Special Session 45: Nonlinear water waves: phenomena and modelling

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The aim of this special session is a discussion of nonlinear phenomena in water wave propagation, such as extreme surface waves in deep water and large internal solitary waves in stratified fluids, involving tools that include laboratory experiments, numerical simulations, perturbation theory, and dynamical systems.

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The formation of rogue waves in NLS models: persistence of homoclinic orbits

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Rogue waves in deep water are investigated in the framework of the nonlinear Schroedinger (NLS) equation and the modified Dysthe (MD) equation. Phase modulation of higher order homoclinic solutions of the NLS equation can produce coalescence of unstable spatial modes, creating waves of maximal amplitude. Numerical simulations of the MD equation indicate that a chaotic regime increases the likelihood of rogue wave formation, and that enhanced focusing occurs due to chaotic phase evolution. In the second part of this two-part talk, we will investigate the persistence of higher order homoclinic structures in perturbed NLS models using Melnikov-type arguments, and show that such persistent structures are close to the observed maximal amplitude homoclinic solutions throughout the chaotic dynamics. Finally, we will discuss a possible relation between the occurrence of phase singularities in the wave form and the persistence results provided by the Melnikov analysis.

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Large internal waves in stratified fluids

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Ann Almgren and Amber Sallerson

Models that are capable of describing the strongly nonlinear internal waves frequently encountered in oceanic flows have recently been developed. Validation with lab experiments and direct numerical simulations will be presented and discussed, with some attention to the onset of instability phenomena with increasing amplitude.

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The Zakharov- Kuznetsov equation as a model for Rossby Waves

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The Zakharov- Kuznetsov (Z-K) equation is a generic envelope equation which balances nonlinear effects, dispersion and weakly two or three dimensionality. In this talk we show that the Z-K equation is also an asymptotic model for large two dimensional coherent structures in the atmosphere and the ocean known as Rossby Waves. We use modulation theory to study the evolution of a coherent structure and how it sheds dispersive radiation to evolve into a soliton solution.

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An internal splash: Levitation of Falling Spheres in Stratified Fluids

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We explore the motion of heavy spheres falling through a sharp salt stratified fluid layer in which an intriguing levitation phenomena is observed: the heavy sphere experiences a transient levitation in which the sphere descends through the sharp transition, stops, and rises back into the layer before ultimately returning to descent. Careful measurements will be presented showing the sphere residence time. The hydrodynamics, which involves a strong coupling between variable density fluid, and moving solid boundary, entrained, turbulently mixed fluid, and strong internal waves will be discussed.

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Highly Nonlinear Soliton Gas in Shallow Water Waves

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We develop new techniques for the numerical solutions of shallow water wave dynamics using the Korteweg-deVries (KdV) and Kadomtsev-PetviashvilWe (KP) equations for periodic/quasi-periodic boundary conditions. The methods are based upon the Riemann theta function solutions of these equations. Essentially the parameters of the theta functions are determined by systems of nonlinear equations based upon "theta functions with characteristics." Numerically one is able to determine the Riemann matrix, frequencies and phases using these equations for particular input space and/or time series. Fast convergence of the method is guaranteed by use of Poincare' series. The speed enhancement in the analysis results from recent development of a fast theta function transform (FTT) which allows vast improvements in computation time. Use of Riemann theta functions provides for a number of advantages over other methods for the study and integration of nonlinear wave equations (integrable and near integrable): (1) The physics is transparent on all levels and includes solitons, coherent and unstable modes, plus the usual Stokes waves and sine waves, etc. (2) Numerical tools allow for the time series analysis of laboratory and ocean wave data. (3) Numerical modeling techniques based upon Riemann theta functions can by considered to be fast in the same sense as the fast Fourier transform. Typical numerical simulations are 3 or 4 orders of magnitude faster than the split-step Fourier approach and are deemed "hyperfast." (4) Modeling of stochastic dynamics of nonlinear partial differential equation can be obtained to infinite order. This contrasts to finite closure arguments which are most often used to obtain and solve kinetic equations related to nonlinear PDEs. We give examples of ocean wave data which have been spectrally analyzed using the structure of the KdV and KP equations to demonstrate the "soliton gas" paradigm for shallow water waves.

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The formation of rogue waves in NLS models: modelling and phase singularities

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Rogue waves in deep water are investigated in the framework of the nonlinear Schroedinger (NLS) equation and the modified Dysthe (MD) equation. Phase modulation of higher order homoclinic solutions of the NLS equation can produce coalescence of unstable spatial modes, creating waves of maximal amplitude. Numerical simulations of the MD equation indicate that a chaotic regime increases the likelihood of rogue wave formation, and that enhanced focusing occurs due to chaotic phase evolution. In the first part of this two-part talk, we will discuss the models, higher order homoclinic solutions of the NLS equation, and show that the formation of extreme waves in random oceanic sea states characterized by JONSWAP power spectra is well predicted by the proximity to homoclinic solutions of the NLS equation. Finally, we will discuss recent work on the relation between wave amplification and phase singularities.

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