Special Session 7: Differential inclusions

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The session is devoted to recent results in the theory of set-valued maps, especially for differential and variational inclusions. In a large way, the session will also include dynamical aspect of set-valued maps, numerical aspects for generalized equations, optimal control related to set-valued maps,...

Tentative topics:

-Fixed points of set-valued maps

-Differential and variational inclusions

-Viability theory

-Application to Mathematical economics, engineering,...

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On the Stability of Noncoercive Variational Inclusions and Applications

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In this talk, we will consider a general class of non-coercive variational inequalities involving a semi-coercive operator. We will give some stability results with respect to small uniform preturbations of the data involved in the problem. Some applications will be discused such as the stability of the elastic equilibrium in unilateral frictional problems in linear elasticity and the Signorini-Fichera problem.

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Dry friction and oscillation: results of stabilization in finite time

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Given a bounded open set $\Omega \subset \mathbb{R}^n$ and a continuous convex function $\Phi : L^2(\Omega) \to \mathbb{R}$, let us consider the following damped wave equation

$$(S) u_{tt} - \Delta u + \partial\Phi(u_t) \ni 0, \quad (t,x) \in \mathbb{R}_+ \times \Omega,$$

under Dirichlet boundary conditions. The notation $\partial\Phi$ refers to the subdifferential of Φ in the sense of convex analysis. The nonlinear term $\partial\Phi$ allows to modelize a large variety of friction models. Among them, the case $\Phi = |.|_{L^1}$ corresponds to a Coulomb friction, equal to the opposite of the velocity sign. Our purpose in this paper is to study the asymptotic properties of the dynamical system (S). We exhibit sufficient conditions which ensure the finite time stabilization of (S) toward some stationary solution u_∞.

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Convergence of the Proximal Point Method for Metrically Regular Mappings

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We study the convergence of a general version of the proximal point algorithm for solving the inclusion

$$T(x) \ni 0, \tag{1}$$

where T is a set-valued mapping acting from a Banach space X to the subsets of a Banach space Y. Choose a sequence of functions $g_n : X \to Y$ with $g_n(0) = 0$ and consider the following algorithm: given x_0 find a sequence x_n by applying the iteration

$$g_n(x_{n+1} - x_n) + T(x_{n+1}) \ni 0 \quad \text{for } n = 0, 1, 2, \ldots \tag{2}$$

We prove in this work that if x is a solution of (1) and the mapping T is metrically regular at x for 0 and with locally closed graph near $(x, 0)$, then, for any sequence of functions g_n that are Lipschitz continuous in a neighborhood U of the origin, the same for all n, and whose Lipschitz constants λ_n have supremum that is bounded by half the reciprocal of the modulus of regularity of T, there exists a neighborhood O of x such that for each initial point $x_0 \in O$ one can find a sequence x_n satisfying (2) which is superlinearly convergent to x in the norm of X.

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A secant–type method for generalized equations

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We present a study of the convergence of secant–type method for solving the abstract generalized equations in Banach spaces

$$0 \in f(x) + G(x) \tag{1}$$

where f is a continuous function from X into Y and G is a set–valued map from X to the subsets of Y with closed graph. X and Y are two Banach spaces. With different assumptions for divided differences, we obtain the quadratic, super–linear and linear convergence.

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Differential inclusions governed by subdifferentials of primal lower nice functions

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The topics involved in this communication are related to nonsmooth and variational analysis, along with theory of differential inclusions. We aim at studying some dynamical systems governed by a subdifferential operator of a non necessarily convex function. Both local and global existence results, as well as uniqueness properties of absolutely continuous solutions are obtained for a special class of lower semicontinous functions, namely, the one of primal lower nice (pln for short) functions in Hilbert spaces introduced by R. Poliquin in 1991. To summ up, we study perturbations of the inclusion

$$\dot{x}(t) + \partial_P f(x(t)) \ni 0 \text{ a.e. } t \geq 0,$$

where f is pln and $\partial_P f$ is the proximal subdifferential of f.

The asymptotic behavior of the related trajectories is also addressed in the (above) homogeneous case, offering connections with optimization theory.

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Existence of Fixed Points in Epilipschitz Sets on Hilbert Spaces

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We provide a new result of existence of fixed points for a single-valued Lipschitz function f on a closed set K of an Hilbert space. We suppose that K is compact locally the epigraph of a Lipschitz functions (such a set is called epilichitz set). The main point of our method lies in the fact that we do not impose that $f(x)$ is an "inward vector" for all point x of the boundary of K. This result is obtained through a technique of deformation of the boundary of the set by suitable differential inclusion and approximations of the Hilbert space by finite dimensionnal spaces.

References

[1] B. Cornet and M.-O. Czarnecki, *Necessary and sufficient conditions for the Existence of (generalized) equilibria on a compact epilipschitzian domain*, Communications on Applied Nonlinear Analysis, N. 7, pp.21-53 (2000).

[2] M. Kamenskii and M. Quincampoix *Existence of Equilibria on Epilipschitz sets without invariance conditions*, Fixed Point Theory and Applications, Vol.2005 , N. 3, pp. 257-279 (2005)

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Gradient flows of non convex functionals: existence and long-time behaviour results

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Giuseppe Savaré, Antonio Segatti and Ulisse Stefanelli

This talk focuses on a class of gradient flow equations for non convex functionals, for which existence results have been obtained in collaboration with G. Savaré . Then, the long-time behaviour of the solutions has been studied from the point of view of global attractors for generalized semiflows in a joint paper with A. Segatti and U. Stefanelli. Such existence and long-time behaviour results can be for instance applied to evolution problems arising in phase transitions.

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Multivalued exponential analysis and reachable sets of differential inclusions

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The exponentiation theory of linear continuous operators on Banach spaces can be extended in manifold ways to a multivalued context. We discuss the use multivalued exponentials in the characterization of reachable sets for differential inclusions

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Solutions set of boundary value problem for differential inclusions

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