Special Session 47: Applications of Dynamical Systems: Celestial Mechanics and Beyond

Marian Gidea, Northeastern Illinois University, USA
Josep Masdemont, Universitat Politecnica de Catalunya, Spain

This special session is devoted mainly to applications of dynamical systems to celestial mechanics, dynamical astronomy, space mission design, etc. The session will also discuss theoretical aspects that are relevant to these topics, such as invariant manifolds, homoclinic and heteroclinic connections, chaotic dynamics, transport, etc. Applications of dynamical systems to other areas, that share similar methods to those involved in the above topics, will also be considered.

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The dynamics around the collinear point L_3 of the RTBP

Esther Barrabés
Universitat de Barcelona, Spain
barrabes@ima.udg.es
J. M. Mondelo and M. Ollé

This talk aims towards the global description of the dynamics around the L_3 equilibrium point of the RTBP. The objects in its center manifold, including its normal behaviour, are computed by purely numerical procedures, in order to avoid the convergence restrictions of semi–analitical ones. Homoclinic phenomena is investigated. Small values of the mass parameter will be considered, in order to detect horseshoe–like motion.

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Low Energy Transfers In Space Using Chaos: Applications to Astrodynamics and Astronomy

Edward A. Belbruno
Princeton University, USA
belbruno@princeton.edu

In 1991 a Japanese spacecraft, *Hiten*, was rescued and brought to the Moon on a revolutionary new type of transfer trajectory that was very low energy. It was determined by utilizing a region about the Moon supporting chaotic motion, where capture can occur with no fuel (ballistic capture) unlike previous transfers which require a lot of fuel. The mathematical theory that accomplished this is called weak stability boundary(WSB) theory. Another transfer from this theory was used in 2004 to get the European spacecraft *SMART-1* to the Moon. The dynamics of the capture mechanism is associated, in part, to a complicated network of manifolds associated with chaotic motions near parabolic motion in the three-body problem. This methodology of determining low energy trajectories has recently been used in astronomy as part of a new theory for the formation of Moon, and also to provide a mechanism for the transfer of material between different solar systems in star clusters.

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Skew products of symplectic maps and almost collision orbits of the 3 body problem

Sergey Bolotin
UW-Madison, USA
bolotin@math.wisc.edu

We prove the existence of chaotic almost collision orbits of the plane restricted elliptic 3 body problem with small mass ratio and eccentricity shadowing chains of collision orbits of Kepler's problem. The proof is based on dynamics of random compositions of nearly integrable symplectic maps.

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Estimation of optimal time for low thrust transfers between elliptic orbits

Alex Bombrun
INRIA, France
alex.bombrun@sophia.inria.fr

We use saturated discontinuous feedbacks derived from Liapunov functions of the Keplerian fisrt integrals to estimate asymptotic behavior o f time optimal trajectory. We prove for all elliptic transfers that the product maximal thrust, optimal time remains bounded when the maximal thrust tends to zero.

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Heteroclinic Bifurcations and Chaotic Transport in the Two-Harmonic Standard Map

Renato C. Calleja

University of Texas at Austin, USA
rcalleja@math.utexas.edu
Hector Lomelí

We study a two-parameter family of standard maps: the so-called two-harmonic family. In particular, we study the areas of lobes formed by the stable and unstable manifolds. Variational methods are used to find heteroclinic orbits and their action. A specific pair of heteroclinic orbits is used to define a difference in action function and to study bifurcations in the stable and unstable manifolds. Using this idea, two phenomena are studied: the change of orientation of lobes and tangential intersections of stable and unstable manifolds.

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Solar Sailing near a collinear point

Ariadna Farres
Universitat de Barcelona, Spain
ari@maia.ub.es
Angel Jorba

In this note we consider the station keeping of a Solar sail near the L_1 point of the Earth-Sun system. As a first model we will use the classical Restricted Three-Body Problem but including the effect of the Solar sail. This effect depends on parameters, that reflect the orientation of the Solar sail. We look at the effects that these parameters have on the geometry of the phase space, and we will discuss how to use these modifications to derive control strategies.

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Dynamical Systems Approach to the Isomerization Problem of a Tri-Atomic Molecule

Frederic Gabern
Universitat Politècnica de Catalunya, Spain
frederic.gabern@upc.edu
W.S. Koon, J.E. Marsden and T. Yanao

In the last 25 years, techniques based in the theory of dynamical systems have been applied to spacecraft mission design and to astronomy problems with a great success. In this talk, we will present some ideas and results on how similar methods can be used to study some simple chemical reactions. In particular, we combine the effective computation of invariant manifolds (tubes) and Monte Carlo methods to determine chemical reaction rates and to study some scattering phenomena. As an example, we apply the methodology to a model for the isomerization problem of a tri-atomic molecule.

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Geometry of homoclinic connections in a planar circular restricted three-body problem

Marian Gidea
Northeastern Illinois University, USA
mgidea@neiu.edu
Josep J. Masdemont

The stable and unstable invariant manifolds associated with Lyapunov orbits about the libration point L_1 between the primaries in the planar circular restricted three-body problem with equal masses are considered. The behavior of the intersections of these invariant manifolds for values of the energy between the one of L_1 and that of the other collinear libration points L_2, L_3 is studied using symbolic dynamics. Homoclinic orbits are classified according to the number of turns about the primaries.

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A note on weak stability boundaries

Gerard Gomez
Universitat de Barcelona, Spain
gerard@maia.ub.es
F. Garcia

This paper is devoted to clarify the algorithmic definition of the weak stability boundary in the framework of the planar Restricted Three Body Problem. The role of the invariant hyperbolic manifolds associated to the central manifolds of the libration points L1 and L2, as boundary of the weak stability region, is shown.

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Computing long lifetime science orbits around natural satellites

Martin Lara
Real Observatorio de la Armada, Spain
mlara@roa.es
Sebastian Ferrer

The dynamics in the vicinity of planetary satellites is closely approximated by full Hill problems that consider the perturbation of the satellite's gravitational harmonics. The long-term behavior of a spacecraft around the satellite can be studied in the realm of perturbation theory. Two Lie transforms average the 3-DOF Hamiltonian to a 1-DOF one that depends on physical and dynamical parameters. For given values of the parameters contour plots of the averaged Hamiltonian represent the flow, which is made of equilibriums and closed curves. Stable relative equilibriums are ideal candidates for locating a spacecraft. However, science missions around natural satellites usually require almost circular, high inclination orbits, which are unstable because of perturbations of the planet. Dynamical systems theory gives assistance in increasing the span of science missions by using the manifolds associated to unstable orbits. Since the Lie-Deprit algorithm provides the transformation equations from the 1- to the 3-DOF problem, a simple inspection of the reduced phase space enables the choice of initial conditions of orbits of the non-averaged problem that, in average, tightly follow the stable-unstable manifold of the selected, unstable, relative equilibrium. An application to a future science mission to the Galilean moon Europa is presented.

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Hamiltonian dynamics of atom-diatomic molecule complexes and collisions

Florence J. Lin
University of Southern California, USA
fjlin@usc.edu

The classical molecule dynamics of an atom-diatomic molecule system is treated as a three-body Hamiltonian dynamical system. Applying techniques of geometric mechanics previously applied to two-body molecular dynamics [1] and to the dynamics of a triatomic molecule [2], the effects of large-amplitude motions are now described for atom-diatomic molecule systems. Just as a geometric phase arises in three-body classical mechanics and is described by the holonomy of a mechanical connection [3], a geometric phase also arises in three-body molecular dynamics and will be described by the holonomy of an analogous molecular connection. Physical consequences of this geometric phase include a contribution due to "internal" motions, such as bends and rotations, to (i) the rotation of a generalized Eckart frame in weakly-bound atom-diatomic molecule complexes and (ii) the scattering angle of the atom in atom-diatomic molecule collisions. The second consequence would be anticipated to be relatively large especially when an intermediate collision complex is relatively long-lived.

References

[1] F. J. Lin and J. E. Marsden, J. Math. Phys., vol. 33, pp. 1281 - 1294 (1992).

[2] F. J. Lin, Phys. Lett. A, vol. 234, pp. 291 - 300 (1997).

[3] J. E. Marsden, R. Montgomery, and T. Ratiu, Mem. Am. Math. Soc., vol. 88, no. 436 (American Mathematical Society, Providence, RI, 1990).

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Applications of invariant manifolds and variational principles in Economics

Hector E. Lomelí
Instituto Tecnologico Autonomo de Mexico (ITAM), Mexico
lomeli@itam.mx

The optimality principle of Bellman is frequently used to solve problems in dynamic programming. The optimal selection of the dynamic control is the optimal policy. An important observation is that there are areas of dynamics that use variational methods similar to the one of Bellman. In particular, it is possible to use a variational principle to approximate and to study an the stable and unstable manifolds of a saddle fixed point. In this work we explore the dynamic properties of the principle of Bellman.

As a starting point, we consider the work of Mackay et. al. in which they consider the problem of optimal scheduling in a periodic environment. This applies for instance to the problem of optimizing a shuttle bus service. Another example is the problem of choosing conditions in a dynamic programming problem so that following the optimal policy will make a system converge to an equilibrium point. In our case, we study the invariant manifolds that appear in connection with this problem and with the so called value function.

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A Methodology for the Computation of Heteroclinic Orbits between Invariant Tori about L_1 and L_2 in the Sun-Earth System

Josep J. Masdemont
Universitat Politecnica de Catalunya, Spain
josep@barquins.upc.edu
L. Arona and P. Roldan

In important space missions, the neighborhood of the so-called libration points of the Sun-Earth system, L_1 and L_2, are considered as privileged nominal places. Among the reasons we can cite their thermal stability, low radiation levels and almost constant geometry with respect to the Sun. In addition, the libration point region has a rich structure of orbits and hyperbolic manifolds which provide low cost channels to approach and leave them, or to transfer from one orbit to another.

Space transportation between libration point orbits must take advantage of homoclinic and heteroclinic orbits that exist between them. As an example, this natural way of transfer greatly benefited the Genesis mission in the return trajectory to Earth. In this talk we present a methodology to compute homoclinic and heteroclinic transfer orbits between invariant tori about L_1 and L_2 for energy values where suitable Lindstedt-Poincaré and normal form expansions can be obtained.

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An Interactive Software System for Mission Design and Trajectory Optimization

Cesar Ocampo
The University of Texas at Austin, USA
cesar.ocampo@mail.utexas.edu

We describe the theory, methods, and architecture for a software system currently capable of facilitating the trajectory design and optimization problem for planet or moon centered trajectories, libration point trajectories, Earth-Moon, or any Planet-Moon transfers, and all types of interplanetary and asteroid/comet missions. The system, called Copernicus, has been under development at the University of Texas for several years and is in its final phase of development. Under a unified framework, it is possible to examine simple single celestial body centered orbit problems, restricted three/four body problems, to more complex interplanetary and natural satellite trajectory tours. The system can be used to design natural orbits such as periodic or quasi-periodic orbits in restricted models, impulsive transfer trajectories, including ballistic capture, and complex powered and controlled trajectories based on either sub-optimal or optimal control methods. Though intended for operational use by NASA, its main sponsor, the system is a useful resource for interactive experimentation for celestial mechanics problems involving the dynamics of a single particle under the influence of one or more gravitating celestial bodies. A key element involves the use multiple coordinate systems, in particular, body-to-body rotating and rotating-pulsating frames, where the visualization of interesting trajectory characteristics are more evident.

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The Generalized Alignment Index (GALI) method: Detecting order and chaos in conservative dynamical systems

Charalampos Skokos
Observatoire de Paris, France
hskokos@imcce.fr

We investigate the dynamics of conservative dynamical systems by studying the evolution of volume elements formed by deviation vectors about their orbits. The behavior of these volumes is strongly influenced by the regular or chaotic nature of the motion. The different time evolution of these volumes can be used to identify rapidly and efficiently the nature of the dynamics, leading to the introduction of quantities that clearly distinguish between chaotic behavior and quasiperiodic motion. More specifically we define the Generalized Alignment Index of order k ($GALI_k$) as the volume of a generalized parallelepiped, whose edges are k initially linearly independent unit deviation vectors from the studied orbit. We show analytically and verify numerically on particular examples of N degree of freedom Hamiltonian systems and $2N$–dimensional symplectic mappings that, for chaotic orbits, $GALI_k$ tends exponentially to zero with exponents that involve the values of several Lyapunov exponents, while in the case of ordered orbits, $GALI_k$ fluctuates around non–zero values for $2 \leq k \leq N$ and goes to zero for $N < k \leq 2N$ following power laws that depend on the dimension of the torus and the number of deviation vectors initially tangent to it. The $GALI_k$ is a generalization of the Smaller Alignment Index (SALI) as $GALI_2 \propto$ SALI. However, $GALI_k$ provides significantly more information on the local dynamics of the system, allows for a faster and clearer distinction between order and chaos than SALI and works even in cases where the SALI method faced difficulties.

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Transport in the solar system - towards robust computations

Bianca Thiere
Universität Paderborn, Germany
thiere@math.upb.de
Kathrin Padberg and Michael Dellnitz

The analysis of transport phenomena in the solar system has received considerable interest in the last decade, in particular since the latest near-impact asteroid sightings. Recently, computational methods have been developed which allow to determine transport rates in a dynamical system, that is, the rate at which particles move from one particular region in phase space to another. These techniques are based on a set oriented approach for the analysis of nonlinear dynamical systems.

In this talk we give a summary of these techniques and demonstrate their applicability to the planar circular restricted three body problem with Sun and Jupiter as primaries. To be more precise, we estimate transport of (quasi-) Hilda asteroids to the Mars region, using fixed energy levels and an appropriately chosen Poincaré section. In a second step, we relax the energy restrictions. Based on these results we propose extensions to our numerical approach to ensure robustness of the transport computations with respect to small variations in the energy.

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