Special Session 43: Non-linear Dynamics and Applications

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This special session provides opportunity for exchanging new ideas and methods in studying non-linear dynamics and its applications. It will consists of talks with emphasis on analytical, geometrical and numerical approaches as well as various applications.

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Non-monotone travelling waves for a scalar reaction-diffusion equation with delay

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We prove the existence of a continuous family of positive and generally non-monotone travelling wave solutions for scalar delayed reaction-diffusion equations $u_t(t,x) = \Delta u(t,x) - u(t,x) + g(u(t-h,x))$, when $g \in C^2(\mathbb{R}_+,\mathbb{R}_+)$ has exactly two fixed points, $x_1 = 0$ and $x_2 = K > 0$. As an example, we consider the diffusive Nicholson's blowflies equation.

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Dynamics of two-strain influenza with isolation and partial immunity

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An ordinary differential equation model is used to study the evolution of influenza A pathogens driven by coevolutionary interactions between human hosts and competing strains of the visus. Oscillatory coexistence of both strains is shown via Hopf-bifurcation theory and confirmed via numerical simulations. We establish that cross-immunity and host isolation lead to recurrent outbreaks in the two-strain system. Sub-threshold conexistence is possible even when the reproductive number of one stain is below one. Conditions that guarantee a winning type or coexistence are established in general.

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Periodic Traveling Wave Solutions for Reaction diffusion Equations with Time Delayed and Non-Local Response

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We develop a singular perturbation technique to obtain the existence of periodic traveling wave solutions of large wave speed for reaction diffusion equations with delayed non-local response. Unlike the classical singular perturbation method, our approach is based on a transformation of the differential equations to integral equations in a Banach space that reduces the singular perturbation problem to a regular perturbation problem. The general result obtained has an application to a model describing the population growth when the species has two age classes and the diffusion of the individual during the maturation process leads to an interesting non-local and delayed response for the matured population.

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Geometric Singular Perturbation Analysis of a Model for Infectious Diseases.

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Abstract: A simple epidemic model for the spread of an infectious disease in a host population is analyzed using perturbation approach. The intrinsic growth rate of the host population is assumed to be a small parameter. The model gives rise to a singularly perturbed system of ordinary differential equations with a turning point. Geometric singular perturbation analysis of the global dynamics establishes the existence of stable relaxation oscillations. Our result suggests a correlation between the intrinsic growth rate of the host population and temporal cyclicity of the disease incidence.

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Multiple solutions for Poisson-Nernst-Planck systems with permanent chrages

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We study the PNP system for two types of ions with three regions of piece-wise constant permanent charge, assuming the Debye number is large. Reservoirs are represented by the outer regions with permanent charge zero. If the reciprocal of the Debye number is viewed as a singular parameter, the PNP system can be treated as a singularly perturbed system that has two limiting systems: an inner and outer systems (termed fast and slow systems in geometric singular perturbation theory). A complete set of integrals for the inner system is presented that provides information for boundary and internal layers. Application of the exchange lemmas from geometric singular perturbation theory gives rise to the existence and (local) uniqueness of the singular boundary value problem near each singular orbit. A set of nonlinear algebraic equations appears in the construction of singular orbits. Multiple solutions of such equations in this or similar problems might explain a variety of multiple valued phenomena seen in biological channels, for example, some forms of gating, and be involved in other more complex behaviors, for example some kinds of active transport.

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Global attractivity for scalar delayed differential equations

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For a scalar delayed differential equations in the general form $x'(t) = f(t, x_t)$, we give sufficient conditions for the global attractivity of its zero solution. It is assumed a 3/2-condition and the following generalized Yorke condition:

(YC) there exist piecewise continuous functions $\lambda_1, \lambda_2 : [0,\infty) \to [0,\infty)$ and a constant $b \geq 0$ such that, for $r(x) := \frac{-x}{1+bx}$, $x > -1/b$, then

$$\lambda_1(t)r(M(\varphi)) \leq f(t,\varphi) \leq \lambda_2(t)r(-M(-\varphi)),$$

for $t \geq 0$, where the first inequality holds for all $\varphi \in C := C([-\tau,0];\mathbb{R})$ and the second one for $\varphi \in C$ such that $\varphi > -1/b \in [-\infty,0)$, and $M(\varphi) := \max\{0, \sup_{\theta\in[-\tau,0]} \varphi(\theta)\}$ is the Yorke's functional.

The hypotheses imposed are weaker than the ones in the recent workers [1] and [2]. The results are applied to obtain several criteria for the global attractivity of the positive equilibrium for some well known "food-limited" population models with delay.

References:

[1] T. Faria, E. Liz, J.J. Oliveira and S. Trofimchuk, On a generalized Yorke condition for scalar delayed population model, *Disc. Cont. Dyn. Systems*, 12(3) (2005), 481-500.
[2] X. Zhang and J. Yan, Stability theorems for linear scalar delay differential equations, *J. Math. Anal. Appl.* 295 (2004), 473-484.

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Orthogonal Integration and Exponential Dichotomy

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In this talk we consider some recent results on backward and forward error analysis for the approximation of Lyapunov exponents and Sacker-Sell spectrum and their implications for determining exponential dichotomies.

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Subharmonic Solutions with Prescribed Minimal Period for a Class of Nonautonomous Hamiltonian Systems

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In this talk, I will present an existence result for subharmonic solutions with prescribed minimal period to a nonautonomous Hamiltonian system. The first existence result in this area goes back to Birkhoff and Lewis in 1933 by a perturbation technique, and a different proof was given by Moser in 1976. Their result was further improved by Rabinowitz, and Clarke and Ekeland in 1980's using calculus of variation techniques. The study of Hamiltonian systems with periodic nonlinearity was initiated by Conley and Zehnder in 1983, who proved existence of subharmonic solutions with minimal period by the use of Morse-Conley theory. Our approach will use a more effective decomposition technique to get a sharper estimation of the energy associated with the periodic solution in terms of its minimal period.

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