# Special Session 15: Nonlinear Evolution Equations, Inclusions and Related Topics

Mitsuharu Otani, Waseda University, Japan Tohru Ozawa, Waseda University, Japan
N. U. Ahmed, University of Ottawa, Canada
S. Migorski, Jagiellonian University, Poland
I. I. Vrabie, Al. I. Cuza University, Romania

This session will focus on the recent developments in the theory of Nonlinear Evolution Equations and Related Topics including the theory of abstract evolution equations in Banach spaces as well as the studies (the existence, regularity and asymptotic behaviour of solutions) of various types of Nonlinear Partial Differential Equations. Some new trends in both abstract linear and nonlinear evolution inclusions as well as concrete applications in Control Theory, Partial Differential Equations, Variational Inequalities, and mathematical models described by evolution inclusions will be disscussed. Open problems and new results in the above mentioned areas based on a interplay between various disciplines as Multi-Valued Analysis, Measure Theory, Fourier Analysis and Spectral Theory, will find their place within this Special Session.

Optimal feedback control for differential inclusions on Banach spaces

Nasir Uddin Ahmed

University of Ottawa, Canada ahmed@site.uottawa.ca

In this paper we consider a class of partially observed semilinear dynamic systems on infinite dimensional Banach spaces governed by differential inclusion representing measurement uncertainty. In the paper we present a result on existence of an optimal operator valued function as the feedback control law minimizing the maximum risk. We also present necessary conditions of optimality for the feedback control law from a class of operator valued functions defined on finite intervals and taking values from the space of compact operators. This is then extended to cover systems with uncertainty in the dynamics itself. The necessary conditions remain valid also for a class of stochastic systems driven by Wiener martingale.

[1] N.U.Ahmed, Optimal Relaxed Controls for Systems Governed by Impulsive Differential Inclusions, Nonlinear Funct. Anal. & Appl., 10(3), (2005), 427-460.

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Doubly nonlinear parabolic equations with variable exponents

Goro Akagi Kobe University, Japan akagi@port.kobe-u.ac.jp

This talk is concerned with doubly nonlinear parabolic equations involving variable exponents. Let  $\Omega$  be a bounded domain in  $\mathbb{R}^d$  with smooth boundary  $\partial\Omega$ . We discuss the following doubly non-

linear parabolic problem (P):

$$\begin{split} \partial_t \left( |u|^{m(x)-2} u \right) - \Delta_{p(x)} u &= 0 \quad \text{ in } \ \Omega \times (0,T), \\ |u|^{m(\cdot)-2} u &= v_0 \quad \text{ on } \ \Omega \times \{0\}, \end{split}$$

where  $\partial_t = \partial/\partial t$ , T > 0,  $v_0 = v_0(x)$  is a given initial data and  $\Delta_{p(x)}$  is the so-called p(x)-Laplacian, a typical example of nonlinearity with variable exponents, together with either the Dirichlet condition

$$u = 0$$
 on  $\partial \Omega \times (0, T)$ ,

or the Neumann condition

 $|\nabla u|^{p(x)-2}\partial_n u = 0 \quad \text{on} \ \partial\Omega \times (0,T),$ 

where  $\partial_n u$  denotes the outward normal derivative of u on  $\partial\Omega$ . The existence of solutions is proved by developing an abstract theory on doubly nonlinear evolution equations governed by gradient operators. In contrast to constant exponent cases, two nonlinear terms have inhomogeneous growth and some difficulty may occur in establishing energy estimates. Our method of proof relies on an efficient use of Legendre-Fenchel transforms of convex functionals and a modified energy method.

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Existence for a class of nonlinear delay reaction-diffusion systems

Monica-Dana Burlica Alexandru Ioan Cuza University Iasi & Gh. Asachi Technical University Iasi, Romania monicaburlica@yahoo.com Daniela Rosu

We consider an abstract nonlinear multi-valued reaction-diffusion system with delay and, using some compactness arguments coupled with metric fixed point techniques, we prove some sufficient conditions for the existence of at least one  $C^0$ -solution. A global

asymptotic stability result is also obtained and some specific examples are analyzed.

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Dynamic analysis of a nonlinear Timoshenko equation

### Jorge Esquivel-Avila

Universidad Autonoma Metropolitana, Mexico jaea@correo.azc.uam.mx

An initial boundary value problem in a bounded domain for the Timoshenko equation with a nonlinear damping and a nonlinear source term is considered. Exploiting an idea of a potential well under suitable assumptions on the damping and the source terms, we prove blow-up as well as convergence to the zero and nonzero equilibria and we give rates of decay to the zero equilibrium.

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Leadership in crowd dynamics: modelling via two-scale interactions

Joep Evers Eindhoven University of Technology, Netherlands j.h.m.evers@tue.nl Adrian Muntean

We describe a two-scale model for the dynamics of a crowd by distinguishing between a number of leaders considered at the micro-scale and a large crowd viewed macroscopically. Leaders are of particular interest in view of crowd control. We encounter the following fundamental questions: - How to describe a leader in mathematical terms? - Does the presence of a leader essentially change the macro-scale behaviour? - Which specific macroscopic patterns can be caused by the leader(s)? - How can we validate our two-scale model with experimental data? The crowd in our model consists of two subpopulations in a corridor, assumed to be two-dimensional. To capture the evolution of the crowd, we use a timedependent mass measure for each subpopulation. Each of the two mass measures satisfies the continuity equation. Hence, their evolution is governed by the partial velocity field, which we have to provide. The actual modelling consists therefore in finding the right velocity field, which we expect to depend functionally on both mass measures. In this talk, I explain the model we use, and the particular form we choose for the velocity field. Furthermore I give a flavour of the crowd-dynamical phenomena that arise from this model.

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Nodal and multiple constant sign solutions for equations with the p-Laplacian

Michael Filippakis University of Piraeus, Greece mfilip@unipi.gr Ravi P. Agarwal, Donal O'Regan, Nikolaos S. Papageorgiou

We consider nonlinear elliptic equations driven by the p-Laplacian with a nonsmooth potential (hemivariational inequalities). We obtain the existence of multiple nontrivial solutions and we determine their sign (one positive,one negative and the third nodal). Our approach uses nonsmooth critical point theory coupled with the method of upper-lower solutions.

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Asymptotic behavior of blow-up solutions for the heat equations with nonlinear boundary conditions

Junichi Harada Waseda University, Japan harada-j@aoni.waseda.jp

We consider the heat equation equation on the half space with nonlinear boundary conditions  $(u_t = \Delta u, \partial_{\nu} u = u^q)$ . Here we discuss the asymptotic behavior of blow-up solutions for the Sobolev subcritical case. In particular, we provide a sufficient condition on the initial data for the single point blow-up at the origin. Furthermore we study the spacial singularities of the blow-up profile.

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Pohozaev-Ôtani type inequalities for weak solutions of some quasilinear elliptic equations in unbounded domains

Takahiro Hashimoto Aichi Medical University, Japan takahi@aichi-med-u.ac.jp

In this talk, we are concerned with the following quasilinear elliptic equations:

(E) 
$$\begin{cases} -\operatorname{div}\left\{a(x)|\nabla u|^{p-2}\nabla u\right\} = b(x)|u|^{q-2}u & \text{ in } \Omega, \\ u = 0 & \text{ on } \partial\Omega \end{cases}$$

where  $\Omega$  is a domain in  $\mathbf{R}^N$   $(N \ge 2)$  with smooth boundary,  $1 < p, q < \infty$ ,  $a(x), b(x) \ge 0$ .

When  $a(x) = b(x) \equiv 1$ , a complementary existence result was founded for the problem of interior, exterior and whole space. The main purpose of this talk is to obtain the nonexistence results via Pohozaev-Ôtani type inequalities for some class of weak solutions of (E). We also discuss the existence result for exterior problem and the regularity of solutions of (E).

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### Fractional Sobolev spaces via Riemann - Liouville derivatives and some imbeddings

Dariusz Idczak University of Lodz, Poland idczak@math.uni.lodz.pl Stanislaw Walczak

Using the Riemann - Liouville derivatives of test functions we introduce fractional Sobolev spaces of functions of one variable, defined on a bounded interval. Next, we characterize elements of these spaces with the aid of the Riemann - Liouville derivatives as well as with the aid of some integral representations. We also study some imbeddings of the introduced spaces and prove their compactness.

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Variational problems associated with Trudinger-Moser inequalities in unbounded domains

Michinori Ishiwata Fukushima University, Japan ishiwata@sss.fukushima-u.ac.jp

In this talk, we are concerned with the existence and the nonexistence of maximizers for variational problems associated with the Trudinger-Moser inequality in unbounded domains. Particularly, we are interested in the variational problems for a version of TM-typeinequality with singular weight functions.

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Well-posedness for anisotropic degenerate parabolic equations with non-homogeneous boundary conditions

Kazuo Kobayasi Waseda University, Japan kzokoba@waseda.jp

We study the well-posedness for anisotropic degenerate parabolic-hyperbolic equations with initial and non-homogeneous boundary conditions. We prove a comparison theorem for any entropy sub- and super-solution, which immediately deduces the  $L^1$ contractivity and therefore, uniqueness of entropy solutions. The method used here is based upon the kinetic formulation and the kinetic techniques developed by Lions, Perthame and Tadmor. By adapting and modifying those methods to the case of Dirichlet boundary problems for degenerate parabolic equations we can establish a comparison property. Moreover, in the quasi-isotropic case the existence of entropy solutions is proved.

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## A new class of nonlinear evolution equations

### Masahiro Kubo

Nagoya Institute of Technology, Japan kubo.masahiro@nitech.ac.jp

Codifying applications of subdifferential evolution equations to variational inequalites, we introduce a new class of evolution equations associate with subdifferentials depending on unknonwn functions. We consider three kinds of evolution equations: parabolic (I), parabolic (II) and hyperbolic evolution equations; and two types of dependence of the subdifferentials on the unknown functions: local and non-local types of dependence. We prove the existence of a solution to a hyperbolic problem with a non-local type dependence on unknown functions and study its singular limit to a solution to a parabolic (II) problem.

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Existence and non existence of solutions to initial boundary value problems for nonlinear evolution equations with strong dissipation

Akisato Kubo Fujita Health University, Japan akikubo@fujita-hu.ac.jp

The main purpose is to investigate existence and nonexistence of global solutions of the initial Dirichletboundary value problem for evolution equations with the strong dissipation. Many authors studied classes for which initial boundary value problems possess global solutions. We consider a related problem and seek global solutions and blow-up solutions of it depending on whether it belongs to such classes or not.

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Existence of sliding motions for nonlinear evolution equations in Banach space

Laura Levaggi Free University of Bolzano, Italy laura.levaggi@unibz.it

In this work the existence problem for solutions of a class of controlled nonlinear evolution equations in a Banach space is presented, where the considered feedback laws can depend nonlinearly on the state variable. The proof is based on the usual Faedo-Galerkin approximation method and the use of techniques related to the monotonicity and coercivity assumptions on the evolution operator. The result is then applied to the existence of the so-called sliding motions for infinite dimensional systems, either under distributed or boundary control. Sliding mode methods are used to control finite-dimensional systems and are based on the idea of constraining the evolution on a manifold, designed in order to attain the control aim. The idea here is to exploit these techniques to define a feedback control that is able to make each Galerkin approximation fulfill approximately the sliding constraint defining the prescribed manifold and show that the limit evolution is viable.

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Fractional du Bois-Reymond lemma and its applications.

# Marek Majewski University of Lodz, Poland marmaj@math.uni.lodz.pl Rafal Kamocki, Stanislaw Walczak

In the paper a fractional du Bois-Reymond lemma for functions of one variable with Riemann-Liouville derivatives of the order  $\alpha \in (n - \frac{1}{2}, n)$  with  $n \geq 1$  is presented. We use this lemma to show that any critical point of a fractional Lagrange functional is a solution to it's Euler-Lagrange equation. The above technique can be apply to the problem of the existence of solutions to the fractional counterpart of classical Dirichlet problem.

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Boundary control and hidden trace regularity of a semigroup associated with a beam equation and non-dissipative boundary conditions

Richard Marchand

Slippery Rock University, USA richard.marchand@sru.edu Irena Lasiecka, Timothy J. McDevitt

New control-theoretic results will be presented for second order (in time) PDE scalar equations with non-monotone feedback boundary conditions. While the analysis of monotone structures is based on a suitable version of monotone semigroup theory, the non-monotone case seems to require detailed microlocal analysis on the boundary, showing that the underlying semigroup is of Gevrey's class. A particular type of "hidden regularity" exhibited by the boundary traces is instrumental for showing wellposedness of the associated control system within a standard finite energy space even when the controls are not necessarily collocated. Although the analysis is applicable to multidimensional problems, the talk will focus on a one-dimensional Euler-Bernoulli beam equation. Theoretical results will be complemented by numerical simulations that illustrate the spectral properties of the system operators.

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Existence of time-periodic solutions for the micropolar fluid equations with the spin-vortex interaction boundary condition

Kei Matsuura Waseda University, Japan kei@kurenai.waseda.jp Mitsuharu Otani

It will be presented that the time-periodic problem for the micropolar fluid equation in a regular bounded domain  $\Omega$  of  $\mathbb{R}^3$  is solvable if the norms of external forces are sufficiently small. As the boundary conditions on the velocity field u and the microrotation field  $\omega$  we impose the no-slip condition  $u|_{\partial\Omega} = 0$  on the velocity and assume that the spin and the vorticity are proportional on the boundary, i.e.,  $\omega = \frac{\theta}{2} \operatorname{curl} u$  on  $\partial\Omega$  with some constant  $\theta \in [0, 1]$ . For the existence of time-periodic solutions the smallness of the parameter  $\theta$  is required as well. The stability and the uniqueness of time-periodic solutions will also be discussed.

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Fuzzy stochastic differential equationsdifferent approaches and recent results.

Mariusz Michta University of Zielona Gora and Opole University, Poland, Poland

m.michta@wmie.uz.zgora.pl

In the talk we present different approaches to the notion of fuzzy stochastic differential equations driven by semimartingales and also to the notion of their solutions. We also present existence and uniqueness theorems to such equations. Presented results extend to stochastic case possible approches known from the theory of deterministic fuzzy differential equations.

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Nonlinear subdifferential inclusions with applications to contact mechanics

Stanislaw Migórski

Jagiellonian University, Krakow, Poland migorski@ii.uj.edu.pl

We consider two classes of nonlinear subdifferential inclusions in a framework of evolution triple of spaces. The first class involves inclusions with a history-dependent term for which we provide an existence and uniqueness result. In the second class, we consider time-dependent possibly nonconvex nonsmooth functions and their Clarke subdifferentials operating on the unknown function. We prove the existence of a weak solution and study the asymptotic behavior of a sequence of solutions when a small parameter in the inertial term tends to zero. We prove that the limit function is a solution of a first order inclusion. Finally, we give applications to quasi-static viscoelastic frictional contact problems.

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Stochastic delay inclusions with noncontinuous multifunctions

Jerzy Motyl University of Zielona Gora, Poland j.motyl@wmie.uz.zgora.pl

The talk deals with the Ito's type delay stochastic differential inclusion

$$dX(t) \in F(X_t)dt + G(X_t)dW(t), \ X_0 = \xi,$$

where W denotes an m-dimensional Wiener process,  $X_t(s) = X(t+s)$  and set-valued functions F, G defined on  $C([-r, 0], \mathbb{R}^d)$  take on closed and convex values. The existence of local and global solutions of the inclusion with so-called "upper separated" multifunctions will be discussed. Some examples of noncontinuous "upper separated" multifunctions will be also presented.

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Regularity and singularities of optimal convex shapes in the plane

Arian Novruzi University of Ottawa, Canada novruzi@uottawa.ca Jimmy Lamboley, Michel Pierre

We focus here on the analysis of the regularity or singularity of solutions  $\Omega_0$  to shape optimization problems among convex planar sets, namely:

$$J(\Omega_0) = \min\{J(\Omega), \ \Omega \text{ convex}, \ \Omega \in \mathcal{S}_{ad}\},\$$

where  $S_{ad}$  is a set of 2-dimensional admissible shapes and  $J: S_{ad} \mapsto \mathbb{R}$  is a shape functional.

Our main goal is to obtain qualitative properties of these optimal shapes by using first and second order optimality conditions, including the infinite dimensional Lagrange multiplier due to the convexity constraint. We prove two types of results:

i) under a suitable convexity property of the functional J, we prove that  $\Omega_0$  is a  $W^{2,p}$ -set,  $p \in [1, \infty]$ . This result applies, for instance, with  $p = \infty$  when the shape functional can be written as  $J(\Omega) = R(\Omega) +$  $P(\Omega)$ , where  $R(\Omega) = F(|\Omega|, E_f(\Omega), \lambda_1(\Omega))$  involves the area  $|\Omega|$ , the Dirichlet energy  $E_f(\Omega)$  or the first eigenvalue of the Laplace-Dirichlet operator  $\lambda_1(\Omega)$ , and  $P(\Omega)$  is the perimeter of  $\Omega$ , ii) under a suitable concavity assumption on the functional J, we prove that  $\Omega_0$  is a polygon. This result applies, for instance, when the functional is now written as  $J(\Omega) = R(\Omega) - P(\Omega)$ , with the same notations as above.

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Splitting methods for semilinear evolution equations with applications to nonlinear Schrödinger equations

Masahito Ohta Tokyo University of Science, Japan mohta.math@gmail.com

We study splitting methods for seminear evolution equation u' = Au + F(u), where A is an m-dissipative operator in a Hilbert space X, and  $F: D(A) \to D(A)$ is a Lipschitz continuous function on bounded subsets of D(A). Under appropriate assumptions on F, we prove the second order convergence of the Strang splitting method in X for initial data in  $D(A^2)$ . This abstract result unifies several known results for nonlinear Schrödinger equations.

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Viability of a time dependent closed set with respect to a semilinear delay evolution inclusion

Marius Popescu Al. I. Cuza University of Iasi, Romania, Romania Marius.Popescu@ugal.ro Mihai Necula

We prove a sufficient condition for a time-dependent closed set to be viable with respect to a delay evolution inclusion governed by a strongly-weakly u.s.c. perturbation of an infinitesimal generator f a  $C_0$ semigroup. This condition is expressed in terms of a natural concept involving tangent sets, generalizing tangent vectors in the sense of Bouligand and Severi.

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Asymptotic behavior of solutions to a coupled system of Maxwell's equations and a controlled differential inclusion

Volker Reitmann St. Petersburg State University, Russia VReitmann@aol.com Dina Kalinichenko, Sergey Skopinov

We consider the parameter-dependent coupled system consisting of the one-dimensional Maxwell's equations and the heat equation which is understood as differential inclusion. This system is called Maxwell's equations with thermal effect introduced by H.M. Yin. The use of a boundary control in the inclusion guarantees the boundedness and convergence of solutions and excludes the presence of a blow-up. The techniques which are used by us are Lyapunov functionals and multiscaling methods.

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Global existence and exponential stability for a nonlinear delay evolution equation with nonlocal initial condition

Daniela Rosu Alexandru Ioan Cuza University Iasi & Gh. Asachi Technical University Iasi, Romania rosudaniela100@yahoo.com Monica-Dana Burlica

We consider a delay evolution equation subjected to a nonlocal initial condition and governed by a nonlinear Lipschitz perturbation of an *m*-dissipative operator. We prove some existence and asymptotic stability results for global  $C^0$ -solutions and we exemplify the abstract results by a delayed porous media equation subjected either to periodic or to anti-periodic conditions.

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Multiple positive solutions for periodic problems with concave terms

Vasile Staicu University of Aveiro, Portugal, Portugal vasile@ua.pt S. Aizicovici, N. Papageorgiou

We consider a nonlinear periodic problem  $(P_{\lambda})$ , driven by the scalar p-Laplacian, with a parametric concave term  $\lambda |x|^{q-2} x$  and a Caratheodory perturbation f(t,x) such that  $x \to f(t,x)$  exhibits a (p-1)-superlinear growth near  $+\infty$  (the convex term). Using variational techniques, based on the critical point theory, and suitable truncation techniques we prove a bifurcation-type theorem describing the nonexistence, existence and multiplicity of positive solutions as the parameter varies. Namely, we show that a critical parameter value  $\lambda^* > 0$  exists such that: for all  $\lambda \in (0, \lambda^*)$ , the problem  $(P_{\lambda})$  has at least two positive solutions; for  $\lambda = \lambda^*$ , the problem  $(P_{\lambda})$  has at least one positive solution, and, for all  $\lambda > \lambda^*$ , the problem  $(P_{\lambda})$  has no positive solutions.

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On a nonstandard nonlinear parabolic problem for the coupling surface – deep ocean temperatures with latent heat and coalbedo terms.

Lourdes Tello

Universidad Politecnica de Madrid, Spain l.tello@upm.es J.I. Diaz, A. Hidalgo

We study a global climate model for the coupling of the mean surface temperature with the deep ocean temperature. The nonlinear model presents some nonstandard facts: the boundary condition, representing the mean surface temperature, is not only of dynamic type (involving the time derivative of

the trace of the solution) but also a surface diffusive term. The model includes also some delicate nonlinear terms such as the coalbedo effect and the latent heat, which here are formulated in terms of suitable (multivalued) maximal monotone graphs of  $\mathbb{R}^2$ . We prove the existence of bounded weak solutions and show some numerical experiences. Other qualitative properties of the solutions will be also presented.

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Robust feedback stabilization of solutions of stochastic evolution equations with delay

Govindan Trivellore Instituto Politecnico Nacional, Mexico tegovindan@yahoo.com N.U. Ahmed

In this talk, we consider the problem of state feedback robust stabilization against uncertainty in the class of relatively A- bounded operators of mild solutions of stochastic evolution equations with delay. An example is included to illustrate the theory.

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Periodic solutions of some double-diffusive convection systems based on Brinkman-Forchheimer equation

Shun Uchida Waseda Universty, Japan u-shun@suou.waseda.jp Mitsuharu Ôtani

We consider the following system:

 $\begin{cases} \partial_t \vec{u} = \nu \Delta \vec{u} - a\vec{u} - \nabla p + \vec{g}T + \vec{h}C + \vec{f_1} \text{ in } \Omega \times [0, S], \\ \partial_t T + \vec{u} \cdot \nabla T = \Delta T + f_2 \text{ in } \Omega \times [0, S], \\ \partial_t C + \vec{u} \cdot \nabla C = \Delta C + \rho \Delta T + f_3 \text{ in } \Omega \times [0, S], \\ \nabla \cdot \vec{u} = 0 \quad \text{in } \Omega \times [0, S], \\ \vec{u}|_{\partial\Omega} = 0; \ T|_{\partial\Omega} = 0; \ C|_{\partial\Omega} = 0, \end{cases}$ 

where N = 2,3 and  $\Omega$  is a bounded domain with smooth boundary. This system describes the doublediffusive convection between the temperature T and the concentration of solute C. The first equation of the system comes from the Brinkman-Forchheimer equation, which describes the behavior of the fluid velocity  $\vec{u}$  and the pressure p in some porous medium. In this talk, we discuss the existence of a solution for this system under the time periodic condition with period S.

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Nonlinear delay evolution inclusions with nonlocal conditions on the initial history

Ioan Vrabie Al. I. Cuza University, Romania ivrabie@uaic.ro

We consider a nonlinear delay evolution differential inclusion, governed by a multi-valued perturbation of an *m*-dissipative operator, subjected to a non-local condition on the initial history. Under some natural assumptions, allowing to handle periodic, anti-periodic and mean-type problems with delay, we prove the existence in the large of at least one  $C^0$ -solution. Two applications concerning nonlinear delay parabolic problems are presented.

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Uniqueness and non-degeneracy of ground states of quasilinear Schrodinger equations

#### Tatsuya Watanabe

Kyoto Sangyo University, Japan tatsuw@cc.kyoto-su.ac.jp

We consider the following quasilinear elliptic problem:

$$-\Delta u - \kappa \Delta (|u|^{\alpha})|u|^{\alpha-2}u = g(u) \quad \text{in } \mathbb{R}^{N}$$

where  $N \geq 3, \kappa > 0$  and  $\alpha > 1$ . This equation can be obtained as a stationary problem of modified Schrodinger equations which appear in the study of plasma physics.

In this talk, we discuss the existence and the variational characterization of the ground state and present our recent results on the uniqueness and nondegeneracy.

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Life span of positive solutions for a semilinear heat equation with non-decaying initial data

Yusuke Yamauchi

Waseda University, Japan yamauchi@aoni.waseda.jp

We consider the following semilinear heat equation:

$$u_t = \Delta u + u^p, \quad (x,t) \in \mathbb{R}^N \times (0,\infty),$$

where p > 1,  $N \ge 1$ . In this talk, we discuss the upper bound of the life span of positive solutions of the equation for initial data having positive limit inferior at space infinity. The proof is based on a slight modification of Kaplan's method.

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