Special Session 44: Differential equations, dynamical systems and related applications

Chao-Nien Chen, National Changhua University of Education , TAIWAN, ROC
Yung-Sze Choi , University of Connecticut , USA

Lots of mathematical models involve differential equations and concepts from dynamical systems. This session plans to bring together researchers with varying backgrounds in theoretical analysis and practical applications. Such interactions intend to push the frontier of scientific understanding and open the doors for new and interesting mathematical research.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Reaction-diffusion systems with skew-gradient structure

Chao-nien Chen
National Changhua University of Education, Taiwan
macnchen@cc.ncue.edu.tw

There are many interesting patterns observed in reaction-diffusion systems of activator-inhibitor type. In this talk, we discuss stability of steady states of skew-gradient systems and related problems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A traveling domain solution

Yung S. Choi
University of Connecticut, USA
choi@math.uconn.edu
Roger Lui

Biological cells like fish keratocyte have been observed to move at (more or less) a steady velocity while keeping its general shape. What kinds of equations and boundary conditions should be prescribed for a moving boundary problem in order to generate such a configuration?

We have recently proved the existence of a domain $\Omega \subset \mathbf{R}^2$ that travels to the right at a constant speed k and a function b which solves an nonlinear elliptic equation in the domain with constant Dirichlet boundary condition. The unknowns include the shape of the domain Ω, the speed k and the solution b. As there is no analogue in the literature, we call them a traveling domain solution. Previously we have proved the well posedness and the existence of a traveling domain solution to a 1D cell motility model first suggested by Mogilner and Verzi. The current model, which involves curvature of the domain, is a generalization to 2D.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Multiplicity results for asymptotically linear second order boundary value problems with indefinite weights.

Francesca Dalbono
CMAF, Universidade de Lisboa, Portugal
dalbono@dm.unito.it

By using a topological approach, we prove the existence and multiplicity of solutions to an asymptotically linear second order equation with Dirichlet boundary conditions. The nonlinearity is required to satisfy asymmetric assumptions involving indefinite weights. The proof is based on some relation between rotation numbers and weighted eigenvalues.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Singular deformation of domains and solution structure of elliptic system

Shuichi Jimbo
Hokkaido University, Japan
jimbo@math.sci.hokudai.ac.jp

I deal with a domain with a very thin subregion and consider solution structre of elliptic system. I give its characterization in the limitting situation by the aid of minimax method.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Transonic 2D compressible potential flows

Eun Heui Kim
California State University Long Beach, USA
ekim4@csulb.edu

It has been conjectured by experiments and numerical studies that the self-similar multidimensional compressible flow changes its type, namely hyperbolic (supersonic) far from the origin and elliptic (subsonic) near the origin. Typically the boundary of the subsonic region is either unknown apriori (free boundary) or sonic (degenerate Dirichlet boundary). We discuss recent developments of

the analysis of such boundary value problems for potential flows.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Networks of Three-Identical Coupled Systems

Maria Leite
Purdue University, USA
mleite@math.purdue.edu
Martin Golubitsky

We consider networks of three-identical coupled systems of ODE's, where which system has at most two couplings. We show that there are 34 distinct networks of three-identical systems at most double coupled as opposed to only two such two-identical coupled systems of ODE's. We also show that, remarkably, transitions from a *synchonous equilibrium* that can occur are determined by the coupling structure of the network.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Note on the embedding properties for Weighted Sobolev spaces in unbounded domains

Hirokazu Ohya
Waseda University, Japan
ohya@aoni.waseda.jp

I am concerned with the embedding properties of certain functional spaces in unbounded domains. In this talk we discuss about Weighted-Sobolev (Banach) spaces with unbounded weight functions, especially for exponentially growing ones. In such cases, Escobedo and Kavian have shown the properties of continuous and compact embedding of Weighted-Sobolev (Hilbert) spaces with simple calculations. By using similar arguments given in the above study, we can derive (or relax) the sufficient conditions on these weight functions where the embedding properties work well. Such kind of properties are usually used in showing the existence of solutions for some quasilinear elliptic problems in unbounded domains. I also talk about some reason why the embedding of Weighted-Sobolev spaces is compact or not.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Rotations in a Car-Following Model

Tilman Seidel
Universität Hamburg, Germany
seidel@math.uni-hamburg.de
Bodo Werner

Consider a car-following model for N cars driving on a circular road. The movement of each car is described by a second order ODE depending on the headway to the car in front. We perturb this system of ODEs (in a way) periodically and show the existence of so-called *ponies on a merry-go-round* solutions.

For this particular type of rotations a stability and bifurcation analysis, connected to the theory of periodic solutions and dynamical systems, can be performed.

The cars on the highway realize the perturbation of the system as: road works.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Elasto-dynamical Systems with Friction Constrained Motions

Liejune Shiau
University of Houston-Clear Lake, USA
shiau@cl.uh.edu
Roland Glowinski

We address problems on modelling elasto-dynamical systems with friction using more sophisticated friction models and novel computational techniques. The new models give a better description of the system behavior particularly when the velocities are close to zero.

These investigations are motivated by the need for more accurate friction models in the software simulating the motion of mechanical systems, such as the remote manipulators of the Space Shuttle or of the International Space Station.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Global behavior of the branch of positive solutions for nonlinear Sturm-Liouville problems

Tetsutaro Shibata
Hiroshima University, Japan
shibata@amath.hiroshima-u.ac.jp

We consider the nonlinear Sturm-Liouville problem

$$-u''(t) + f(u(t)) = \lambda u(t), \;\; u(t) > 0, \quad t \in (0,1),$$
$$u(0) = u(1) = 0,$$

where $\lambda > 0$ is an eigenvalue parameter. To understand well the global behavior of the bifurcation branch in $\mathbf{R}_+ \times L^2(0,1)$, we establish the precise asymptotic formulas for $\lambda(\alpha)$, which is associated with eigenfunction u_α with $\|u_\alpha\|_2 = \alpha$, as $\alpha \to \infty$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nonexistence of eventually positive solutions of quasilinear elliptic systems

Hiroyuki Usami
Hiroshima University, Japan
usami@mis.hiroshima-u.ac.jp

We consider quasilinear elliptic systems in an exterior domain. Sufficient conditions for the nonexistence of eventually positive solutions of the systems are given. The proof is based on comparison principles and asymptotic analysis of ordinary differential systems associated with the elliptic systems.

$\longrightarrow \infty \diamond \infty \longleftarrow$