Special Session 31: Convex/Nonconvex Dynamical Systems and Computational Mechanics with Applications in Physics and Engineering

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Nonconvex dynamics is a newly developed, multi-disciplinary research area, which involves a powerful combination of theoretical analysis in mathematical modelling of natural systems, finite deformation theory, fluid mechanics, material sciences, nonlinear partial differential equations, variational methods, dynamical systems, global optimization, mathematical biology, logistic networks, numerical algorithms and scientific computation etc. Nonconvex phenomena appear naturally in many applications, such as chaotic dynamics, soliton, post-buckling of thin-walled structures, phase transitions of modern materials, and certain biological processes like DNA dynamics, etc. Due to the nonconvexity of the total potential energy of the system concerned, traditional analysis and related numerical methods for solving these problems have proven to be very difficult, or even impossible. In scientific computation, many nonconvex minimization problems are NP-hard. Local analytic methods and the application of the standard optimization procedures for these kinds of problems can not guarantee the identification of the global minima. In nonconvex dynamical systems, numerical methods may produce the so-called chaotic solutions.

This Symposium will emphasize on some recent progresses in the study of nonconvex dissipative dynamical systems and related nonlinear differential equations, and their applications to mechanics, biological and physical problems et al. The symposium will bring together mathematicians, engineers, computational scientists...

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Theoretical optimization of finite difference schemes

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We presently propose general optimization methods for finite difference schemes used to approximate linear differential equations. The specific case of the transport equation

$$\frac{\partial u}{\partial t} + c\,\frac{\partial u}{\partial x} = 0 \qquad (1)$$

with the initial condition $u(x,0) = u_0(x)$ is exposed.
First, finite differences schemes are grouped into classes corresponding to specific values of a given set of parameters, which can be handled through symbolic calculus. Second, we show that finite difference problems can be solved matricially, without loops. This new resolution method enables to minimize the symbolic expression of the error as a function of the scheme parameters.

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asymptotic behavior of the Burgers-Korteweg-de Vries equation

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The Burgers-Korteweg-de Vries equation has wide applications in physics, engi- neering and fluid mechanics. The phase plane analysis reveals that the Burgers-Korteweg-de Vries equation has neither nontrivial bell-profile traveling solitary waves, nor periodic waves. In the present work, we apply the Lie group analysis to analyze the Burgers-Korteweg-de Vries equation and an asymptotic behavior in Banach space will be illustrated.

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New Way to Understand Chaos: Canonical Duality Approach

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Chaotic phenomena appear naturally in many complex systems. In Newtonian mechanics, the chaotic behavior is mainly due to nonconvexity of the total potential energy concerned in the system, and this nonconvexity leads to multi-solutions of the governing equations at each time-space coordinate. Mathematically speaking, each of these solutions represents a critical point of the total action. How to identify extremality conditions of these critical points is a challenge task in nonconvex analysis. Traditional analytic methods and direct approaches can not be used to clarify global minimizers, and this is the main reason that in global optimization and computational science, many nonconvex problems are NP-hard.

In this talk, the speaker will present a potentially powerful canonical dual transformation method and the associated triality theory for solving a large class of nonconvex/nonsmooth variational problems. He will show that by using this method, many nonlinear differential equations can be transformed into so-called DAEs (differential-algebraic equations). The chaotic trajectories in phase space form an invariant set in dual phase space. The triality theory discovered recently can be used to identify both local and global minimizers. Therefore, a chaotic criterion is presented to control chaotic behavior of the nonconvex systems. Based on this triality theory, a powerful primal-dual algorithm is suggested. Applications will be illustrated by nonconvex problems in damped Duffing systems and phase transitions governed by Landau-Ginzburg equation. Complete solutions to certain very difficult nonconvex problems in finite dimensional systems will be announced.

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Pseudo-potential and Some Exact Solutions of Two-dimensional Flow

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Based on the conception "pseudo-potential" of incompressible fluid, we present a technique for finding some exact particular solutions of two dimensional flow of incompressible viscous and inviscid fluid. One of them describes infinitely many stationary vortexes doubly periodically distributed in the plane. Based on this solution, the chaotic motion of the flow may be discussed with the method of Melnikov function.

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Lie group stability study of finite difference schemes

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Differential equations arising in fluid mechanics are well known to be invariant with respect to a group of point transformations in the space of dependent and independent variables. The variety of finite difference schemes used to approximate them should thus be classified on a special grid according whether they respect or not the invariance properties of the original equation.

Denoting respectively x, t the space and time independent variables, u the dependent variable, h, k the space and time steps, let us consider a differential equation of the form:

$$F\left(x,t,u,\frac{\partial^a u}{\partial x^a},\frac{\partial^b u}{\partial t^b}\right)$$

and its finite difference approximation:

$$F\left(x,t,u,\frac{\partial^a u}{\partial x^a},\frac{\partial^b u}{\partial t^b}\right) + R\left(x,t,u,h,k,\frac{\partial^\alpha u}{\partial x^\alpha},\frac{\partial^\beta u}{\partial t^\beta}\right) = 0$$

R being the first error term.

We presently propose to determine the groups of transformations of (E_j) $j=1,2$, by means of the infinitesimal operators:

$$X_1 = \xi(x,t,u)\frac{\partial}{\partial x} + \zeta(x,t,u)\frac{\partial}{\partial t} + \eta(x,t,u)\frac{\partial}{\partial u}$$

$$X_2 = X_1 + \theta(x,t,u,h,k)\frac{\partial}{\partial h} + \phi(x,t,u,h,k)\frac{\partial}{\partial k}$$

ξ, ζ, η, θ being the infinitesimals of the transformation groups which is admitted by (E_j) $j=1,2$.

The necessary and sufficient condition for the invariance of (E_j) $j=1,2$ is given by:

$$\tilde{X_j}^{(l)}\, E_{j|E_j=0} = 0$$

$\tilde{X_j}^{(l)}$ being the l^{th} prolongation of X_j $(j=1,2)$, and l the highest derivative order.

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Rapid Fluctuation of Chaotic maps on fractal sets

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For a Lipschitz mapping f on a bounded subset X of $\mathbb{R}^d$ or a general complete metric space and a $s-$set X_0 in X, we characterize the complexity behavior of f acting on the fractal set X_0 by rapid fluctuation. We call that f has rapid fluctuation of dimension s if there exists a $s-$set X_0 in X such that the total variation of f^n on X_0 grows exponentially as n tends to infinity. Here the total variation of f on a fractal set is defined in term of Hausdorff measure. In this paper, for some kinds of interval maps with positive entropy and symbolic dynamical system, we prove that for any $s \in (0,1]$, they have rapid fluctuation of dimension s. For plane systems with horseshoe we prove that for any $s \in (0, \frac{\ln 4}{\ln 3})$ it has rapid fluctuation of dimension s, this shows that the horseshoe map has high dimensional chaos.

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Dynamic complex logistics information networks

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Abstract: Recently, as information technology progresses at a rapid rate and Internet infrastructure improves observably, the information remands of logistics industry have proliferated continuously. On the other hand, all logistics networks have gradually developed, but now we cannot be optimistic in spite of our semicentennial efforts, because the efficient logistics information networks are out of our sights. Taking both into account, we propose a theoretic model and dynamic analysis for the dynamic and complex logistics information networks. Especially, we get so many interesting results about the description and simulation of real logistics information networks and their behaviors and mechanisms. Keywords: Logistics, Logistics information networks, Complex networks, Node transition

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Special Bäcklund transformations and nonlinear superpositions for nonintegrable ϕ^4 field model

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Some special Bäcklund transformation (BT) theorems and a particular nonlinear superposition theorem are established to find exact solutions for a *nonintegrable* model, the (N+1)-dimensional ϕ^4 scalar field. Some new types of exact solutions such as the conoid periodic-periodic interaction waves and the periodic-solitary wave interaction solutions are explicitly given. The interaction solutions possess abundant structures thanks to the intrusion of some arbitrary functions in the expressions of the special solutions.

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Bifurcation and Synchronization of A Synaptically Coupled FHN Neurons Model with Time Delay

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This paper is focused on the dynamics of a coupled nonidentical FHN neurons model with synaptic connection both analytically and numerically. It is shown that rich bifurcation behavior is exhibited with the variation of the coupling strength. However, a transition from the original chaotic motions to periodic ones accompanied with complex bifurcation scenario can be observed. At the same time, the frequency synchronization of the coupled neurons is studied in terms of their mean frequencies. It is also shown that a small time delay can induce new period windows with the coupling strength increasing. Moreover, it is found that the synchronization of the coupled neurons can be achieved in some parameter ranges related to bifurcation transitions. The parameter regions of different dynamic behavior are clarified.

Keywords: FHN neuron, time delay coupling, bifurcation, synchronization

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Modelling dynamics of nonlinear thermomechanical phase transformations in multidimensional shape memory alloy samples

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In this contribution, we construct a new mathematical model to simulate martensitic phase combinations in shape memory alloy samples. Governing equations are obtained on the basis of the momentum balance law and a modified Landau-Ginzburg free energy functional, a generalization of the 1D Falk non-convex free-energy function. These equations are then supplemented with a kinetics model allowing us to describe the dynamics of multivariant phase transformations in three-dimensional samples. Finally, by using the Ginzburg-Landau modeling framework, we provide a methodology for linking the developed microscopic model with the atomistic reodering process that gives rise to self-accommodating microstructures. Results of numerical simulations, based on the finite-element implementation of the developed methodology, are presented.

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A Minimax Result with Applications to Adhesive Contact Problems

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We show that the description in terms of unilateral Mechanics of a general model with adhesive contact naturally leads to an associated variational–hemivariational inequality. In turn, this formulation can be expressed as a critical point problem for the general class of nonsmooth functionals $I : X \to R \cup \{+\infty\}$ on a real Banach space X satisfying the structure hypothesis $I = \Phi + \Psi$, where $\Phi : X \to R$ is locally Lipschitz and $\Psi : X \to R \cup \{+\infty\}$ is convex, proper, lower semicontinuous. Here an element $u \in X$ is said to be a critical point if $\Phi^0(u; x-u) + \Psi(x) - \Psi(u) \geq 0$ for all $x \in X$. The notation $\Phi^0(u; v)$ designates the generalized directional derivative in the sense of Clarke. We focus on a recent result representing the nonsmooth version of the minimax theorem due to Brezis and Nirenberg with local linking and under presence of the splitting for the space X. This nonsmooth version was recently obtained in the paper:

R. Livrea, S. Marano and D. Motreanu, Critical points for nondifferentiable functions in presence of splitting, *J. Differential Equations*, to appear.

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DEA Analysis of Reverse Logistics of Supply Chain Integration Project Choice

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Abstract: logistics has a good relationship with the economic utility of supply chain and the sustainable development of environment. It has been the considerable link of supply chain research. This paper analyzes and appraises the e??ectivity of di??erent supply chain integration projects and whether considers reverse logistics or not, by establishing DEA model of supply chain integration project that is on the basis of order and synthetically considered forward and reverse logistics, .provides the growth e??ciency model of input and output with preference supply chain integration project for the decision-maker. Key words: Reverse Logistics, Supply Chain, DEA

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On the accurate modeling of film flows down inclined planes

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Besides their importance for engineering applications (*e.g.* evaporators or chemical eactors), the interest of thin films down inclines mainly stems from the fact that their evolution is amenable to thorough theoretical analysis. The systems behaves first like a selective noise amplifier, then evolves after inception into a weakly chaotic dynamics of interacting solitary pulses whose amplitude and speed statistics increase continuously but slowly with the location on the plane. Two-dimensional film flows are modelled using weighted-residual approach to eliminate the cross-stream variable. A refined two-equation model is next obtained by applying an algebraic Padé-like approximant technique. This model accounts for inertia effects due to the deviations of the velocity profile from the parabolic shape, and closely sticks to the asymptotic long-wave expansion in the appropriate limit. Comparisons of two-dimensional wave properties with experiments and direct numerical simulations show good agreement. ange of parameters where a two-dimensional wavy motion is reported in experiments.

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Axisymmetric Ivantsov type traveling waves in generalized 3-D Mullins-Sekerka equation

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In this talk, the existence problem of axisymmetric Ivantsov type solutions for generalized 3-dimensional Mullins-Sekerka equation is discussed. The generalized two-phase Mullins-Sekerka equation models a general class of phase transitions with anisotropic surface energy. The Ivantsov type solutions are traveling waves whose needle-shaped interfaces are moving along the axis at a constant velocity within a cylindrical domain. The existence of at least one axisymmetric needle solution is obtained through a Leray-Schauder argument of a singular integral transformation.

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Second order optimality conditions in optimal control

with applications to spaceflight mechanics

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The aim of this talk is to present algorithms to compute conjugate times in smooth optimal control, based on recent theoretical geometric developments. We describe computable second order sufficient optimality conditions, that are implemented into a software called COTCOT (Conditions of Order Two and COnjugate times), available on the web. We present some applications and numerical simulations in spaceflight mechanics.

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Front propagation into unstable states: a general perspective

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The problem of front propagation in the Fisher/KPP equation, or generalizations thereof, is well known in the mathematics community. What is less well known is that the problem of front propagation can be posed more generally for large classes of dynamical equations that admit fronts propagating into a linearly unstable state. Examples occur in higher order partial differential equations like the Swift-Hohenberg equation or Kuramoto-Sivashinsky equation, and in systems of partial differential equations, difference equations etcetera. Fronts in such cases can be uniformly translating (like in the Fisher-KPP equation), coherent pattern generating fronts, or lead to incoherent dynamics. In this talk I will give an overview of a recently developed general framework to address this type of front propagation into unstable states. We show that such fronts generally come in two classes, pulled fronts which propagate with the so-called linear spreading velocity, and pushed fronts whose speed is larger than the linear spreading speed. We present an important exact result for the asymptotic rate of convergence of the speed of a pulled front to its asymptotic value.

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Numerical Methods for Computing Nonlinear Eigenpairs

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This project is to develop numerical methods for solving nonlinear variational eigenpair problems for multiple solutions following an (instability) order. The problems are classified into three categories: iso-homogeneouse, homogeneous and non homogeneous. For iso-homogeneous eigenpair problems, a Rayleigh-local minimax method is devised; for the other two types, an active Lagrange-local minimax method is proposed in order to form a local mountain pass structure for the variational functional and to apply the local minimax method developed by this research group. The methods are numerically implemented to solve several nonlinear eigenpair problems for multiple solutions, such as the p-Laplacian in non Newtonian flows/materials and the Gross-Pitaevskii problem in the Bose-Einstein condensates. Numerical results will be presented. Some theoretical results related to the convergence of the algorithms and an instability order of eigenpairs computed by the algorithms will be discussed.
This research is supported in part by NSF Grant DMS-0311905.

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