. Ruiz Goldstein, J.A. Goldstein and S. Romanelli, Fourth order operators with Wentzell boundary conditions, Rocky M. J. Math. (to appear)

[2] A. Favini, G. Ruiz Goldstein, J.A. Goldstein and S. Romanelli, Higher order degenerate operators on Sobolev spaces (preprint)

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Well-posedness and uniform decay rates at the L2-level for the Schrodinger equation with non-linear boundary dissipation

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Semigroup well posedness and sharp energy decay rates at the L2-level are shown for the Schrodinger equation with a physically attractive non-linera boundary dissipation.

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Wave equation with second.order non-standard dynamical boundary conditions.

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We study the well–posedness of the evolution problem consisting of the standard wave equation posed in a bounded regular domain of $I\!R^N$, supplied with a non–standard second order dynamical boundary condition. More precisely, we consider the initial–boundary value problem

$$\begin{cases} u_{tt} - \Delta u = 0 & \text{in } I\!R \times \Omega, \\ u_{tt} = k u_\nu & \text{on } I\!R \times \Gamma, \\ u(0,x) = u_0(x), \quad u_t(0,x) = v_0(x) & \text{in } \Omega, \end{cases}$$

where $u = u(t,x)$, $t \in I\!R$, $x \in \Omega$, Ω is a bounded regular (C^∞) open domain of $I\!R^N$ ($N \geq 1$), $\Gamma = \partial\Omega$, ν is the outward normal to Ω, k is a positive constant.

We prove that the problem is ill–posed when $N \geq 2$, while it is well posed in $H^2(-R,R) \times H^1(-R,R)$ in the one dimensional case, when $\Omega = (-R,R)$. The main ingredients in the proofs are the analysis of the related elliptic eigenvalue problem

$$\begin{cases} -\Delta u = \lambda u, & \text{in } \Omega, \\ \Delta u = k u_\nu & \text{on } \Gamma \end{cases}$$

already developed by the authors, basic semigroup theory, and a detailed study of the case $N = 1$.

In particular in the one dimensional case a delicate characterization of the domain of the square root of the Laplacian operator on a suitable subspace of $H^1(-R,R)$ (where it has no negative eigenvalues) is required.

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