Special Session 46: Stochastic evolution equations with spatial structure and applications, from micro to macro scales

Roberto Camassa, UNC, USA
Brenton LeMesurier, Department of Mathematics,College of Charleston, Charleston SC, USA

Current plans are for this session to cover topics such as
- Nonlinear waves in molecular systems subject to random effects from Brownian motion
- Stochastic effects, turbulence and transport
- Experiments and modeling of dynamically similar flows: micro to macro fluidics of spinning cilia and rods
- Selected aspects of stochastic PDE's in financial mathematics

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nonlinear localization of light in disorderd optical fiber arrays

Alejandro B. Aceves
The University of New Mexico, USA
aceves@math.unm.edu
Alejandro B. Aceves and Gowri Srinivasan

Recent experiments in wave propagation along arrays of coupled nonlinear optical fibers have shown that the behavior is affected by non-linearity and randomness. The randomness is caused by imperfections in manufacturing that affect nearest neighbors coupling constants. The competition of nonlinear focusing, discrete diffraction and disorder determines if localization of light in a single output occurs. We present here both the theoretical and numerical studies of such light localization phenomenon for inputs at a single fiber and also at multiple fibers.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Evolution of passive scalar distributions in some basic deterministic fluid flows

Roberto Camassa
University of North Carolina, USA
camassa@amath.unc.edu
Neil Burrell and Rich McLaughlin

Mixing and transport of passive scalars is an important physical problem. Observed distributions of scalars in convection, the stratosphere and the ocean have heavy tails – large probabilities of large fluctuations. Mathematically, heavy tails have been found in random flows with and without chaos. Here we investigate distributions of scalars in deterministic flows. We see heavy tails for shear flows, cellular flows and chaotic flows in certain parameter regimes and take a first step towards explaining the appearance of such distributions by setting up an associated eigenvalue problem for simple shear flows.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Sensitivity analysis of financial options in jump-diffusion models

David Delphine
Université de la Rochelle, France
delphine.david@univ-lr.fr

Using Malliavin weights in a jump-diffusion model we obtain an expression for Theta (the sensitivity of an option price with respect to the time remaining until exercise), with application to European and Asian options with non-smooth payoff function. In time inhomogeneous models our formula applies to the derivative with respect to the maturity date T, and its proof can be viewed as a generalization of Dupire's PDE method to arbitrary payoff functions. In the time homogeneous case, our result applies to the derivative with respect to the current date t, but our representation formula differs from the one obtained from the Black-Scholes PDE in terms of Delta and Gamma. Optimal weights are computed by minimization of variance and numerical simulations are presented.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Wave energy localization by self-focusing in large molecular structures: a damped stochastic discrete nonlinear Schrödinger equation model

Brenton J. Lemesurier
Department of Mathematics, College of Charleston, South Carolina, USA
lemesurierb@cofc.edu
Barron Whitehead

Wave self-focusing in molecular systems subject to thermal effects, such as thin molecular films and long biomolecules, can be modeled by stochastic versions of the Discrete Self-Trapping equation of Eilbeck, Lomdahl and Scott, and this can be approximated by continuum limits in the form of stochastic nonlinear Schrödinger equations.

Previous studies directed at the SNLS approximations have indicated that the self-focusing of wave energy to highly localized states can be inhibited by phase noise (modeling thermal effects) and can be restored by phase damping (modeling heat radiation).

Here it is shown that the continuum limit is probably ill-posed in the presence of spatially uncorrelated noise, at least with little or no damping, so that discrete models need to be addressed directly. Also, as has been noted by other authors, omission of damping produces highly unphysical results.

Numerical results are presented here for the first time for the discrete models including the highly nonlinear damping term, and new numerical methods are introduced for this purpose.

Previous conjectures are in general confirmed, and the damping is shown to strongly stabilize the highly localized states of the discrete models. It appears that the previously noted inhibition of nonlinear wave phenomena by noise is an artifact of modeling that includes the effects of heat, but not of heat loss.

⟶ ∞◇∞ ⟵

Closed form expressions of the probability density function for passive scalar advection by random winds and shears

Zhi Lin
University of North Carolina at Chapel Hill, USA
zlin@email.unc.edu
Jared Bronski, Roberto Camassa and Richard McLaughlin

We explore the evolution of the Probability Density Functions (PDF) for the random Green's functions for a diffusing passive scalar, which is also advected by a velocity field with stochastic components. These stochastic components are rapidly varying in time and they can be spatially dependent such as shears. Different analytical methods are presented to compute closed-form expressions for these PDF's or their asymptotic limits. Although computed for simple flows, these exact solutions retain mechanisms which are physically relevant. Moreover, these solutions can be effectively used as tests for more general situations where one has to resort to numerical studies of the PDF dynamics because only moment information is available.

⟶ ∞◇∞ ⟵

Spinning Rods and Passive Tracers, from Nanoscale to Table-Top scale: coherent fluctuations in the presence of thermal noise

Richard M. Mclaughlin
Mathematics, University of North Carolina, USA
rmm@email.unc.edu
Roberto Camassa, Jing Hao, Terry Jo Leiterman, Richard Superfine, Leandra Vicci and Jonathan Toledo

We present experimental measurements and mathematical predictions concerning the hydrodynamics induced by spinning rods in viscous fluids. Recent advances in nano-technology have enabled controlled manipulation of nanoscale objects immersed in fluids. Such advances allow for new biological measurements (such as physical properties of cell membranes) on length scales smaller than the wavelength of visible light, and direct observations are challenging. Moreover, on such scales, the hydrodynamics are thermally fluctuating, and observational tracers experience strong Brownian signals on top of coherent motion induced by nanoscale manipulation. As such, predictive mathematical theories are essential to interpret the observations. We discuss the hydrodynamic solutions we have developed for rods sweeping upright cones in viscous fluids. These quasi steady, three dimensional, exact and asymptotic solutions of the Stokes equations are used to study the motion of passive tracers. These predictions are shown to quantitatively match low Reynolds number experiments performed on a scaled up, table top version of the nanoscale measurements performed. In turn, the predictions on the nano-scale are considered where agreement is good, but not as good as on the macro scale. The uncertainty of the nanoscale measurements as regards missing observational information and stochastic dynamics will be discussed.

⟶ ∞◇∞ ⟵

Evaluating First Passage Times in Stochastic Evolution Equations with Jump-Diffusions and Applications in Finance

Roderick Melnik
WLU, Waterloo, Canada
rmelnik@wlu.ca
Di Zhang

The first passage time problem has important applications ranging from modelling in biology, physics, and chemistry to the financial market analysis. In this contribution, we focus on the first passage time problem for jump-diffusion processes described by an evolutionary stochastic differential equation with constant drift and volatility and a Poisson-like discontinuous part. It is known that a standard Monte-Carlo run for the solution of this problem requires many thousands of time steps for each process path, while the evaluation of the density function itself would typically require many thousands of generated paths carried out by Monte-Carlo runs. In this talk we analyze a faster procedure, provide several test examples, and apply the developed methodology to pricing barrier options.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Stochastic Dynamics of Integrable, Nonlinear Partial Differential Wave Equations

Alfred R. Osborne
Universita' di Torino, Italy
al.osborne@gmail.com
M. Onorato and M. Serio

I discuss the stochastic dynamics of the Korteweg-deVries (KdV) and Kadomtsev-Petviashvili (KP) equations. Essentially the problems are cast with periodic/quasi-periodic boundary conditions and Riemann theta functions provide the spectral decomposition of these nonlinear PDEs. The stochastic nature of the equations is elicited by a technique common for linear PDEs in which the ordinary linear Fourier phases are taken to be uniformly distributed random numbers. In the nonlinear case the phases of the Riemann theta functions are taken to be random in the same way. Each set of random numbers thus provides a stochastic realization of a wave train which is characterized by a particular Riemann matrix. Each component in the Riemann spectrum is a "cnoidal wave" and therefore includes solitons, Stokes waves and sine waves in the formulation. A generalization of the correlation function is developed in terms of theta functions and the exact solution of the KdV and KP equations are evoked in terms of the time evolution of the power spectrum (derived analytically for the theta function solution of KdV) evolving from a particular Cauchy initial condition. The evolution of the power spectrum is carried out to infinite order, in contrast to the usual approach in which (gaussian) closure arguments are used to derive a kinetic equation which is then solved numerically to get the power spectrum evolution. I give numerical examples contrasting the finite closure kinetic equation solutions with the exact, infinite-order evolution for the KdV and KP equations. The finite closure arguments are found to destroy solitons, primarily because the individual soliton phase information is thrown away in the finite closure schemes. Instead, the infinite-order closure based on Riemann theta functions preserves the solitons perfectly. I discuss how the approach can also be applied to the study of "asymptotically integrable" evolution equations.

$\longrightarrow \infty \diamond \infty \longleftarrow$