Special Session 40: Nonlinear Partial Differential Equations

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The main topics of the session include, but not limited to, qualitative and quantitative analysis of solutions of nonlinear elliptic or parabolic partial differential equations and systems, as well as their applications. The purpose of this session is to get together experts and young researchers in this area to report their recent development and to exchange their ideas in research.

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Fractional degree vortices for a spinor Ginzburg-Landau model

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Recent papers in the physics literature have introduced spin-coupled (or spinor) Ginzburg–Landau models for complex vector-valued order parameters in order to account for ferromagnetic or antiferromagnetic effects in high-temperature superconductors and in optically confined Bose–Einstein condensates. These models give rise to new types of vortices, with fractional degree and non-trivial core structure. By studying the associated system of equations in $\mathbb{R}^2$ which describes the local structure of these vortices, we show some new and unconventional properties of these vortices. We illustrate the various possibilites with some specific examples of Dirichlet problems in the unit disk.

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Eigenvalue, maximum principle and regularity for fully nonlinear operators

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The main scope of this talk is to extend the notion of principal eigenvalue for fully non linear operators in bounded domains that are elliptic and homogenous of degree α greater than -1 in the gradient. This, and some existence results, will be exposed, the proofs use the maximum principle and some Hölder estimates.

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A generalized sup + inf inequality for $-\Delta u = R(x)e^u$

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In establishing the a priori estimates for the semi-linear elliptic equation $-\Delta u = R(x)e^u$ in R^2, Brezis, Li, and Shafrir obtained the following inequality

$$\sup_K u + \inf_\Omega u \leq C,$$

where Ω is a domain in R^2 and K is a compact subset of Ω. They assumed that $R(x)$ is positive and bounded away from zero.

This inequality has become a powerful tool in estimating the solutions of semi-linear elliptic equations either in Euclidean spaces or on Riemannian manifolds.

We use the method of moving spheres to extend this inequality to the case where $R(x)$ is allowed to have zeros which are non-degenerate minima. We will also illustrate how this inequality can be applied to obtain a priori estimates for the solutions of prescribing Gaussian curvature equations.

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On a semilinear PDE with a singular nonlinearity

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I will discuss some recent progress on the semilinear elliptic problem $-\Delta u = \lambda f(x)(1-u)^{-2}$ on a smooth bounded domain Ω of $\mathbb{R}^N$ with an homogeneous Dirichlet boundary condition. This equation models a simple electrostatic Micro-Electromechanical System (MEMS) device and has been studied recently by Pelesco, by Guo-Pan-Ward, and by Guo-Ghoussoub. Guo-Ghoussoub pointed out that dimension N=7 is critical for this problem: the branch of minimal (or semi-stable) solutions is compact up to a certain critical value $\lambda*$ provided $1 \leq N \leq 7$. In this talk, I will present a general compactness result concerning the higher branches of unstable solutions in the same low dimensions. A previous result had been obtained for the second branch (of "mountain pass" solutions) in collaboration with Ghoussoub and Guo.

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Bubble tower solutions of slightly supercritical elliptic equations and application in symmetric domains

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We construct solutions of the semilinear elliptic problem

$$\begin{cases} \Delta u + |u|^{p-1}u + \varepsilon^{\frac{1}{2}} f & = 0 \qquad \text{in } \Omega \\ u & = \varepsilon^{\frac{1}{2}} g \qquad \text{on } \partial\Omega \end{cases}$$

in a bounded smooth domain $\Omega \subset \mathbb{R}^N$ $(N \geq 3)$, when the exponent p is supercritical and close enough to $\frac{N+2}{N-2}$. As $p \to \frac{N+2}{N-2}$, the solutions have multiple blow up at finitely many points which are the critical points of a function whose definition involves Green's function. As applications, we will give some existence results, in particular, when Ω are symmetric domains perforated with the small hole and when $f = 0$ and $g = 0$.

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Nonrelativistic limit in the Abelian Chern-Simons model

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The Abelian Chern-Simons model describes charged vortices with gauge field dynamics governed only by the Chern-Simons term without the Maxwell term. These vortices are charged both electrically and magnetically, which are important in anyonic superconductivity. The equation under consideration is

$$\Delta u = \frac{4q^4}{\kappa^2 c^4} e^u (e^u - \sigma^2) + 4\pi N \delta_0$$

with the nontopological boundary condition: $u(x) \to -\infty$ as $|x| \to \infty$ in $\mathbb{R}^2$. We verify the nonrelativistic limit for the radial solutions by varying each parameter suitably and keeping the shooting constant of radial solutions to be fixed in the limit $c \to \infty$.

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Global Well-Posedness of Equations of Fluid Type

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Tom Hou

I will present the joint work with Tom Hou on the global well-posedness of the viscous incompressible Boussinesq equations in two spatial dimensions and some realted equations of Fluid Type. Using sharp and delicate energy estimates, we prove global existenc and strong regularity of this viscous Boussinesq system for general initial data in H^m with $m \geq 3$. Similar results on some related equations of fluid type will also be discussed.

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Boundary-value problems for light near a caustic

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A quasilinear system of elliptic-hyperbolic partial differential equations arising from the uniform asymptotic approximation of solutions to the Helmholtz equation is discussed. These equations produce a model for high-frequency waves which is valid on both sides of a smooth, convex caustic. We prove the existence of strong solutions to boundary-value problems for an arbitrarily small lower-order perturbation of such a system in its hodograph linearization. The boundary is allowed to extend into both the elliptic and hyperbolic regions of the equations. This extends work by Kravtsov and Ludwig, who independently developed the asymptotic approximation in the 1960s, and also recent work by Magnanini and Talenti, who showed the existence of solutions for the case in which the boundary is restricted to the elliptic region of the equations.

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Elliptic problems: inverse square potential versus dependence on power of the gradient

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We analize the joint effect of Hardy potential and terms of the form $|\nabla u|^p$ related to complete blow-up and to break down resonance.

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On the existence of sign changing solutions to some critical problems

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Let Ω be a bounded smooth domain in R^N, $N \geq 3$ and $p = \frac{N+2}{N-2}$. We are interested in existence and multiplicity of sign changing solutions to the slightly subcritical problem

$$(1) \qquad -\Delta u = |u|^{p-1-\varepsilon} u \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega,$$

and to the Bahri-Coron's problem

$$(2) \qquad -\Delta u = |u|^{p-1} u \text{ in } \Omega_\varepsilon,\ u = 0 \text{ on } \partial\Omega_\varepsilon,$$

when $\Omega_\varepsilon = \Omega \setminus B(0,\varepsilon)$. In both cases ε is a small positive parameter. We prove that, problem (1) has at least N pairs of solutions which change sign exactly once. Moreover, the nodal surface of these solutions intersects the boundary of Ω, provided some suitable conditions are satisfied ([1]). When Ω is symmetric and contains the origin, we construct sign changing solutions to problems (1) ([3]) and (2) ([2]) with multiple blow up at the origin. These solutions have, as ε goes to zero, more and more annular-shaped nodal domains.
References:
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[2] M. Musso, A. Pistoia, Sign changing solutions to Bahri-Coron's problem in pierced domains, (preprint).
[3] A. Pistoia, T. Weth, Sign changing bubble tower solutions in a slightly subcritical semilinear Dirichlet problem, *Ann. Inst. H. Poincaré Anal. Non Linéaire* (to appear).

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A priori bounds and the Ambrosetti-Prodi problem for nonlinear elliptic systems

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We report on some recent results on a priori bounds and existence of solutions for systems of semilinear elliptic equations. These results depend on new Liouville type theorems for the systems in the whole space or in a half-space, which in turn are proven through a monotonicity result in unbounded domains. We also consider the so-called Ambrosetti-Prodi problem, for general elliptic operators in non-divergence form and power-growth nonlinearities. We obtain new existence results, both in the case of a scalar equation and the case of a system.

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On a Source-type Solution for a Nonlinear Parabolic Equation

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In this paper we study a nonlinear parabolic equation subject to a Direc-delta type of initial value. It is showed that the existence or nonexistence of such a solution depends on the nonlinearity of convection in the equation. The basic idea is to derive a precise decay estimate by using Moser's iteration technique.

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some gradient estimates on solutions to the heat equation on domains and manifolds

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First, we establish qualitatively sharp upper bound for the gradient of log Dirichlet and Poisson heat kernels on bounded smooth domains. Second, we obtain some new pointwise bounds for the time dependent heat equation on noncompact manifolds.

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