Special Session 36: Topological Methods for Boundary Value Problems

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This session will concentrate on the use of topological methods in the study of various boundary value problems. This will include using degree theory, fixed point index theory, monotone iterative techniques and numerical methods to establish existence and multiplicity of positive solutions.

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Positive Solutions of Second Order Differential Equations With Integral Boundary Conditions

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We will discuss the existence and multiplicity of positive solutions of the following boundary value problem

$$\begin{aligned} u''(t) &= f(t,u(t),u'(t)), \\ u(0) &= au'(0) + \int_0^1 g_0(s)u(s)ds, \\ u(1) &= bu'(1) + \int_0^1 g_1(s)u(s)ds \end{aligned}$$

where $f \in C([0,1], \times\mathbb{R}^2, \mathbb{R}$, $g_0, g_1 \in C([0,1], \mathbb{R}_+)$, $a, b \geq 0$.

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Stationary states for a competition-diffusion system with inhomogeneous Dirichlet boundary conditions

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For sufficiently large positive values of the competition parameter k, all non-negative solutions of the competition-diffusion system $u_t = \Delta u + f(u) - kuv$, $v_t = \Delta v + g(v) - \alpha kuv$ in Ω, with inhomogeneous Dirichlet boundary conditions $u = m_1 \geq 0$, $v = m_2 \geq 0$ on $\partial\Omega$ approach stationary states as $t \to \infty$, provided $\alpha m_1 - m_2 \not\equiv 0$ on $\partial\Omega$, and all stationary solutions of the limit problem $-\Delta w = \alpha f(\alpha^{-1}w^+) - g(-w^-)$ in Ω, $w = \alpha m_1 - m_2$ on $\partial\Omega$ are non-degenerate. Here Ω is bounded, f and g are positive on $(0,1)$ and negative elsewhere and $\alpha > 0$. After describing this time-dependent result and briefly discussing the methods used to establish it, we will show how index theory can be used, for sufficiently large k, to establish the existence of a unique positive stationary solution of the $k-$dependent system close to $(\alpha^{-1}w_0^+, -w_0^-)$ where w_0 is a non-degenerate stationary solution of the limit problem. We will also discuss the non-degeneracy assumption, which does not always hold, but for which some genericity results can be proved.

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POSITIVE SOLUTIONS OF A THIRD ORDER NONLOCAL BOUNDARY VALUE PROBLEM

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The authors consider the nonlocal boundary value problem consisting of the third order nonlinear ordinary differential equation

$$u'''(t) = g(t)f(u(t)), \quad 0 \leq t \leq 1,$$

together with boundary conditions

$$u(0) = u'(p) = \int_q^1 w(t)u''(t)dt = 0,$$

where $g : [0,1] \to [0,\infty)$ and $f : [0,\infty) \to [0,\infty)$ are continuous functions, $g(t) \not\equiv 0$ on $[0,1]$, $\frac{1}{2} < p < q < 1$ are constants, and $w : [q,1] \to [0,\infty)$ is a nondecreasing continuous function with $w(t) > 0$ for $q < t \leq 1$. Sufficient conditions for the existence and nonexistence of positive solutions of this problem are obtained.

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Positive solutions of some nonlinear boundary value problems involving singularities and integral boundary conditions

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In this talk we shall discuss the existence of positive solutions of some nonlocal boundary value problems subject to integral boundary conditions and where the involved nonlinearity might be singular.

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Optimal constants arising from some boundary value problems

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Some optimal constants arising from some boundary value problems are studied and estimated. These optimal values are of importance in the study of the existence of positive solutions for the boundary value problems.

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Multiple positive solutions for a fourth order equation of Kirchhoff type

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One considers the existence of positive solutions for a class of nonlocal fourth order ordinary differential equations modeling deformations of extensible elastic beams. Existence and multiplicity results are obtained by using fixed points theorems in cones of positive functions.

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Higher Order Two-Point Boundary Value Problems with Asymmetric Growth

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In this work it is studied the higher order nonlinear equation

$$u^{(n)}(x) = f(x, u(x), u'(x), \ldots, u^{(n-1)}(x))$$

with $n \in \mathbb{N}$ such that $n \geq 2$, $f : [0,1] \times \mathbb{R}^n \to \mathbb{R}$ a continuous function, and the two-point boundary conditions

$$\begin{aligned} u^{(i)}(0) &= A_i, \quad A_i \in \mathbb{R}, \ i = 0, \ldots, n-3, \\ u^{(n-1)}(0) &= u^{(n-1)}(1) = 0. \end{aligned}$$

From one-sided Nagumo-type condition, allowing that f can be unbounded, it is obtained an existence and location result, that is, besides the existence, given by Leray-Schauder topological degree, some bounds on the solution and its derivatives till order $(n-2)$ are given by well ordered lower and upper solutions. Applications to some models of beams and suspension bridges will be presented.

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Mild almost automorphic solutions to some semilinear boundary evolution

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We will use the extrapolation methods to study the existence and uniqueness of almost automorphic mild solutions to the semilinear boundary differential equation

$$x'(t) = A_m x(t) + h(t, x(t)),$$

$$Lx(t) = g(t, x(t)),$$

where $A := A_m | \ker L$ generates a hyperbolic C_0-semigroup on a Banach space X and h, g are almost automorphic functions which take their values in X and in a 'boundary space' ∂X respectively. These equations are an abstract formulation of partial differential equations with semilinear terms at the boundary, like population equations, retarded differential equations and boundary control systems. We illustrate our abstract result with an application to retarded differential equations.

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Another topological approach to boundary value problems

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Existence results for the 3th order singular differential equation x”’(t)=a(t)f(t,x(t)), 0¡t¡1, satisfying some three-point boundary value conditions, are given.

In addition, we prove the existence of a sequence of such solutions tending in norm to the zero.

The emphasis in this paper is mainly to use as our base the powerful Knaster-Kuratowski-Mazurkiewicz's principle in the very simple case of two-dimensional plane. That is, our approach is based on the Sperner's Lemma, using in this way an alternative to the classical methodologies based on fixed point or degree theory and results the use of similar quite natural hypothesis and at the same time, we eliminate at all the related monotonicity assumption on the nonlinearity. In addition, we don't use the corresponding Green's function, the positivity of which usually implies some restriction on the class of the above BVPs and so, we are able to remove such a restriction. Generally, our principal tool is a combination of an analysis of the corresponding vector field on the face-plane along with Knesser's type properties of the solutions funnel, the Sperner's Lemma and the well known fixed point Krasnosel'kii's theorem applied on a new very simple cone of $\mathbb{R}^2$. This method have been applied several times, for solving more general BVP, like focal, conjugate or vector two-point's ones.

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Population Models with Diffusion and Strong Allee Effect

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We study the steady state distributions of reaction diffusion equations with strong Allee effect type growth in heterogeneous bounded habitats with hostile boundary. We establish existence and multiplicity results via the method of sub super solutions. We also study the effects of contant yield harvesting.

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Periodic solutions of differential equations with weak singularities

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To prove the existence of a positive periodic solution of a scalar equation $\ddot{x} = f(t,x)$ with a repulsive singularity in the origin ($f(t,x) \to +\infty$ as $x \to 0^+$), the classical technique of a priori bounds and topological degree requires a "strong force" assumption (the potential becomes unbounded near the origin). Our aim is to report on some recent strategies of proof that do not need this assumption and to point out some interesting open problems. Also, we will show that, in some cases, some additional information about the location of the solution can be exploited to study its stability.

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Positive Solutions of Nonlinear Systems of Differential Equations

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Nonlinear systems of differential equations are natural links between mathematics and a growing number of related disciplines such as physics, biology, etc. In this talk I will discuss some concepts of superlinearity and sublinearity for system of equations, and then illustrate that the number of positive solutions of nonlinear systems of differential equations can be determined by appropriate combinations of superlinearity and sublinearity assumptions.

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On the unique principal eigenvalue of nonlocal boundary value

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In the study of nonlocal boundary value problems, existence of positive solutions can be shown if the nonlinearity 'crosses' the principal eigenvalue, the eigenvalue corresponding to a positive eigenfunction. In one result, shown by this writer and K. Q. Lan, it was necessary to know that such an eigenvalue is unique. This was well-known to be the case for symmetric problems but was unclear in the generality studied. I show that old results due to Krasnosel'skii can be applied to see that the nonlocal problems which have been well studied over the last few years do have a unique principal eigenvalue.

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On the Existence of Fixed-sign Solutions for a System of Generalized Right Focal Problems with Deviating Arguments

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We consider the following system of third-order three-point generalized right focal boundary value problems

$$u_i'''(t) = f_i(t, u_1(\phi_1(t)), u_2(\phi_2(t)), \cdots, u_n(\phi_n(t))), \quad t \in [a,b]$$

$$u_i(a) = u_i'(z_i) = 0, \qquad \gamma_i u_i(b) + \delta_i u_i''(b) = 0$$

where $i = 1,2,\cdots,n$, $z_i \in (\frac{1}{2}(a+b), b)$, $\gamma_i \geq 0$, $\delta_i > 0$, and ϕ_i are deviating arguments. By using different fixed point theorems, we establish the existence of one or more *fixed-sign* solutions $u = (u_1, u_2, \cdots, u_n)$ for the system, i.e., for each $1 \leq i \leq n$, $\theta_i u_i(t) \geq 0$ for $t \in [a,b]$, where $\theta_i \in \{1, -1\}$ is fixed.

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On positive solutions to coincidence equations

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Donal O'Regan

We discuss the existence of positive solutions to the coincidence equation $Lx = Nx$, where L is a linear Fredholm mapping of index zero and N is (in general) a nonlinear operator. Using the properties of cones in Banach spaces and Leray-Schauder degree we obtain some refinements of the results established in [1] and [2]. Some applications to the periodic problem for first order differential equation will also be given.

[1]. R. E. Gaines, J. Santanilla , A coincidence theorem in convex sets with applications to periodic solutions of ordinary differential equations, Rocky Mountain J. Math. 12 (1982), 669–678.
[2]. J. Santanilla, Some coincidence theorems in wedges, cones, and convex sets, J. Math. Anal. Appl. 105 (1985), 357–371.

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