Special Session 18: Qualitative Theory of Evolutionary Equation and its Application

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The geometric theory of the semilinear parabolic equations has seen a rapid progress and new applications in recent years, in which the study of steady states and traveling wave solutions as well as pattern formation has achieved fruitful results. The purpose of this special section is to provide experts a platform to exchange ideas on new trends and developments as well as new applications of solutions (steady state and traveling wave solutions) of evolutionary equations; to enhance interactions/collaborations, and to provide graduate students and junior researchers opportunities to learn the frontier work in these directions and interact with experts. In particular, the following subareas are included: existence and multiplicity of solutions of semilinear elliptic problems; existence and stability of special solutions (steady-state, traveling solutions, etc) of reaction-diffusion (dispersion) problems; applications to geometry, physics, and biology, medical sciences.

Limiting behavior dynamics of a two-predator one-prey population system with a Beddington-DeAngelis functional response

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We consider a population system of two competing predator species that exploit their consumption of a single renewable prey. In the system, Beddington-DeAngelis functional responses model each predator-prey interaction. We investigate on how the parameters can determine the limiting behavior of any solution in the given system. We solve differential inequalities to obtain parametric conditions that suffice for a given predator species to independently become extinct, hence being an "inadequate" competitor. We also show that competitive exclusion holds whenever the losing competitor is inadequate. Given that at least one competitor becomes extinct, we analyze subsystems and utilize the fluctuation lemma with Barbălat's Lemma, to calculate the limiting values of the solution. In case neither competitor is inadequate, we establish conditions where all three species survive. Specifically, by solving differential inequalities, we find that a competitor can survive at a very low density. In addition, we use a Lyapunov function to establish coexistence through a global stability of the unique interior equilibrium point.

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Flat stationary solutions of the Vlasov-Poisson system (flat galaxies)

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The "flat" Vlasov-Poisson system of partial differential equations describes the evolution of an aggregation of mass particles under their mutual attraction in the plane under the preservation of the 1/r singularity of the Newtonian potential of the three-dimensional case. The study of existence of flat stationary solutions leads to a nonlinear singular integral equation of convolution type on all over R_2. We present a new direct approach to the existence of its solutions beyond results of G. Rein, Comm.Math.Phys. 1999 (joint work with E.Jörn and Yi Li).

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Traveling waves for nonlocal evolution systems

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The existence, uniqueness and stability of traveling wave solutions for nonlocal evolution equations with bistable nonlinearity are discussed. The spectrum of the operator obtained by linearizing about a monotone traveling wave is also studied.

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Traveling waves in high Lewis number combustion model

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High Lewis number combustion model is a model that is used to describe propagation of fronts in the process of burning of high density liquid fuels. Both existence and stability of such fronts are important issues in applications and, at the same time, are challenging mathematical problems. I will talk about the existence of fronts and show that, depending on the value of the exothermicity parameter, these fronts can be either convectively or absolutely unstable.

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Traveling fronts for a nonlocal reaction diffusion system

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We study the long term effect of competitions between two species with nonlocal diffusion, and the competition is of a strong type. Using a new monotone iteration scheme we derive the existence of the front solutions. The uniqueness of the front solution corresponding to each propagation speed is proved by sliding domain method. We also derive the asymptotics of the fronts with critical and noncritical wave speeds. The asymptotic stability of the fronts are shown by a new method of spectral analysis in weighted Banach spaces. The results illustrate that under certain conditions the weaker competitor will die and the stronger one will survive in the long run, and a small change of the initial environment will not change the outcome of the competition.

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Multiple solutions to an elliptic problem related to vortex pairs

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Let Ω be a bounded domain in $\mathbb{R}^N$ $(N \geq 2)$, φ is a harmonic function in $\overline{\Omega}$. In this paper we study the existence of solutions to the following problem arising in the study of vortex pairs

$$(P_\lambda) \qquad \begin{cases} -\Delta u = \lambda (u-\varphi)_+^{p-1}, & x \in \Omega, \\ u = 0, & x \in \partial\Omega. \end{cases}$$

The set $\Omega_p = \{x \in \Omega,\ u(x) > \varphi\}$ is called "vortex core". Existence of solutions whose "vortex core" consisting of one component and asymptotic behavior of "vortex core" were studied by many authors for large λ recently. Under the condition that φ has k strictly local minimum points on the boundary $\partial\Omega$, we obtain in this paper that for λ large enough, (P_λ) has a solution with "vortex core" consisting of k components by a constructive way.

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Traveling waves of thermal diffusivity system-existence and stability

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In this talk, I shall discuss some of the recent results on existence and stability of Traveling Waves to Thermal Diffusivity System with non-KPP type of nonlinearity. In particular, we show how to get sharp estimates on the minimum speed by using a novel approach.

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Spreading speed, traveling waves and linear determinacy for STDs models

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In the talk, I will discuss a Susceptible-Infected-Susceptible (SIS) model for the spatial-spread of sexually transmitted diseases (STD), in spatially mobile heterosexually active populations. First, we will consider a single strain SIS reaction-diffusion model with density dependent recruitment rates. Then, we will reformulate the model to include multiple competing strains of the same pathogen. I will demonstrate the existence of a minimal speed at which the disease spreads to a non-infected region in the form of a traveling wave. Finally, I will discuss a single-strain model with multiple stages of infections.

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A Graph-Theoretic Approach to Global Stability Problems in Some Discrete Diffusion Models

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Spatially discrete diffusion models can be regarded as coupled systems on networks. A new graph-theoretic approach is developed to guide the constructions of Lyapunov functions for coupled systems on networks, and can be applied to investigate global stability problems for some discrete diffusion models, such as a single-species diffusion model and a predator-prey model with prey movement.

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Turing instability in a three species food chain model

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In this paper, we study a strongly coupled reaction-diffusion system describing three interacting species in a food chain model, where the third species preys on the second one and simultaneously the second species preys on the first one. We first show that the unique positive equilibrium solution is globally asymptotically stable for the corresponding ODE system. The positive equilibrium solution remains linearly stable for the reaction diffusion system without cross diffusion, hence it does not belong to the classical Turing instability scheme. We further proved that the positive equilibrium solution is globally asymptotically stable for the reaction diffusion system without cross diffusion by constructing a Lyapunov function. But it becomes linearly unstable only when cross-diffusion also plays a role in the reaction-diffusion system, hence the instability is driven solely from the effect of cross diffusion. Our results also exhibit some interesting combining effects of cross-diffusion, intra-species competitions and inter-species interactions.

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Some estimates of solutions to a quasilinear elliptic Dirichlet problem with large diffusion

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In this talk, some estimates of solutions to a quasilinear elliptic Dirichlet problem with large diffusion will be presented and it will be shown that the L-infinity norm has an algebraic growth as the diffusion coeffient increases, which is quitely different from the corresponding Neumann case.

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Non pattern formation in a chemo-repulsion problem

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In contrast to diffusion (random diffusion without orientation), chemotaxis is the biased movement of cells/particles toward the region that contains higher concentration of beneficial or lower concentration of unfavorable chemicals. The former often refers to the attractive chemotaxis and latter to the repulsive chemotaxis. Chemotaxis has been advocated as a leading mechanism to account for the morphogenesis and self-organization of a variety of biological coherent structures such as aggregates, fruiting bodies, clusters, spirals, spots, rings, labyrinthine patterns and stripes, which have been observed in experiments. In this talk, I will present some recent development on the rigorous analysis of a partial differential equation model arising from repulsive chemotaxis which is a system of conservation laws consisting of nonlinear and coupled parabolic and hyperbolic type PDEs. In particular, global well-posedness, large-time asymptotic behavior of classical solutions to such model are obtained which indicate that chemorepulsion problem of this type exhibits strong tendency against pattern formation. The results are consistent with general results for classical repulsive chemotaxis models.

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Time periodic traveling wave solutions for periodic advection-reaction-diffusion systems

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In this presentation, we will talk about time periodic traveling wave solutions to a class of periodic advection-reaction-diffusion systems. Among the basic questions, we shall concentrate on the existence of time periodic waves, the determination of critical wave speed, and the stability of traveling waves. Under certain conditions, we show that there exists a maximal wave speed c^* such that for each wave speed $c \leq c^*$, there is a time periodic traveling wave connecting two periodic solutions of the corresponding kinetic system. It is shown that such a traveling wave is unique modulo translation and is monotone with respect to its co-moving frame coordinate. We also show that the traveling wave solutions with wave speed $c \leq c^*$ are asymptotically stable in certain sense.

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