Special Session 5: Nonlinear Evolution Equations and Related Topics

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This session will focus on the recent developments in the theory of Nonlinear Evolution Equations and Related Topics including the theory of abstract evolution equations in Banach spaces as well as the studies of several types of nonlinear partial differential equations (the existence , regularity and asymptotic behavior of solutions).

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Periodic solutions of classes of abstract evolution equations

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We discuss the existence of periodic solutions to classes of first and second order evolution equations. Various applications are included.

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On certain fully nonlinear parabolic equation associated with the infinity-Laplacian

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This talk concerns a fully nonlinear parabolic equation of the form $u_t = \Delta_\infty u$, where Δ_∞ stands for the so-called infinity-Laplacian given by $\Delta_\infty u = \sum_{i,j=1}^N u_{x_i} u_{x_j} u_{x_i,x_j}$. The infinity-Laplacian was introduced by Aronsson to investigate the absolutely minimizing Lipschitz extensions into Ω of functions defined only on the boundary $\partial\Omega$, and there exists a great number of contributions to elliptic problems including $\Delta_\infty u$. However, parabolic problems associated with the infinity-Laplacian have not yet studied except a couple of works. We discuss various properties of solutions as well as the existence and uniqueness.

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Baire category and evolution differential inclusions

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The Baire's method has proven to be a useful tool to prove existence results for certain classes of nonlinear problems. This method will be used to establish the existence of mild solutions for the Cauchy problem, in a reflexive and separable Banach space E, of the form

$$(C_F) \qquad \dot{x} \in Ax(t) + F(t, x(t)) \quad x(t_0) = a \in E.$$

Here A is the infinitesimal generator of a C_0 semigroup of linear bounded operators on E and F is a multifunction defined on $[t_0, t_1] \times E$ with nonempty closed and bounded values contained in E.

Under appropriate assumptions on F we present some results of existence, relaxation and bang-bang for the Cauchy problem (C_F). The case in which $\overline{co}F(t,x)$ has nonempty interior is also considered. In our setting the approach is based on the following two steps: construction of a suitable nonempty closed subset M of mild solutions of the convexified problem $(C_{\overline{co}F})$, and proof that "most" mild solutions $x \in M$ are actually solutions of (C_F).

Some relations with the Gromov's method based on convex integration will be discussed as well.

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On the structure of attractors for a class of degenerate reaction-diffusion systems

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This talk deals with some properties of a quasi-linear degenerate parabolic system of partial differential equations arising in the continuous deterministic modelling of bacterial biofilm communities. In particular the structure of the global attractors of this class of systems is discussed. It is shown that the long time behaviour of solutions depends merely of one specific parameter, reducing in some sense this behaviour to that of the porous media equation.

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Nonexistence of global solutions of nonlinear Schödinger equations in non star-shaped domains

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In this talk, we discuss the nonexistence of global solutions of the following nonlinear Schödinger equations:

$$\text{(NLS)} \quad \begin{cases} i\dfrac{\partial u}{\partial t} &= \Delta u + |u|^{q-2}u \quad \text{in } \mathbf{R}\times\Omega, \\ u(0,x) &= u_0(x) \qquad\qquad x\in\Omega, \\ u|_{\partial\Omega} &= 0. \end{cases}$$

where Ω is a domain in $\mathbf{R}^N$ $(N \geq 1)$ and $q > 2$.

When $\Omega = \mathbf{R}^N$, there are many results concerning the nonexistence of global solutions (or existence of blow-up solutions) for the equation (NLS). For the case $\Omega \neq \mathbf{R}^N$, there are few studies of blowing-up conditions for (NLS). The main purpose of my talk is to discuss the nonexistence of global solutions of (NLS) with a deformed tube-shaped domain which is not a star-shaped domain.

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Dynamics of partially damped wave equations

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In this talk, we consider the global stability of the dynamics of partially damped wave equations. In particular, we show that the wave equation damped in the interior of the interval]0,1[and the wave equation damped on the boundary of]0,1[satisfy the Morse-Smale property (which implies in some sense the global stability of dynamics), generically with respect to the non-linearity. We also prove that the wave equation with boundary damping is the singular limit of the wave equation with an interior damping concentrating on the boundary. We study the convergence of the corresponding attractors and compare the dynamics restricted to the attractors.

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Modeling Quorum Sensing and Cell-Cell Communication

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Many bacteria developed a possibility to recognize aspects of their environment or to communicate with each other by chemical signals. One important case is the so-called Quorum Sensing (QS), a regulatory mechanism for the gene expression. The common belief is that the bacteria measure their own cell density by means of this signaling pathway. One of the best-studied species using QS is the marine luminescent bacterium *Vibrio fischeri* which is considered here as a model organism.

The dynamics of the two main regulatory pathways (LuxI/LuxR and AinS) is modeled by a ODE system. This system is analyzed thoroughly, considering bifurcation and hysteresis behavior and the biological meaning. This modeling approach can be applied to concrete experimental data and allows the estimation of model parameters. The experimental observation of very small amounts of signal substance densities can be explained by a stochastic model.

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Exponential attractors for a quasilinear parabolic equation

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Let Ω be an open bounded subset of $\mathbf{R}^\mathbf{N}$ with smooth boundary. We consider the following quasilinear parabolic equation:

$$u_t - \operatorname{div}\{(|\nabla u|^2+\varepsilon^2)^{(p-2)/2}\nabla u\} + f(u) = g(x),$$

where $\varepsilon > 0$ and $\max\{1, 2N/(N+2)\} < p < 2$, with the homogeneous Dirichlet boundary condition and the initial condition $u(\cdot,0) = u_0$.

Assume that $f(u) := f_1(u) + f_2(u)$ where f_1 is a possible multi-valued monotone nondecreasing function with $f_1(0) = 0$, and f_2 is locally Lipschitz continuous, and $g \in L^\infty(\Omega)$.

In the above settings, we construct an exponential attractor associated with this equation in the phase space $L^2(\Omega)$.

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Asymptotic analysis for Kirchhoff equation

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We will find asymptotic profiles for the Cauchy problem to Kirchhoff equation. More precisely, it will be shown that there exists a solution which has a decomposition into a free wave, a non-free wave and a remainder term, where the non-free wave is meant by the function which is not asymptotically free.

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Asymptotic Behavior of Solutions for Some Semilinear Heat Equations in Exterior Domains

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We consider the initial-boundary problem of some semilinear heat equations with subcritical nonlinear terms. The domain Ω to be considered here is the external domain $\Omega = \{x \in \mathbb{R}^N; \mathbb{R}^N \setminus \omega\}$, where ω is a star-shaped domain with respect to the origin of $\mathbb{R}^N$, and our interest here is the asymptotic behavior of the solutions for this problem. More precisely, we construct the stable set and the unstable set for this problem.

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Periodic problems of quasilinear elliptic-parabolic variational inequalities with time-dependent constraints

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Masahiro KUBO

In this talk we consider the periodic problems of quasilinear elliptic-parabolic variational inequalities with time-dependent constraints of the following form:
Problem (P)

$$u(t) \in K(t), \qquad t > 0,$$

$$(b(u)_t, u - v) + \int_\Omega a(x, b(u), \nabla u) \cdot \nabla (u - v) dx$$

$$\leq (f, u - v), \quad v \in K(t), \quad t > 0,$$

$$b(u(0,\cdot)) = b_0 \qquad \text{in } \Omega.$$

Here Ω is a bounded domain in $\mathbf{R}^N$ $(N \geq 1)$, $b : \mathbf{R} \to \mathbf{R}$ is a given bounded, nondecreasing and continuous function, the term $a(x,s,p)$ is a quasi-linear elliptic vector field satisfying some structure condition, in particular we assume $a(x,s,p) = \partial_p A(x,s,p)$ for a potential function $A : \Omega \times \mathbf{R} \times \mathbf{R}^N \to \mathbf{R}$, and $f(t,x)$ is a given function on $[0,\infty) \times \Omega$.

Assuming that the constraint $K(t)$ changes periodically in time, we prove the existence of periodic solutions. Moreover, we apply our general results to models of flows in partially saturated porous media.

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Attractors for the complex Ginzburg-Landau equation

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Noboru Okazawa

We consider the initial-boundary value problem for the complex Ginzburg-Landau equation (problem (CGL) for short). The equation has the form of the nonlinear heat equation with linear and nonlinear terms multiplied by complex coefficients, and so it is a mixed type model of the nonlinear heat equation and the nonlinear Schrödinger equation.
The first purpose of this talk is to establish general results on the existence and uniqueness of global strong solutions to problem (CGL). The results include the smoothing effect on the initial data. The second purpose is to discuss the existence of global attractors to problem (CGL). Our method here is based on an abstract theory formulated in terms of subdifferential operators.

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