Special Session 3: Theory and Applications of Hysteresis Modeling

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This session focuses on modeling and analysis of hysteresis phenomena arising in mechanics, thermodynamics, or electromagnetism. Macroscopic hysteresis is observed in systems with microstructure possessing a large number of possible equilibria, when the processes on the microscale are substantially faster that the observer's time scale, and the changes in microstructure are irreversible. In the limit case, the reaction to a slowly varying input is almost instantaneous, and this leads to the idea of 'rate-independence' as a characteristic feature of hysteresis. Elastoplastic or ferromagnetic constitutive laws are well-known examples for such a behavior. Strong hysteresis effects in combination with other phenomena can also be observed in phase transitions, deformations of shape memory alloys and piezoceramics, soil hydrology, or muscle mechanics. In practice, hysteresis can either be desirable, like in magnetic data storage devices, or undesirable, like in material fatigue or real time control of piezoelectric sensors and actuators.

The concept of hysteresis operator is based on replacing complicated considerations about a large number of microstates by introducing additional macroscopic memory variables to describe the memory effects. This is somehow similar to the transition from particle mechanics to continuum mechanics and thermodynamics, where hysteresis constitutive laws represent a compatible part.

The main mathematical issues involve the qualitative and numerical analysis of hysteresis models, theory and control of ordinary and partial differential equations with hysteresis operators, including questions of convergence and stability of numerical methods.

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On some P.D.E.s with hysteresis

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We present some P.D.E.s containing a continuous hysteresis operator. We introduce a weak formulation in Sobolev spaces for a Cauchy problem; under suitable assumptions on the hysteresis operator, we prove its well-posedness as well as some regularity results. When possible, we also present some results of asymptotic behaviour.

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Emergent Hysteretic Behavior in Systems of Interconnected Relays

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We consider model systems that consist of many similar bi-stable relays interacting with each other. Such models can be viewed as prototypes of magnetic, economic, social, biological and other types of complex systems. In the absence of interactions the system is described by the Preisach model. When interactions are introduced the hysteretic behavior can change qualitatively. Our goal is to investigate possible relationship between patterns in hysteretic processes and model parameters describing the interaction strength, sign and topology (mean-field, near-neighbour, random, etc.). We investigate numerically and analytically emergence of various types of training effects where minor hysteresis loops do not close at the end of the very first cycle. We also study dependence of hysteretic losses on the interactions. We demonstrate that training effects and losses display patterns that are related to the extent of disorder in the system as well as strength, sign and topology of interactions.

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Asymptotic behavior for a phase-field model for thermo-visco-plasticity involving outwards pointing hysteresis operators

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As a mathematical model for nonlinear thermo-visco-plastic developments a phase-field system involving hysteresis operators will be considered, and a large-time asymptotic result for this system will be presented.

It will be shown, how the method of invariant regions, which was used before only to derive uniform estimates for evolution equations involving superposition operators, can be generalized for dealing with hysteresis operators, at least if the hysteresis operators are *pointing outwards*.

This assumption will be discussed in this talk, and for some kind of hysteresis operators sufficient and necessary conditions for satisfying this assumption will be presented.

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Hysteresis and semigroups

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By a suitable formulation of a large class of hysteresis operators, several differential equations with hysteresis can be written as systems of equations containing an accretive operator. Results of the theory of nonlinear contractions can then be applied, yielding in particular the well possedness of the corresponding Cauchy problems. The main advantage of this approach is that it can be used for continuous as well as discontinuous hysteresis operators. Several examples will be given.

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A thermodynamically consistent temperature-dependent Preisach hysteresis model

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Rate independent dissipation processes in real materials are in typical cases strongly temperature-dependent. It is known that even in relatively simple models of, say, thermoelastoplasticity, the dependence on temperature cannot be prescribed in an indifferent way. The main issue is to define the state space properly and to find conditions on the parameters of the process to make the model compatible with the thermodynamic principles. In this contribution, we focus on the Preisach model and its potential applications in materials sciences.

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Compensation of parameter-dependent complex hysteretic actuator nonlinearities in smart material systems

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In many mechatronic applications we find solid-state actuators with complex hysteretic characteristics between the electrical input quantity and the mechanical output quantity. In the case of giant magnetostrictive materials this characteristic varies with a second input quantity as for example a variable prestress in the actuator material. This paper describes the design of a new parameter-dependent threshold-discrete representation of the Preisach hysteresis operator and the realisation of an inverse model to compensate the parameter-dependent complex hysteretic characteristic in real time.

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Equations with time derivatives of the Preisach operator

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A new class of closed systems with hysteresis where the Preisach hysteresis operator enters equations under the highest derivative has been recently suggested as a modeling tool in several application areas. We discuss some properties of solutions of such equations related to uniqueness/sensitivity with respect to perturbations of initial data. In particular, surfaces where the derivative of a solution jumps and nonuniqueness (bifurcation) points are identified. We also consider periodic solutions.

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Three state relays

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Consider a person that chooses between two states, s_0 and s_1, based on the values x_0 (x_1) of being in the state s_0 (s_1). Being in the state s_0 one chooses to switch to s_1 only if $x_1 - x_0 > 0$, provided there are no switching costs. If there is a cost $-a$ of switching from s_1 to s_0 and a cost b; $b > -a$ of switching from s_0 to s_1, the current state of the person is described by the output of the non-ideal relay $R[a,b]$ with thresholds a, b and input $x_1 - x_0$. Imagine now that we choose one of three states, provided there are transition costs between any two of them. We call the resulting hysteresis nonlinearity a three-state relay. In this talk we discuss the memory properties of the hysteresis model that is defined as an aggregation of all possible

three-state relays with some measure μ on the parameter space. This model is analogous to the Preisach model, which can be interpreted as an aggregation of non-ideal relays, but its properties are completely different.

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BV-extension of rate independent operators

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Rate independent operators naturally arise in the mathematical analysis of hysteresis. Among rate independent operators, the locally monotone ones are those better suited for the study of PDE's with hysteresis. We prove that a rate independent operator $\mathsf{R} : F(0,T) \longrightarrow F(0,T)$, with $Lip(0,T) \subseteq F(0,T) \subseteq BV(0,T) \cap C(0,T)$, which is locally monotone and continuous with respect to the strict topology of BV admits a unique continuous extension $\widetilde{\mathsf{R}} : BV(0,T) \longrightarrow BV(0,T)$. This result provides a general extension theorem which applies to several concrete hysteresis operators. For a number of these operators the existence of a continuous extension was previously known at most to the space $BV(0,T) \cap C(0,T)$.

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Rate-independent models of isothermal hysteretic response of shape-memory alloys.

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Models of shape-memory alloys based on a continuum-mechanical description of the stored energy having a multi-well character giving rise to microstructures and of the dissipated energy giving rise to hysteresis in stress/strain response will be presented. The configuration can be described either conventionally in terms of displacements or by special gradient Young measures combined with displacement to reflect better a multiscale character of the problem. Basic mathematical and numerical analysis accompany both sort of models, and also isothermal computational simulations with NiMnGa or CuAlNi single crystals have been done.

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Prandtl-Ishlinskii hysteresis operators and 1D elastoplasticity

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In this lecture, which reports joint work with P. Krejčí (WIAS Berlin), we derive the 1D equation governing the transversal vibrations of an elastoplastic beam from a general 3D system. Starting from the single-yield 3D von Mises plasticity model *without hardening*, we arrive after a dimensional reduction at a multi-yield 1D model of Prandtl-Ishlinskii type *with kinematic hardening*. In this sense, the occurrence of Prandtl-Ishlinskii hysteresis operators is quite natural in the framework of elastoplasticity.

It turns out that the weight function associated with the resulting Prandtl-Ishlinskii operator can even be calculated explicitly. We also study the question of well-posedness of the resulting PDE system with hysteresis that governs the vibrations of the beam.

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Hysteresis in congested networks

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The phenomenon of premature loss of stability for certain classes of communication networks is known since early 1990s. For instance, a simple two-node priority network N studied by Rybko and Stolyar can be unstable under stationary inflows whose intensity lay well below the linear traffic conditions. We consider a large homogenous network composed of multiple instances of N, with additional feedback (all the customers within the network are permanent), and with random cross-flows. As the number of instances tends to infinity and the population of the network grows proportionally, we get a so-called mean field limit which is a deterministic dynamical system on the space of probability measures on the states of a single network N. This limit demonstrates a surprising oscillatory behavior (the Poisson hypothesis does not hold for certain sets of parameters). Moreover, large homogenous networks have two metastable regimes: a laminary and a turbulent one. The phase transition between these regimes, under slow evolution of parameters, demonstrates typical hysteresis loops.

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