Special Session 38: Nonlinear Analysis, Trends and Applications, Special Session Celebrating the Sixtieth Birthday of J.R.L. Webb

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This special session celebrates the sixtieth birthday of Professor J.R.L. Webb with some invited contributions on some recent developments in the area of Nonlinear Analysis and its applications.

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Global branches of periodic solutions for delay differential equations on compact manifolds

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We study the nonlinear delay differential equation

$$\dot{x}(t) = \lambda f(t, x(t), x(t-1)), \quad \lambda \geq 0,$$

in the following assumptions: given a smooth manifold (possibly with boundary) embedded in $\mathbb{R}^k$, $f : \mathbb{R} \times M \times M \to \mathbb{R}^k$ is a continuous map, T-periodic in the first variable and tangent to M in the second one; that is, $f(t + T, p, q) = f(t, p, q) \in T_pM$ for all $(t, p, q) \in \mathbb{R} \times M \times M$, where $T_pM \subseteq \mathbb{R}^k$ denotes the tangent space of M at p.

Using a topological approach based of the fixed point index we obtain global bifurcation results for periodic solutions of the above problem.

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Existence and uniqueness of solutions to a super-linear three-point boundary value problem

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In previous papers, degree theory for nonlinear operators has been used to study a class of three-point boundary value problems for second order ordinary differential equations having a super-linear term, and existence of a sequence of solutions has been shown. In this paper, we forgo the previous approach for the shooting method, which gives a drastically simpler existence theory, with less assumptions, and easy calculation of solutions. We even obtain uniqueness in the simplest case.

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Eigenvalues of homogeneous gradient mappings in Hilbert space

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Any non-trivial linear compact self-adjoint operator acting in a Hilbert space possesses at least one non-zero eigenvalue. We present a generalization of this to nonlinear mappings as in the title.

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Some Topological Results for the Semilinear A-Spectrum

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Using the semilinear A-spectrum, we obtain a semilinear analogue of the Birkhoff-Kellogg theorem and other results.

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A class of maps related to the semilinear spectrum and its applications

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In this paper, a class of nonlinear maps, the (a, q)-L-stable solvable maps for the Fredholm operator L, is introduced. Closely related to the spectrum of semilinear operators, the (a,q)-L-stable solvable maps are generalization of the (a,q)-stably solvable maps that was defined by Appell, Giorgieri and Väth previously. We prove properties for the new class of operators including the continuation principle and eigenvalues. We also show its application in the study of solvability for differential equations.

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Existence results for differential equations on unbounded domains

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We shall present new sufficient conditions to guarantee the existence of nontrivial solutions for ordinary differential equations on unbounded intervals.

The results will be based on the study of the properties of the Green function for a related linear problem and fixed point index theory.

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Switching in a nematic liquid crystal device

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We present a mathematical model of a bistable nematic LCD device and show that using a flexoelectric liquid crystal in the construction, we can ensure energy-efficient switching between stable states. Mathematically, this amounts to an analysis of a semilinear parabolic equation with a nonlinear dynamic boundary condtion. We will also highlight open problems in this area of applications.

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Iterative Solutions for Zero of Accretive Operators

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We define iterative schemes to approach zeros of m-accretive operators in Banach spaces.

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The spectrum of the periodic p-Laplacian

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We consider one dimensional p-Laplacian eigenvalue problems of the form

$$-\Delta_p u = (\lambda - q)|u|^{p-1}\operatorname{sgn} u, \quad \text{on } (0,b),$$

together with periodic or separated boundary conditions, where $p > 1$, Δ_p is the p-Laplacian, $q \in C^1[0,b]$, and $b > 0$, $\lambda \in \mathbb{R}$. The structure of the spectrum of this problem is well known and understood in the following cases:

- the general separated case for all $p > 1$;
- the general periodic case with $p = 2$;
- the periodic case with $p \neq 2$ and $q = 0$.

In contrast, we show that when $p \neq 2$ and $q \neq 0$, the structure of the spectrum in the periodic case can be completely different, and considerably more complicated than in any of these cases.

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Recent results in nonlinear spectral theory

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Let E be a (real or complex) Banach space, $f : U \to E$ a continuous map defined on an open subset of E, and p a point in U. In a joint work with A. Calamai and M. Furi, we introduce the concept of *spectrum of the map f at the point p*, denoted $\sigma(f,p)$. This new spectrum shares several properties with the asymptotic spectrum introduced by Furi, Martelli and V. in the year 1978. For instance, it is always closed and coincides with the ordinary spectrum in the linear case. However, it is related to the Fréchet derivative of f at p, whenever defined, rather than to its asymptotic derivative. This new notion of spectrum was made possible by the recent introduction due to Calamai of appropriate numerical characteristics for nonlinear operators. Applications to bifurcation theory are given.

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Projection Algorithms for Solving the Multiple-Set Split Feasibility Problem

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Let H_1 and H_2 be two Hilbert spaces. The multiple-set split feasibility problem (MSSFP) is to find a point x such that

$$x \in C = \cap_{i=1}^{N} C_i \text{ and } Ax \in Q = \cap_{j=1}^{M} Q_j, \tag{1}$$

where $N \geq 1$ and $M \geq 1$ are positive integers, and $\{C_i\}_{i=1}^{N}$ and $\{Q_j\}_{j=1}^{M}$ are closed convex subsets of H_1 and H_2, respectively, and $A : H_1 \to H_2$ is a bounded linear operator.

The MSSFP (1) includes the convex feasibility problem (CFP) as a special case. The CFP is to find a point x such that $x \in \cap_{i=1}^{N} C_i$. The MSSFP (1) also includes the split feasibility problem (SFP) which corresponding to the case of (1) with $N = 1 = M$.

The SFP is solved by the *CQ* algorithm of Byrne which generates a sequence $\{x_n\}$ recursively by

$$x_{n+1} = P_C(x_n - \gamma A^*(I - P_Q)Ax_n), \quad n \geq 0. \tag{2}$$

Censor *et al* proposed the following algorithm to solve the MSSFP (1)

$$x_{n+1} = P_\Omega\Big(x_n - \gamma\big(\sum_{i=1}^{N} \alpha_i(x_n - P_{C_i}x_n) + \sum_{j=1}^{M} \beta_j A^*(Ax_n - P_{Q_j}Ax_n)\big)\Big) \tag{3}$$

where Ω is an extra closed convex subset of H_1, and where $\alpha_i > 0$ for all i and $\beta_j > 0$ for all j and such that $\sum_{i=1}^{N} \alpha_i + \sum_{j=1}^{M} \beta_j = 1$.

It is the purpose of this paper to present some new projection algorithms to solve the MSSFP (1) which generate a sequence $\{x_n\}$ either by

$$x_{n+1} = (1 - t_{n+1})P_{C_{[n+1]}}\left(x_n - \gamma \sum_{j=1}^{M} \beta_j A^*(I - P_{Q_j})Ax_n\right) \tag{4}$$

where $\{t_n\}$ is a sequence in (0,1), or by

$$x_{n+1} = \sum_{i=1}^{N} \lambda_i P_{C_i}\left(x_n - \gamma \sum_{j=1}^{M} \beta_j A^*(I - P_{Q_j})Ax_n\right) \tag{5}$$

where $0 < \gamma < 2/\|A\|^2$, where $\lambda_i > 0$ for all i such that $\sum_{i=1}^{N} \lambda_I = 1$, and where $\beta_j > 0$ for all j such that $\sum_{j=1}^{M} \beta_j = 1$. We show that the algorithm (4) converges strongly to the minimal-norm solution of the MSSFP (1), and (5) weakly to a solution of the MSSFP (1).

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