Contributed Session 09: PDEs and Applications

Solving certain PDE's using a generalized Hankel transform

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We introduce a generalized form of the Hankel transform, and study some of its properties. A partial differential equation associated with the problem of transport of a heavy pollutant (dust) from the ground level sources within the framework of the diffusion theory is treated by this integral transform. The pollutant concentration is expressed in terms of a given flux of dust from the ground surface to the atmosphere.

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Applications of mixed boundary conditions and Sturm Liouville equations in atherosclerosis

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We have solved the problem of the existence and uniqueness of the solutions for the pressure of the blood flow of Laplace's equation with the Dirichlet and Robin mixed boundary conditions. This mathematical problem is given as a model of the steady state diffusion of a large tracer molecule or molecular aggregate from the blood, across the vessel lining, comprised of a monolayer of endothelial cells that are tightly bound to one another, and into the vessel wall. The geometry of the problem is a rectangle in the Euclidean plane that is divided into two rectangles: media, and intima with the two solutions. By expanding the boundary conditions in terms of the eigenfunctions of Bessel functions used to represent the solution and applying the given boundary conditions, we found a relationship between the coefficients of these expansions. By using Brower's fixed point theorem for an operator equation we showed that the vector of coefficients of the solutions converges in the Banach space which proved the existence of an unique solution. Our proof supports a surprising result that was confirmed experimentally that the convergence rate increases as the size of the leaky juncture increases.

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Topological methods for boundary value problems involving discrete vector ϕ-Laplacians

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In this talk, using Brouwer degree arguments, we present some existence results for nonlinear problems of the type

$$-\nabla[\phi(\Delta x_m)] = g_m(x_m, \Delta x_m) \quad (1 \leq m \leq n-1),$$

submitted to Dirichlet, Neumann or periodic boundary conditions, where $\phi(x) = |x|^{p-2}x$ $(p > 1)$ or $\phi(x) = \frac{x}{\sqrt{1-|x|^2}}$, and $g_m : \mathbb{R}^N \to \mathbb{R}^N$ $(1 \leq m \leq n-1)$ are continuous nonlinearities satisfying some additional assumptions.This is a joint work with Cristian Bereanu.C. Bereanu, D.Gheorghe, Topological methods for boundary value problems involving discrete vector ϕ-Laplacians, *Topol. Methods Nonlinear Anal.* 38 (2011), 265-276.

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Poincare Sobolev equations in the hyperbolic space

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We have studied the a priori estimates, existence/nonexistence of radial sign changing solution, and the Palais-Smale characterisation of the problem $-\Delta_{B^N} u - \lambda u = |u|^{p-1}u, u \in H^1(B^N)$ in the hyperbolic space B^N where 1

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On solvability of balance equations for atmosphere dynamics

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Numerical atmosphere modeling includes computational solution of the PDEs expressing the laws of continuum media in the rotating reference frame. The governing equations support both relatively slow energy dominant processes and fast gravity and acoustic waves of small amplitude. Solution of such

a stiff system requires, in particular, definition of appropriate initial conditions, which usually are not adjusted dynamically, that gives rise to fast oscillations of great amplitude, which are not observed in the real atmosphere. An adjustment of the initial data for the atmospheric models usually leads to a set of diagnostic PDEs representing balance relations. One general problem in solving such diagnostic relations is non-ellipticity of the PDEs for some real atmospheric conditions that does not allow to formulate well posed boundary value problems. For example, the nonlinear balance equation by Charney is of the Monge-Ampere type and as such it is non-elliptic for given pressure function in the regions where anticylconic activity is rather strong. In this study, we present ellipticity conditions for more complex differential systems of nonlinear adjustment. Based on these results, we show distributionof non-elliptic regions in the gridded data of the actual atmospheric fields for different forms of the balance equations.

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Global regularity for the initial value problem of a 2-D Kazhikhov-Smagulov type model

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We prove global-in-time existence of regular solution to the initial value problem of a 2-D Kazhikov-Smagulov type model for incompressible nonhomogeneous fluids with mass diffusion.

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Regularization of ill-posed evolution problems in Hilbert space

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In recent literature, many results have been proved concerning the regularization of the backwards heat equation, the classic ill-posed problem. Such initial value problems may not have unique solutions for sufficiently many initial data and or solutions may not depend continuously on the initial data. The regularization of the backwards heat equation and other ill-posed problems then involves approximating a known solution of the problem for a given initial value by the solution of an approximate well-posed problem with the same initial data. In this paper, we generalize this process to prove regularization for abstract evolution problems in a Hilbert space H of the form $du/dt = A(t, D)u(t) + h(t)$, $0 \leq s \leq t \leq T$ with initial value $u(s) = x$, where $h(t)$ is an H-valued function and $A(t, D)$ is an operator depending on both $t \in [0, T]$ and a positive, self-adjoint operator D in H. We use operator theory to prove the regularization and apply the results to the inhomogeneous backwards heat equation with a time-dependent diffusion coefficient and other higher-order partial differential equations in $L^2(\mathbb{R})$ where $D = -\Delta$.

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Nonlinear hyperbolic balance laws coupled with ordinary differential equations

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Systems composed by PDEs and ODEs can be used for the description of complex phenomena, whose evolution has both macroscopic and microscopic behaviors. Similar multiscale models are used, for example, in the simulation of the blood flow in the human body: the main parts of the circulatory system are modeled by PDEs, while the remaining parts by ODEs. Other possible applications of such models are: traffic flows, supply chain, particles inside fluids. In the talk we present a nonlinear hyperbolic system of balance laws coupled with a system of ordinary differential equations. More precisely, the ordinary differential equations influence the solution to the balance laws by means of the boundary term and, at the same time, the balance laws modify the vector field of the ordinary differential equations. We discuss existence and well posedness of the Cauchy problem of this coupled system. This is a joint work with R. Borsche and R. M. Colombo.

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Some results in tomography

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We present recent work on the inverse problem of analytically inverting the attenuated x-ray transform on curves in two-dimensional regions of Euclidean space and more generally, simple Riemannian manifolds. The work is based on recasting the problem as a Beltrami equation with analytic dependence on a parameter to find a suitable integrating factor for the corresponding stationary transport equation. The problem originated in the medical imaging modality SPECT and has also recently arisen in the unique reconstruction of permittivity and permeability parameters of a conductive body made from external measurements.

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Temporal dispersion along uniform flow in one-dimensional porous media

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The use of water has increased year to year and the consumption rate varies among countries. Over exploitation of water resources without commensurate recharge has resulted in fall in groundwater table. Further, leaching of pollutants from disposal garbage sites, pesticides and fertilizers into the aquifer has degraded groundwater quality. In the present study, analytical solutions are obtained for temporal dispersion along uniform flow velocity in a one-dimensional semi-infinite domain. Initially the domain is not solute free. It is combination of exponentially increasing function of space variable and ratio of zero order production and first decay which are inversely proportion to dispersion coefficient. Retardation factor is also considered. Using Laplace transform technique to obtained the solutions. Numerical examples are given with different graphs.

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The boundary Harnack principle for second order elliptic equations with unbounded drift

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We derive the Carleson type estimates and the boundary Harnack principle for positive solutions of non-divergence, second order elliptic equations $Lu = a_{ij}D_{ij}u + b_iD_iu = 0$ in a bounded domain $\Omega \subset \mathbb{R}^n$. We assume that $b_i \in L^n(\Omega)$, and Ω is a twisted Hölder domain (THD) of order $\alpha \in (0, 1]$. In addition, Ω satisfies a strong regularity condition if $\alpha \in (0, 1/2]$, and a weak regularity condition if $\alpha \in (1/2, 1]$.

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Analytical approach of one - dimensional solute transport through inhomogeneous semi-infinite porous domain for unsteady flow: dispersion being proportional to square of velocity

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In the present work, analytical solution of one-dimensional solute transport for a pulse type point source for uniform nature along the unsteady flow through inhomogeneous semi-infinite porous medium along longitudinal directions. According to Scheidgger (1957), dispersion is considered directly proportional to the square of velocity as the linear spatially dependent function defining the inhomogeneity and temporally dependent function. It is expressed in degenerate form. Initially the domain is solute free. The input condition is considered pulse type and introduced at the origin of the domain. Other condition is considered flux type at the end of the domain. Certain new independent variables are introduced through separate transformation to eliminate the variable coefficients of Advection Diffusion Equation (ADE) into constant coefficients. Then Laplace transform technique (LTT) is used to get the analytical solution of ADE. It is illustrated for exponentially decreasing and increasing time dependent functions and concentration profiles are shown graphically.

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Evaluation of the stochastic modeling on options

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Modern option pricing techniques are often considered among the most mathematically complex of all applied areas of financial engineering. In particular these techniques derive their impetus from four milestone of option pricing models: Bachelier model, Samuelson model, Black-Scholes-Merton model and Levy model. In this paper we evaluate all related option pricing models based on these milestones, by comparing the corresponding stochastic differential equations and option pricing formulas. In addition we also include some simulations to make the comparisons more transparent.

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Group classisfication of nonlinear equations on different surfaces

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A complete group classification for the (1 + n)-dimensional Klein-Gordon equationwhen n = 2, 3 is presented. Then using these results we extend the classification tothe general case when n is arbitrary. It is shown that there is a class of functionswhich is independent of the number of independent variables for which the groupclassification exists. Symmetry generators, up to equivalence transformations,

arecalculated for each f(u) when the principal Lie algebra extends. Moreover the groupclassification of the (1 + 2)-dimensional Klein-Gordon equations on the sphere (S2)and torus (T2) are discussed. Classifications of these symmetry algebras are obtainedup to conjugacy classes and similarity reduction for each class is given. The effect off(u) and the underlying space on the infinitesimal generators is also discussed. Wealso comment on the group classification of the (1 + n)-dimensional Klein-Gordonequation on Sn and Tn.

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A complex Noether approach for derivation of conservation laws for partial differential equations in complex field

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We study scalar complex partial differential equations with two real independent variables which admit variational formulations. Such a complex partial differential equation, via a complex split of the dependent variable, splits into a system of two real partial differential equations. The decomposition of the Lagrangian of the complex partial differential equation in the real domain yields two real Lagrangians for the split system. The complex Maxwellian distribution equation, transonic gas flow equation, Maxwellian tail, wave equation with dissipation and Klein-Gordan equation are considered. The Noether symmetries and gauge terms of the split system corresponding to both Lagrangians are constructed by the Noether approach. We compare the Noether symmetries and gauge terms of the split system with the split Noether-like operators and gauge terms of the given complex partial differential equation in the real domain. We conclude that the split Noether-like operators and gauge terms of the complex partial differential equation are not in general the same as the Noether symmetries and gauge terms of the split system of real partial differential equations. They are the same when all the Noether symmetries of the complex partial differential equation has either pure real or pure imaginary form. Furthermore, the split conserved vectors of the complex partial differential equation are the same as the conserved vectors of the split system of real partial differential equations in the case of coupled systems.

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Attractors for semilinear parabolic problems with concentrated and oscillating terms on the boundary

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In this talk we analyze the dynamics of the flow generated by a nonlinear parabolic problem when some reaction and potential terms are concentrated in a neighborhood of the boundary. We assume that this neighborhood shrinks to the boundary as a parameter ϵ goes to zero. Also, we suppose that the "inner boundary" of this neighborhood presents a highly oscillatory behavior. Our main goal here is to show the continuity of the family of attractors with respect to ϵ. Indeed, we use abstract results from [A. L. Pereira and M. C. Pereira, *Continuity of attractors for a reaction-diffusion problem with nonlinear boundary conditions with respect to variations of the domain*, Journal of Differential Equations **239** (2007), 343-370] to extend results from [A. Jiménez-Casas and A. Rodríguez-Bernal, *Asymptotic behaviour of a parabolic problem with terms concentrated in the boundary*, Nonlinear Analysis: Theory, Methods & Applications **71** (2009), 2377-2383] to a parabolic problem in which the "inner boundary" of ω_ϵ presents a highly oscillatory behavior, and assuming hyperbolicity of the equilibria of the limit problem, we also obtain results on the lower semicontinuity of the attractors.

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Energy decay of Klein - Gordon - Schrödinger type with linear memory term

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This paper is concerned with the existence, uniqueness and uniform decay of the solutions of a Klein-Gordon-Schrödinger type system with linear memory term. The existence is proved by means of the Faedo-Galerkin method and the asymptotic behavior is obtained by making use of the multiplier technique combined with integral inequalities.

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Note on "common fixed point results for noncommuting mappings without continuity in cone metric spaces"

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In 2007, Huang and Zhang introduced the concept of the cone metric space, replacing the set of real numbers by an ordered Banach space, and they showed some fixed point theorems of contractive type mappings in cone metric spaces. The concept of w-distance in complete metric spaces was defined by Kada et al. in 1996. Then Cho et al. Wang and Guo defined a concept of the c-distance in a cone metric space, which is a cone version of the w-distance and proved some fixed point theorems in ordered cone metric spaces. In this talk, we investigate to a common fixed point theorem by using the generalized distance in a cone metric space. Our theorems extend some results of Abbas and Jungck [M. Abbas, G. Jungck, Common fixed point results for noncommuting mappings without continuity in cone metric spaces, J.Math. Anal. Appl. 341 (2008) 416-420] and Cho et al. [Y.J. Cho, R. Saadati, S.H. Wang, Common fixed point theorems on generalized distance in ordered cone metric spaces, Comput. Math. Appl.61 (2011) 1254-1260].

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On ratio-dependent predator-prey systems with disease in the prey

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In this talk, we consider ratio-dependent predator-prey systems with disease in the prey under Neumann boundary conditions. we investigate the properties of nonnegative solutions to the reaction-diffusion systems. Furthermore, we provide sufficient conditions for the existence of non-constant positive steady-state solutions.

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Spectrally matched optimal grids for receiver-targeted PDE problems

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In this talk, we will present an introduction to the construction of non-linear spectrally matched optimal grids for solving receiver-targeted PDE problems such as in geophysical exploration. This involves a clever rational approximation of the NtD (Neumann to Dirichlet) map. The error convergence rate at the receiver locations will be investigated for a 2-d isotropic and a 1-d anisotropic problem. We will look at an application of this method to Laplace's equation on a rectangle with unbounded spectrum for the Neumann data will be illustrated along with numerical evidence.

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A homogenization problem for a thin domain with a highly oscillating boundary

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"A homogenization problem for a thin domain with a highly oscillating boundary"We combine methods from linear homogenization theory to obtain convergence rates for solutions of the Poisson Equation with Neumann boundary conditions posed in an one parameter family of two dimensional domains with highly oscillating boundary and which collapse on one dimensional set as the parameter goes to 0.

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Dynamics in the nonlinearly excited 6th-order phase equation

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Excitable media behaving in oscillatory manner can often be simulated by a single active-dissipative partial differential equation for the phase of oscillations. The equation reduces the complex phenomenon into a relatively concise form, examples being the Kuramoto-Sivashinsky equation describing unstable flame fronts and the Nikolaevskiy equation describing resonances in seismic waves in fluid-saturated rocks. The both equations incorporate excitation by a linear mechanism. This mechanism has been studied quite extensively by now, whereas the nonlinear mechanism remains under-explored. In the present work we study three-dimensional dynamics in the nonlinearly excited phase equation where dissipation is represented by the 6th-order spatial derivative. Numerical solutions are obtained for the phase evolving in a rectangular spatial domain. Regular and chaotic modes of the evolution are discussed.

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Approximate analytical solution of reaction-diffusion Brusselator system with fractional time derivative

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Canan Unlu, Sunil Kumar, Yasir Khan, Ahmet Yildirim

In this letter, we used homotopy perturbation method(HPM) to obtain approximate analytical solutions of reaction-diffusion Brusselator system with fractional time derivative. Fractional reaction-diffusion Brusselator system has important applications in chemical reaction-diffusion processes. We present numerical results with graphically and we see that the homotopy perturbation method is an effective and convenient method to solve fractional-order reaction diffusion Brusselator system.

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Influence of gravity and initial stress on the Torsional wave propagation in a Substratum over a dry sandy gibson half space

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The propagation of Torsional surface waves in an inhomogeneous anisotropic substratum lying over a dry sandy gravitating gibson half space in the presence of initial stress have been studied analytically and computed numerically. The dispersion equation has been derived in a closed form using Whittaker's function and its derivative. The influence of various inhomogeneity parameters on the propagation of Torsional surface wave has been described under the effect of gravity and compressive initial stress by means of graphs. The influence of Biot's gravity parameter and sandy parameter has also been shown on the Torsional surface wave propagation. Dispersion equations are in perfect agreement with the standard results when derived for some particular cases. It is observed that the presence of gravity field always allow the Torsional surface wave to propagate. It has also been concluded that the Torsional surface wave propagates more smoothly in the layer when the lower gibson half space is elastic in comparison to dry sandy gibson half space. Further, anisotropy has also much effect in enhancing the velocity of Torsional surface wave.Graphical user interface (GUI) software has been developed using MATLAB to generalize the effect of various parameter discussed.

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Sharp blow-up for semilinear wave equations with non-compactly supported data

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In this talk, I will introduce a correction of Asakura's observation on semilinear wave equations with non-compactly supported data by showing a sharp blow-up theorem for classical solutions. We know that there is no global in time solution for any power nonlinearity if the spatial decay of the initial data is weak, in spite of finite propagation speed of the linear wave. Our blow-up theorem clarifies the final criterion on such a phenomenon. I will discuss the assumption on the data in various forms.

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