Contributed Session 02: ODEs and Applications

Monte - Carlo Galerkin approximation of fractional stochastic integro-differential equation

Abdallah Badr
Alexandria University, Egypt
badrzoo@yahoo.com
Hanan Salem El-Hoety

A stochastic differential equation, SDE, describes the dynamics of a stochastic process defined on a space-time continuum. This paper reformulates the fractional stochastic integro-differential equation as a SDE. Existence and uniqueness of the solution to this equation is discussed. A numerical method for solving SDEs based on the Monte - Carlo Galerkin method is presented.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Geometry of invariant surfaces of Lotka-Volterra systems

Stephen Baigent
UCL, England
s.baigent@ucl.ac.uk

Three-dimensional strongly competitive Lotka-Volterra (LV) systems have globally attracting invariant surfaces known as carrying simplices. Using methods similar to those used for studying the evolution of interfaces, we study the geometry of these invariant surfaces by following the evolution of an initially planar surface under the LV flow. The evolving surface converges to the carrying simplex. By monitoring the evolution of the surface's 2nd fundamental form and normal map, we give examples where the entire carrying simplex is convex, concave or saddle-like. We also discuss how the geometry of the carrying simplex relates to the stability of fixed points of the flow, and how our results extend to more general Kolomogorov systems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Split-Lyapunov stabilty of Lotka-Volterra system

Stephen Baigent
UCL, England
s.baigent@ucl.ac.uk
Zhanyuan Hou

The Split Lyapunov method was introduced by M-L Zeeman and E.C. Zeeman to study global stability of interior fixed points of competitive Lotka-Volterra systems. Their approach uses the carrying simplex, an invariant manifold that attracts all points except the unstable origin. Here we describe two advancements: (i) We extend the Split Lyapunov method to deal with non-competitive systems, and (ii) we also deal with the stability of boundary fixed points. Moreover, our method does not rely upon carrying simplices, but does rely on permanence and partial permanence. We also show briefly how these stability results for strongly competitive systems - where the carrying simplex is known to exist - are related to the Gaussian curvature of the carrying simplex.

$\longrightarrow \infty \diamond \infty \longleftarrow$

New class of exact solutions for the equations of motion of a chain of n rigid bodies

Dmitriy Chebanov
City University of New York, USA
dchebanov@lagcc.cuny.edu

One of the classical directions of investigations in the theory of motion of a system of several coupled rigid bodies is concerned with particular cases of integrability of the equations of motion of the system. In comparison with the Euler and Poisson equations describing the dynamics of a single rigid body about a fixed point, the analytical study of mathematical models of a system of hinge-connected rigid bodies is a much more complicated problem due to the fact that the increase in the number of bodies constituting the system leads to the increase of both the number of mechanical parameters characterizing the system and the number of differential equations describing its motion.In this paper we construct a new class of nonstationary exact solutions for the equations of motion of a classical model of multibody dynamics – a chain of n heavy rigid bodies that are sequentially coupled by ideal spherical hinges. We establish sufficient conditions for the existence of the solutions and show how the motion equations can be integrated in the case when these conditions are fulfilled. The new class generalizes most of the exact solutions to the problem of chain's motion known so far.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence and uniqueness of linear functional differential equations with anticipation

Duane Chin-quee
Florida Institute of Technology, USA
dchinque@irsc.edu
Gnana Bhaskar Tenali

We consider functional differential equations of the form $\dot{u}+Mu = r$, $t \geq t_0$, $t_0 \in \mathbb{R}$. The anticipation operator M is a linear mapping $M : C(\mathbb{R}, \mathbb{E}) \to B(\mathbb{R}, \mathbb{E})$ where $\mathbb{E}$ is a Banach Space and B is the linear space of

all measurable functions $f : \mathbb{R} \to \mathbb{E}$ that are Bochner-Integrable on each compact interval. We establish an existence and uniqueness result with prescribed boundary conditions. An illustrative example will be presented.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Geometrical dissipation for dynamical systems

Dan Comanescu
West University of Timisoara, Romania
comanescu@math.uvt.ro
Petre Birtea

On a Riemannian manifold (M, g) we consider the $k+1$ functions $F_1, ..., F_k, G$ and construct the vector fields that conserve $F_1, ..., F_k$ and dissipate G with a prescribed rate. We study the geometry of these vector fields and prove that they are of gradient type on the regular leaves corresponding to $F_1, ..., F_k$. By using these constructions we show that the cubic Morrison dissipation and the Landau-Lifschitz equation can be formulated in a unitary form. The stability problem in the presence of the geometric dissipation is also presented.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Equality problems in a class of conjugate means

Judita Dascal
University of Luxembourg, Luxembourg
judita.dascal@uni.lu
Pál Burai, Zoltán Daróczy

A common generalization of the well-known arithmetic, geometric, harmonic, quasi-arithmetic and other classical means is the so-called L-conjugate mean introduced by Daróczy and Páles and it has the form

$$\varphi^{-1}(p\varphi(x) + q\varphi(y) + (1-p-q)\varphi(L(x,y))),$$

where φ, called a generating function, is a continuous strictly monotone function defined on an interval I, L is a fixed strict mean and p, q are parameters from the interval $]0,1]$.When characterizing a class of mean values, it is natural to ask under what necessary and sufficient conditions do two functions generate the same mean. This problem is called the equality problem in a given class of mean values. We will solve the equality problem of L-conjugate means of two variables, when L is a quasi-arithmetic mean. The investigations lead to solving composite functional equations and differential equations, sometimes making use of the software *Maple*® 14 to simplify the computation.

$\longrightarrow \infty \diamond \infty \longleftarrow$

The parametrization method of research and solving of boundary value problems for integro-differential equations

Dulat Dzhumabaev
Institute of Mathematics MES of Kazazkstan, Kazakhstan
dzhumabaev@list.ru

In report a linear two-point boundary value problem for Fredholm integro-differential equation is considered. The method bases on dividing interval and introducing of additional parameters is proposed. The necessary and sufficient conditions of solvability and unique solvability considered two-point boundary value problem are established.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Criterion of well-posed solvability of linear semi-periodical boundary value problem for system of loaded hyperbolic equations

Zhazira Kadirbayeva
Institute of Mathematics MES of Kazakhstan, Kazakhstan
apelman86pm@mail.ru
Dulat Dzhumabaev

The report is devoted to finding of necessary and sufficient conditions for well-posed solvability of linear semi-periodical boundary value problem for system of loaded hyperbolic equations with mixed derivative. In addition to the original problem appropriate family of periodical boundary value problems for systems of ordinary differential equations is considered. In terms of families of periodical boundary value problems the criterion of well-posed solvability of initial semi-periodical boundary value problem is obtained.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Constrained mechanics and idealized models for aquatic locomotion

Scott Kelly
University of North Carolina at Charlotte, USA
scott@kellyfish.net

The geometric structure underlying the mechanics of finite-dimensional systems subject to nonholonomic constraints has been detailed in recent years, particularly in the context of systems exhibiting symmetries, and control strategies exploiting this structure have been applied successfully to problems in wheeled robotic locomotion. Many features of such systems are paralleled in idealized models for aquatic locomotion in which propulsive vortex shedding is driven by the concurrent enforcement of velocity constraints (like Kutta conditions) and conservation laws. We examine aspects of this parallelism and their implications for motion control, focusing on single-input locomotion systems in which velocity constraints couple equations for the evolution of rotational momentum and translational momentum.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A fitted numerical method to solve a mathematical model describing TB dynamics

Kailash C. Patidar
University of the Western Cape, So Africa
kpatidar@uwc.ac.za

Mathematical models described by the autonomous systems of nonlinear ordinary dierential equations arising in biology are sometimes so complex that they cannot be solved analytically and one has to rely on efficient numerical integrators. However, conventional numerical methods like Euler, Runge-Kutta and even some popular MATLAB ode solvers fail to solve these nonlinear systems in the sense that they generate oscillations, chaos, and false steady states. In this talk, we will discuss the design and implementation of a new class of fitted finite difference methods to solve a mathematical model describing Mycobacterium tuberculosis transmission dynamics. The dynamics of this model are studied numerically using the qualitative theory of dynamical systems. We analyze this method for stability. Furthermore, we show that this method also preserves positivity of the solution which is one of the essential requirements when modeling epidemic diseases. To show the power of the proposed method, numerous comparison are made with the method that are commonly used by other researchers in the field.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Limit cycle existence and uniqueness in elementary piecewise linear continuous systems

Enrique Ponce
University of Sevilla, Spain
eponcem@us.es
J. Llibre, M. Ordoñez

Some techniques to show the existence and uniqueness of limit cycles, typically stated for smooth vector fields, are extended to continuous piecewise-linear differential systems. New results are obtained for systems with three linearity zones without symmetry and having one equilibrium point in the central region. We also revisit the case of systems with only two linear zones giving shorter proofs of known results.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Uniform asymptotic expanisons of solutions of a class of singularly perturbed boundary value problems

Susmita Sadhu
Southwest Minnesota State University, USA
susmita.sadhu@smsu.edu
J.B. McLeod

We will consider a class of singularly perturbed boundary value problems (BVPs):

$$\varepsilon y'' + 2y' + f(y) = 0, \quad y(0) = 0,\ y(A) = 0,$$

where $f \in C^2[0,\infty)$ is a positive function satisfying certain conditions. It can be shown that the BVP admits at most two solutions depending on A. The main goal of this talk is to rigorously prove a uniform asymptotic expansion of the "smaller" solution using an integral equation method, whenever the problem admits two solutions. To achieve our goal, we will prove an existence result that will ensure a uniform bound on the "smaller" solution and that would lead us to the asymptotics.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Criteria for determining the limit point case and limit circle case for singular Sturm-Liouville differential operators

Kishor Shinde
Sant Gadge Baba Amravati University, Maharashtra, India
shinde_kj1805@yahoo.co.in
S. M. Padhye

We extend a result on the criteria for determining the limit point and limit circle case for Sturm-Liouville differential operators. This forms a comparison theorem for limit point and limit circle case for Sturm-Liouville differential operators. These results can be used to analyze limit point case and limit circle cases at 0 for the function $q(x) = \frac{k}{x^p}$, for all k and p. We determine the values of k and p such that the Sturm-Liouville differential operator $\tau u = -u'' + qu$ is in limit point case or limit circle case at zero, where $q(x) = \frac{k}{x^p}$. τ is in the limit circle case when (i) $p < 0$ and for all k (ii) $p = 0$ and for all k (iii) $p > 0$ and $k \leq 0$ (iv) $p = 2$ and $0 < k < \frac{3}{4}$ (v) $0 < p < 2$ and for all $k > 0$ and τ is in the limit point case when (i) $p = 2$ and $k \geq \frac{3}{4}$ (ii) $p > 2$ and $k > 0$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Diffusion approximations for metapopulation models

Andrew Smith
University of Queensland, Australia
a.smith23@uq.edu.au
Ross McVinish, Phil Pollett

A metapopulation is a population that occupies several geographically distinct patches. Stochastic models are often employed in order to capture the randomness inherent in these populations. However, realistic models, which account for patch-specific dynamics and arbitrary spatial arrangement of patches, can be complicated and are often intractable. For this reason, we propose to approximate model behaviour using a diffusion process. We adopt a technique developed by Thomas Kurtz, which enables us to identify an appropriate approximating deterministic dynamical system. This system is then analysed to give qualitative results that relate back to the original stochastic model. Conditions under which the population persists, or otherwise becomes extinct, are presented.

$\longrightarrow \infty \diamond \infty \longleftarrow$