Special Session 32: Magnetohydrodynamics in Astrophysics and Geophysics: advances in dynamo theory.

Emmanuel Dormy, CNRS/ENS/IPGP, France
Manuel Nunez, University of Valladolid, Spain

Magnetohydrodynamic (MHD) provides important challenges both in Astrophysics and in Geophysics. As research progresses in MHD, it yields new issues relevant to applied mathematics (both analytical and numerical). This special session will gather applied mathematicians and physicists to present the state of the art in MHD and in dynamo theory for natural applications (planets, stars, galaxies). The emphasis will be placed on the present opened issues relevant to applied mathematics.

⟶ ∞ ◇ ∞ ⟵

Force-free magnetic fields in the solar corona

Jean-jacques Aly
CEA Saclay, France
jjaly@discovery.saclay.cea.fr

Force-free magnetic fields – i.e., fields for which the Lorentz force vanishes, the electric currents flowing along the magnetic lines – are widely used for modeling the solar corona, which is an extended region around the Sun in which the magnetic energy density is much larger than the energy density of the plasma. Two types of problems have been considered for this type of fields. The first one is the so-called reconstruction problem, and it consists to set up a boundary value problem allowing to recover at some given time the field inside the corona (not directly measurable) from the measurement of its three components effected at its basis (the photosphere). The second one is the problem of the quasi-static evolution of the field which is driven by the slow changes (footpoints shearing by the dense plasma, flux emergence and submergence, ...) continuously occurring on the photosphere. The main issue here is to determine if such an evolution may lead to some form of nonequilibrium process which could be used to explain the triggering of the huge eruptive phenomena occuring sporadically in some regions of the corona. In my talk, I shall review the results obtained up to now on these two problems by the solar physicists, and I shall point out the numerous mathematical problems which are yet to be solved.

⟶ ∞ ◇ ∞ ⟵

Magnetohydrodynamic evolution of solar coronal magnetic field

Tahar Amari
CNRS. Ecole Polytechnique, France
amari@cpht.polytechnique.fr

We discuss some problems and their numerical constraints relevant to the MHD evolution of coronal (low beta) configurations such as those occurring in the solar atmosphere for the triggering of large scale eruptive events. Those are introduced as different classes of boundary value problems driven by the conditions set at the surface of the astrophysical object.

We then introduce an approach in which this evolution is driven by the MHD evolution inside the object, such as in the convection zone. We discuss in particular the attempt to construct a domain decomposition alternative approach to avoid the difficult interface problem between these two domains, between which strong variations of the physical parameters occur.

⟶ ∞ ◇ ∞ ⟵

The Magnetorotational Instability in Nature and in the Laboratory

Steven Balbus
Ecole Normale Superieure, France
Steven.Balbus@lra.ens.fr

In the last 15 years, the magnetorotational instability (MRI) has received widespread interest in the astrophysical community. Its primary application has been to accretion disks around normal or collapsed stars, in which it is thought to mediate the transition form laminar Keplerain flow to MHD turbulence. Despite its importance, the MRI had not been seen in the laboratory until very recently. Its apparent detection in a liquid sodium experiment in 2004 has sparked interest amongst geophysicists in the possibility that the MRI may play a role in our understanding of planetary dynamos. In this talk, I will review the essentials of the MRI in both analytic lineary theory and fully nonlinear simulations, and discuss how the MRI might manifest itself in an MHD regime appropriate to the geodynamo.

⟶ ∞ ◇ ∞ ⟵

Entropy satisfying approximate Riemann solvers for compressible MHD built via Suliciu relaxation

Francois Bouchut
CNRS & ENS Paris, France
Francois.Bouchut@ens.fr
Christian Klingenberg and Knut Waagan

The Suliciu relaxation system has been recently used as a way to derive and justify the HLLC approximate Riemann solver for gas dynamics. In particular, this approach provides a tool to analyze positity and entropy inequalities, leading to a sharp choice of the wave speeds. In this work, we use the same Suliciu relaxation approximation applied to the compressible MHD system. It enables again to justify positivity and entropy conditions. Two solvers are derived: one with three waves, and one with seven waves that enables the exact resolution of all contact waves (material and Alfven).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Mathematical Study of small scale dynamos

David Gerard-varet
CNRS, Ecole Normale Superieure, France
dgerardv@dma.ens.fr

We will present some recent results on the equations of magnetohydrodynamics. These results are connected to the dynamo effect, which is an instability phenomenon due to the movement of a conducting fluid. We will recover mathematically some small-scale dynamo mechanisms, due to oscillation and concentration in fluid flows.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Antidynamo Theorems

Ralf Kaiser
Universität Bayreuth, Germany
Ralf.Kaiser@uni-bayreuth.de

Antidynamo theorems exclude dynamo action for certain classes of magnetic and/or flow fields. The most famous example is Cowling's theorem ruling out dynamo action for axisymmetric or two-dimensional magnetic fields. Another class of antidynamo theorems constrains the velocity field rather than the magnetic field. Its most prominent representative is the toroidal velocity theorem, stating that no dynamo action is possible for a purely toroidal velocity field in a sphere. These well-known results as well as their generalizations and variants reduce the problem to an uncoupled parabolic second order differential equation for the poloidal or toroidal scalar of the magnetic field. With appropriate assumptions the decay in time of this scalar (in a suitable norm) can then be proved. On the other side there is a conjecture due to Vainshtein that in all cases where such a reduction is not possible and, consequently, no antidynamo theorem is available dynamo action can actually be found, at least in some part of the parameter space.

I review in this talk the classical antidynamo theorems, refute Vainshtein's conjecture, discuss some more recent results, and close with some open problems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Highly Supercritical Convection in a Strong Magnetic Field

Edgar Knobloch
University of California at Berkeley, USA
knobloch@px1.berkeley.edu

In this talk I will review some of the results obtained through the use of a different type of asymptotic procedure that employs the small horizontal scale of motion as an expansion parameter. The technique applies naturally to systems in which strong restraints such as a strong magnetic field or rapid rotation lead to high wavenumbers at the onset of convective instability, and leads to reduced partial differential equations valid outside of passive boundary layers. A regime with exact, fully nonlinear, single-mode solutions is identified. These are determined from the reduced pdes reformulated as a nonlinear eigenvalue problem whose solution also gives, for each Rayleigh number, the time-averaged Nusselt number and oscillation frequency together with the mean temperature profile in the vertical. Solutions of the resulting nonlinear eigenvalue problem for both uniform and depth-dependent fluid properties will be presented. These reveal a tendency to form isothermal cores at highly supercritical Rayleigh numbers. Various mathematical challenges presented by this approach will be discussed.

References:
K. Julien, E. Knobloch and S. Tobias. Physica D 128, 105-129 (1999).
K. Julien, E. Knobloch and S. Tobias. J. Fluid Mech. 410, 285-322 (2000).
M. Sprague, K. Julien, E. Knobloch and J. Werne. J. Fluid Mech., in press.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Saturation of the Magnetorotational Instability

Edgar Knobloch
University of California at Berkeley, USA
knobloch@px1.berkeley.edu

K. Julien

An analytical theory[1] that describes asymptotically exactly the saturation of the magnetorotational instability in astrophysical[2] and laboratory systems[3] will be presented. The theory applies in the strongly nonlinear regime, and shows that the instability saturates by modifying the shear responsible for it. The saturation process requires both viscous and ohmic dissipation. The theory also describes the approach from small amplitude perturbations to the final strongly nonlinear saturated state.

[1]Knobloch, E. & Julien, K., 2005 Phys. Fluids, 17, 094106. [2]Balbus, S.A. & Hawley, J.F., 1998 Rev. Mod. Phys., 70, 1. [3]Sisan, D.R., et al., 2004 Phys. Rev. Lett., 93, 114502.

⟶ ∞◇∞ ⟵

Effects of turbulence on the dynamo instability

Francois Petrelis
LPS-ENS, France
petrelis@lps.ens.fr
S. Fauve

Recent experimental observations of the dynamo instability in Karlsruhe and Riga have shown that (a) the onset of dynamo is very close to the one predicted using a kinematic code in which the average in time velocity field is used (b) a magnetic field of the order of hundred Gauss is created at roughly 10 percent above criticality

We will discuss a perturbative expansion that allows to compute the onset shift driven by the turbulent fluctuations. Small scale turbulent fluctuations of moderate amplitude and weak helicity are shown to have no effect on the dynamo onset. However, we will show that the non-linear saturated magnetic field can only be predicted if one take into account the inertial term of the Navier-Stokes equation.

⟶ ∞◇∞ ⟵

Reversals in nature and the nature of reversals

Frank Stefani
Forschungszentrum Rossendorf, Germany
F.Stefani@fz-rossendorf.de
Uwe Günther and Gunter Gerbeth

One of the most interesting features of Earth's magnetic field reversals is their asymmetric shape including a rather slow decay of a given axial dipole and a very fast recreation of the dipole with opposite polarity. With focus on this asymmetry, we consider a simple mean-field dynamo model with a spherically symmetric isotropic helical turbulence parameter α that is quenched by the magnetic field energy and subjected to some noise. With an appropriate radial dependence of $\alpha(r)$ (including at least one sign change), this dynamo model exhibits typical features of reversals. The asymmetric shape and the very fast recreation are attributed to the dynamical behaviour in the vicinity of a branching point of square root type (exceptional point) of the spectrum of the non-selfadjoint dynamo operator. Other features, like the possible correlation of magnetic field amplitude and reversal rate, the bimodal field distribution, and the inhibition time are also addressed within our simple model. We discuss a tendency of highly supercritical dynamos to self-tune into reversal prone states, and hypothesize that reversing dynamos might be much more common in nature than what could be expected from a purely kinematic perspective.

⟶ ∞◇∞ ⟵

A Landau fluid description of collisionless plasmas

Pierre-louis Sulem
CNRS, Observatoire de la Cote d'Azur, France
sulem@obs-nice.fr
T. Passot

In many astrophysical plasmas such as the solar wind, the terrestrial magnetosphere or the interstellar medium at small enough scales, collisions are negligible. When interested in the large-scale dynamics, an hydrodynamic description is nevertheless advantageous, in that numerical simulations are much easier than when using kinetic approaches, and also in that it usually provides a deeper understanding of cross-scale nonlinear couplings. It is thus of great interest to construct fluid models that extend the usual magnetohydrodynamic (MHD) equations to collisionless situations, by retaining main kinetic effects, such as Landau damping (the only fluid-particle resonance that can affect the large scales) and finite Larmor radius (FLR) corrections. Retaining Landau damping in a fluid formalism cannot be fully rigorous, but this effect is conveniently captured by fitting with the kinetic theory in various asymptotic regimes. Introduced by Snyder, Hammett and Dorland [Phys. Plasmas 4, 3974 (1997)] in the limit of very large scales, this approach was recently extended by retaining dominant FLR corrections, in order to address the dynamics of dispersive MHD waves and in particular their modulational and parametric instabilities. Furthermore, in environments with high beta and strong proton temperature anisotropy, a mirror instability can develop, with a maximal growth rate that increases with the transverse wavenumber, up to scales comparable with the ion Larmor radius. For a well-posed formulation of the nonlinear regime, FLR corrections at small transverse scales are thus retained to ensure the quenching of the instability at small scales, as predicted by the kinetic theory. Results on the existence of mirror mode structures, based on a generalized equation of state for the quasi-static regime, are also presented.

$\longrightarrow \infty\diamond\infty \longleftarrow$

A High-Order Godunov Scheme with Constrained Transport and AMR for Ideal MHD

Romain S. Teyssier
CEA Saclay, France
romain.teyssier@cea.fr

We discuss the application of the well-known Godunov method to the MHD equations, showing that it leads naturally to the Constrained Transport scheme with a 2D Riemann solver on cell edges. We have applied our scheme to several kinematic dynamos problems using Adaptive Mesh Refinement. New results on astrophysical MHD problems will be presented.

$\longrightarrow \infty\diamond\infty \longleftarrow$

Magnetohydrodynamics in a finite cylinder

Laurette Tuckerman
LIMSI-CNRS, France
laurette@limsi.fr
Piotr Boronski

We discuss some of the computational challenges in constructing a 3D hydrodynamic or magnetohydrodynamic code in a finite cylindrical geometry, such as that of the VKS experiment. We generalize techniques used in the spherical geometry, notably the the toroidal-poloidal decomposition to construct velocity and magnetic fields which are divergence-free, i.e. incompressible and free of magnetic monopoles, respectively. While straightforward in the spherical geometry, in this case boundary conditions couple the two potentials and extra equations, analogous to a gauge condition and to a constant of integration, must be imposed. We have also generalized a method which couples the magnetic field inside the cylinder to that in the external vacuum.

$\longrightarrow \infty\diamond\infty \longleftarrow$