Special Session 37: Dynamical Systems of Multiple Time Scales

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The focus of this special session is on various phenomena of singularly perturbed dynamical systems and their applications.

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Ramping Through a Hopf Bifurcation: New Insights into the Memory Effect

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In many experiments modeled mathematically as bifurcation problems, the bifurcation parameter varies naturally with time (e.g., elliptic bursting in nerves) or the parameter is deliberately varied by the experimenter. The delay effect is a well known phenomena that occurs when a parameter slowly ramps through a Hopf bifurcation point and the onset of large amplitude oscillations occurs beyond the critical value predicted by treating the parameter as static, i.e., the dynamic aspect of the slowly changing parameter leads to a delay in the onset. In this talk we obtain new insights into a phenomena called the memory effect. The memory effect refers to how the delay in onset is dependent on the initial state of the system. Here we explore numerically and analytically the nature of the memory effect. We apply our results to better understand the dynamics underlying neuronal elliptic bursting.

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Metastability and stochastic resonance in slow–fast systems with noise

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Dynamical systems perturbed by weak noise often display metastable behaviour, that is, typical sample paths spend exponentially long time spans near local equilibria of the unperturbed system. When combined with a time-periodic forcing, this phenomenon can give rise to large-amplitude close-to-periodic oscillations between attractors, the so-called stochastic resonance. This mechanism may be responsible, for instance, for repeated warming events observed during the last Ice Age.

Building on new techniques allowing to determine the sample-path behaviour of stochastic slow–fast systems, we derive several quantitative measures of resonance, such as threshold noise intensities and residence-time distributions. Some of these quantities exhibit non-trivial power-law dependences on forcing amplitude and frequency, and noise intensity.

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On maximum bifurcation delay in real planar singularly perturbed vector fields

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In singularly perturbed vector fields, where the unperturbed vector field has a curve of singularities (a critical curve), orbits tend to be attracted towards or repelled away from this curve, depending on the sign of the divergence of the vector field at the curve. When at some point, this sign bifurcates from negative to positive, orbits will typically be repelled away immediately after passing the bifurcation point (turning point). A-typical behaviour is nevertheless observed as well, when orbits follow the critical curve for some distance after the turning point, before they repel away from it. One says a delay in the bifurcation is present. Interesting are systems that have a maximum bifurcation delay, i.e. there is a point on the critical curve beyond which orbits cannot stay close to the critical curve. This behaviour is known to appear in some systems in dimension 3, and it is commonly believed that it is not an issue in (real) planar systems. Beside making the observation that it does appear in non-analytic planar systems, it is shown that whenever it appears, bifurcation delay has no non-trivial maximum for analytic planar vector fields. The proof is based on the notion family blow up at the turning point, on formal power series in terms of blow up variables, the study of their Gevrey properties and analytic continuation of their Borel transform.

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Qualitative Study to A Reaction-Diffusion Equation

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There is the widespread existence of wave phenomena in physics, chemistry and biology. This clearly necessitates a study of traveling waves in depth and of the modeling and analysis involved. In the present talk, we study a nonlinear reaction-diffusion equation, which can be considered as a generalized Fisher equation. We present a traveling wave solution and an oscillatory solution by means of the Cole-Hopf transformation and the Lie group method.

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Concentration of sample paths in stochastic slow–fast systems

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We consider the effect of noise on dynamical systems evolving on two well-separated timescales. These systems are described by singularly perturbed stochastic differential equations.

A constructive method, introduced recently, allows to describe the typical behaviour of solutions of the noisy system in the neighbourhood of asymptotically stable manifolds of the deterministic system. It is based on the observation that for sufficiently small noise intensity, typical sample paths remain in metastable equilibrium near the deterministic solution with the same initial condition for exponentially long time spans. The probability of exceptional paths is proved to be exponentially small in the small parameters characterising the system such as noise intensity and adiabatic parameter.

We shall conclude by discussing how to control the effect of noise in the case of the deterministic system undergoing a bifurcation.

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Geometric singular perturbations for multiple turning points

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We consider singularly perturbed systems with multiple sets of turning points. Two types of related results contributing to geometric singular perturbation theory are obtained. The first result establishes a new class of invariant manifolds. The second result consists of several exchange lemmas that characterize the evolution of an invariant manifold passing through the vicinity of the slow manifold.

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Multimodal oscillations in systems with strong contraction

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A one-parameter family of flows in R^3 near an Andronov-Hopf bifurcation (AHB) is investigated in this work. We identify conditions on the global vector field, which yield a rich family of multimodal orbits passing close to a weakly unstable saddle-focus and provide a detailed asymptotic description of these orbits near the saddle-focus. The analysis covers both cases of sub- and supercritical AHB. If the AHB is subcritical, it is accompanied with appearance of the multimodal orbits, which consist of long series of small amplitude nearly harmonic oscillations separated by large amplitude spikes. We analyze the dependence of the interspike intervals (which can be extremely long) on the control parameters present in the system. In particular, we show that the interspike intervals increase significantly as the boundary between the regions of sub- and supercritical AHB bifurcation is approached from the subcritical region. Furthermore, we identify a window of complex dynamics near the boundary between the regions of sub- and supercritical AHB. This work is motivated by the numerical results for a finite-dimensional approximation of a free boundary problem modeling solid fuel combustion. This is a joint work with Yun Yoo.

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Metastability and dispersive shock waves in Fermi-Pasta-Ulam system

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We show the relevance of the dispersive analogue of the shock waves in the FPU dynamics. In particular we give strict numerical evidences that metastable states emerging from low frequency initial excitations, are indeed constituted by dispersive shock waves travelling through the chain. Relevant characteristics of the metastable states, such as their frequency extension and their time scale of formation, are correctly obtained within this framework, using the underlying continuum model, the KdV equation. http://arxiv.org/nlin.SI/0511026

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Noisy neuronal bursting activities

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Bursting and oscillation phenomenon are common in multiple-time scale dynamical systems and they are typically subject to various noisy backgrounds. In this talk, we examine elliptic bursting with and without noise. Particular attention is given to Hodgkin-Huxley neuron models that exhibit such a behavior in response to physiological changes.

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