Special Session 4: Global and Exponential Attractors for Dissipative Dynamical Systems

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In these last years, a number of new results about the large time behavior of infinite-dimensional dissipative dynamical systems has been established. In particular, a deeper understanding of complex systems generated by nonlinear (and even nonautonomous) PDE has been reached. This session wants to be an up-to-date presentation of the state-of-the-art, with particular emphasis on the existence and stability properties of global and/or exponential attractors.

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Large time behavior of solutions to a dissipative boussinesq system

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We consider a damped forced Boussinesq equation, that model the propagation of a water wave. We prove that the dynamical system associated this equation features a compact global attractor that is smooth.

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Synchronization of random attractors for a stochastic reaction-diffusion system on a thin two-layer domain

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A system of semi-linear parabolic stochastic partial differential equations with additive space-time noise is considered on the union of thin bounded tubular domains $D_{1,\varepsilon}$ $:= \Gamma \times (0,\varepsilon)$ and $D_{2,\varepsilon} := \Gamma \times (-\varepsilon,0)$ joined at the common base $\Gamma \subset \mathbb{R}^d$ where $d \geq 1$. The equations are coupled by an interface condition on Γ which involves a reaction intensity $k(x',\varepsilon)$, where $x = (x',x_{d+1}) \in \mathbb{R}^{d+1}$ with $x' \in \Gamma$ and $|x_{d+1}| < \varepsilon$. Random influences are included through additive space-time Brownian motion, which depend only on the base spatial variable $x' \in \Gamma$ and not on the spatial variable x_{d+1} in the thin direction. Moreover, the noise is the same in the both layers $D_{1,\varepsilon}$ and $D_{2,\varepsilon}$. We prove existence of global random attractors and establish some limiting properties of them as the thinness parameter of the domain $\varepsilon \to 0$, i.e. as the initial domain becomes thinner, when the intensity function possesses the property $\lim_{\varepsilon\to 0}\varepsilon^{-1}k(x',\varepsilon) = +\infty$ In particular, the limiting dynamics is described by a single stochastic parabolic equation with the averaged diffusion coefficient, and nonlinearity term, which essentially indicates synchronization of the dynamics on both sides of the common base Γ.

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Dissipative equations in locally uniform spaces

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Following joint works [1], [2], [3], Cauchy problems in R^N are considered for a semilinear strongly damped wave equation, certain partly dissipative system, and second order parabolic equations. A dissipative semigroup of global solutions is constructed in the locally uniform spaces under the assumptions on the nonlinear term similar to those in bounded domains. Existence of a suitable notion of attractor is discussed.

References:
[1] J. M. Arrieta, J. W. Cholewa, Tomasz Dlotko, A. Rodriguez-Bernal, Linear parabolic equations in locally uniform spaces, Math. Models Methods in Appl. Sci. 14 (2004), 253-294.
[2] J. M. Arrieta, J. W. Cholewa, Tomasz Dlotko, A. Rodriguez-Bernal, Dissipative parabolic equations in locally uniform spaces, Math. Nachr., to appear.
[3] J. W. Cholewa, Tomasz Dlotko, Strongly damped wave equation in uniform spaces, Nonlinear Anal. 64 (2006), 174-187.

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Singular limit of differential systems with memory

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We consider differential systems with memory terms, expressed by convolution integrals which account for the past history of one or more variables. Focusing on the reaction-diffusion equation with memory as a model, we analyze the passage to the singular limit when the memory kernel collapses into a Dirac mass, leading formally to the classical heat equation. In particular we discuss the convergence of solutions on finite time-intervals and we establish convergence results of the global and the exponential attractors.

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Asymptotic compactness and attractors for models of compressible fluids

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The problems of the existence of bounded absorbing sets, asymptotic compactness, and attractors are discussed for various models of compressible viscous fluids. The role of the second law of thermodynamics is emphasized.

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Navier-Stokes limit of Jeffreys type flows

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This talk is devoted to a Jeffreys type model ruling the motion of a viscoelastic polymeric solution with linear memory in a two-dimensional domain with nonslip boundary conditions. For fixed values of the concentrations, we describe the asymptotic dynamics and we prove that, when the scaling parameter in the memory kernel (physically, the Weissenberg number of the flow) tends to zero, the model converges in an appropriate sense to the Navier-Stokes equations.

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Dissipative waterwaves equations

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We consider some asymptotical models for the propagation of waterwaves as nonlinear Schrodinger equations. We supplement these equations with a damping and a forcing term, and we discuss the issues of the existence of a global attractor for these equations, and of the asymptotical smoothing effect for the associated dynamical system.

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Attractors for a Cattaneo Model

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We study a semilinear Cattaneo system modelling a correlated random walk with reactions as introduced by Hillen. Under suitable assumptions one can show the existence of a global attractor. We discuss some modifications which are motivated by the applications from biology.

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Damped/driven Navier–Stokes system on large domains

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We consider the damped/driven 2D Navier–Stokes system in the two-dimensional periodic domain $\Omega = [0, L/\alpha] \times [0, L]$, $\alpha \leq 1$. The damping term μu is the Rayleigh (or Ekman) friction which plays an important role in geophysical models of atmospheric and oceanic circulation.

We obtain sharp estimates for the number of the degrees of freedom for this system (expressed in terms of the fractal dimension of the global attractor and the number of determining modes and nodes) as both $\alpha \to 0^+$ and $\nu \to 0^+$.

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Long time behavior of semilinear wave and plate equation with nonlinear dissipation and critical exponents.on the boundary.

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Semilinear wave and patae equation with nonlinear dissipation located either in the interior of the domain or on the boundary are considered. Distinct feauture of the problem is that both semilinear sources as well as nonlinear

dissipation are of ”critical exponents”. Thus, the problem under considertaion is hyperbolic like, without any source of compactnness and with strong nonlinear dissipation. The main results to be presented incliude: (i) existence of global attractors, (ii) uniform decay rates of trajectories to points of equilibria.In addition, under the condition that the dissipation at the origin has non-vanishing derivative, regularity and finite-dimensionality of attractors is established.

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Trajectory and global attractors for evolution equations with memory

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The long time behavior of equations with memory has been much studied in the recent years. The main difficulty, in order to treat such a problem, is that, due to the presence of the memory term (in general the time convolution of a linear operator applied to the unknown function with a suitable memory kernel), the system is nonlocal; furthermore, the values of the unknown function are known for all negative times.

A first method allowing to overcome these difficulties is based on adding a new variable, called past history, accounting for all the past values of the unknown function, so to obtain an autonomous system in an extended phase space, for which the existence of the global attractor can be studied.

Recently, a second approach, based on the notion of a trajectory attractor has been proposed. The idea, roughly speaking, consists in working on the phase space suggested by the initial data, namely, a space of semi-trajectories. One then considers the translation semigroup acting on this phase space and calls the trajectory attractor for the problem with memory the global attractor associated with the translation semigroup.

Although this second approach may work for more general forms of equations with memory, it is natural, when both approaches can be applied to a given problem, to study their mutual relationships. More precisely, our aim in this talk is to examine in detail the connection between the two types of attractors obtained by both approaches.

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Exponential attractor for ODEs with infinite delay

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We consider a system of ODEs with infinite delay. Under very general assumptions on the nonlinearity, we prove the existence of an exponential attractor and also estimate its dimension. Our main application is the system of ODEs with infinite delay which can be obtained from the 2d Navier-Stokes equations after suitable projection. We also briefly discuss the possible extensions to dissipative PDEs with infinite delay.

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Attractors for doubly nonlinear equations

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We shall investigate the equation $\alpha(u_t) - \Delta u + W'(u) = f$, where α is a smooth monotone function with $\alpha' \geq \sigma > 0$, f a source term, and W represents the derivative of a possibly nonconvex potential in u. The equation is complemented with the initial and (either Dirichlet or Neumann) boundary conditions.

After recalling some known results, we shall present some new developments in the study of the above problem, especially related to regularity of solutions and their long time properties (in particular, attractors). In particular, we shall deal with the (more difficult) case when α is fastly growing at infinity.

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Generalized semiflows and global attractors for evolution systems without uniqueness

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This talk reports on a research program, partly in collaboration with R. Rossi and U. Stefanelli, on the long time behaviour of evolution systems without a unique solution. Using the recent theory of Generalized Semiflows by John. M. Ball, we discuss about the construction of the global attractor for the Hyperbolic relaxation of the Cahn-Hilliard equation in 3-D, and for gradient flows of nonconvex functionals in Hilbert and in metric spaces.

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Global attractors for 2D Navier-Stokes equations in a strip in the class of spatially non-decaying solutions

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A weighted energy theory for Navier-Stokes equations in 2D strips is developed. Based on this theory, the existence of a solution, its uniqueness, dissipativity and existence of a global attractor in the uniformly local phase spaces (without any assumptions on spatial decay of the corresponding solutions) are verified. In particular, these phase spaces contain all of the 2D Poiseuille flows.

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Global Attractors for a Klein-Gordon-Schrödinger Type System

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We present certain contemporary trends in the theory of Klein-Gordon-Schrödinger type Systems. Then we give some resent results on the existence, uniqueness and asymptotic behavior of solutions for the following evolution system of Klein-Gordon-Schrödinger type

$$\begin{gathered} i\psi_t + \kappa\Delta\psi + i\alpha\psi = \phi\psi, \\ \phi_{tt} - \Delta\phi + \phi + \lambda\phi_t = -Re\psi_x, \\ \psi(v,0) = \psi_0(v),\, \phi(v,0) = \phi_0(v), \\ \phi_t(v,0) = \phi_1(v), \\ \psi(v,t) = \phi(v,t) = 0, v \in \partial\Omega,\ \ t > 0, \end{gathered}$$

where $x \in \Omega,\ t > 0,\ \kappa > 0, \alpha > 0, \lambda > 0$ and Ω is a bounded subset of $\mathbb{R}^n$, with $N \leq 3$. This certain system describes the nonlinear interaction between high frequency electron waves and low frequency ion plasma waves in a homogeneous magnetic field. We prove the existence of a global attractor in the strong topology of the space $(H_0^1(\Omega) \cap H^2(\Omega))^2 \times H_0^1(\Omega)$ which attracts all bounded sets of $(H_0^1(\Omega) \cap H^2(\Omega))^2 \times H_0^1(\Omega)$ in the norm topology and establish certain energy decay estimates under some parametric restrictions. Finally, we are going to present some upper estimates for the Hausdorff and Fractal dimensions of the global attractor.
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Global Regularity of the 3D Primitive Equations of Large Scale Ocean and Atmosphere Dynamics

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In this talk I will show the global well-posedness of the three-dimensional Primitive Equations of large scale ocean and atmosphere dynamics. I will also show that the long-term dynamics of this model is finite dimensional.

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