Special Session 2: Nonlinear Evolution PDEs and Interfaces in Applied Sciences

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Applied sciences often pose problems in terms of interfaces that can be studied through nonlinear evolution equations. Among these are phase field or diffusive interface problems, where Allen-Cahn and Cahn-Hilliard type equations play a basic role. This session will mainly focus on such models and their mathematical properties, namely, well-posedness, regularity, and stability and asymptotic behavior (considered here in a broad sense). Their implications for applications will also be an issue.

Stationary free surface flows in three dimensions

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We consider an incompressible, viscous, free surface flow down a three dimensional channel. In the absence of surface tension, we prove the existence of a unique stationary solution in weighted Sobolev spaces. The result is based on a thorough study of the pseudodifferential operator relating the normal velocity of the fluid and the normal component of the associated stress tensor along the free surface, and requires the use of the Nash-Moser Implicit Function Theorem. Extensions to other geometries are discussed.

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Diffuse interface Cahn-Hilliard-Ladyzhenskaya models with singular potentials

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In order to describe the evolution of a mixture of two incompressible and (partially) immiscible fluids, a diffuse interface model consists of the Ladyzhenskaya-Navier-Stokes type equations for the (average) fluid velocity coupled with a convective Cahn-Hilliard equation with a singular (e.g., logarithmic) potential. In this talk we will present some recent results concerning such systems endowed with no-slip boundary conditions for the velocity and no-flux boundary conditions on the order parameter field. We shall discuss existence of weak solutions in dimension three, some regularity results and the existence of a trajectory attractor both in a weak as well in a strong topologies. Further issues and open questions, concerning e.g. uniqueness, will also be addressed.

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Consistent hierarchy of Cahn-Hilliard systems

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In this work, we investigate n-component Cahn-Hilliard systems. Our goal is to derive models in a consistent way: we require that a solution of the (n-1)-component model is a solution to the n-component model. Properties of such models are given in some cases and numerical illustrations are proposed. We also show that this consistency property is important in applications, in particular in the framework of phase-field modeling.

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Phase field models and the connection between microscopic and macroscopic anisotropy

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Anisotropy at the microscopic level is manifested at the macroscopic level that is on a vastly larger scale. The phase field approach can facilitate our understanding of this complex process. We use a formulation in which we work directly with an integral equation for the phase variable. The technique allows one to derive macroscopic conditions at the interface from the microscopic potentials. Differential geometry and asymptotic analysis yield interface conditions, in arbitrary spatial dimension, for interactions that include anisotropy as well as non-local potentials. The interface condition can be expressed in various mathematical formulations, e.g., in terms of the principal curvature directions of the interface, or the second order directional derivatives of the interface and the Hessian of the surface tension. This work is in collaboration with X. Chen and E. Esenturk.

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Long-time behavior of the Caginalp system with singular potentials and dynamic boundary conditions

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This talk deals with the Caginalp phase-field model equipped with both singular potentials and dynamic boundary conditions. Classical solutions (in the sense of distributions) are not assured in this case. Thanks to a suitable definition of solutions, we are able to get dissipative estimates, leading to the existence of the global attractor with finite fractal dimension, as well as of an exponential attractor.

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Approximation of multivalued attractors and applications to the Navier-Stokes equations

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We present the theory of multivalued dynamical systems, with particular emphasis on the approximation of global attractors of continuous-time semigroups by discrete ones. As an application, we focus on a fully implicit time-discretization of the two-dimensional Navier-Stokes equations, establishing new uniform bounds on the time-step parameter. As a byproduct, we obtain a general long-time stability result and we prove the convergence of the discrete attractors to the continuous one as the time-step approaches zero.

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Congestion in cell migration models

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This work concerns a class of macroscopic models for cell migration in a saturated medium for two-species mixtures. Those species tend to achieve some motion according to a desired velocity, and congestion forces them to adapt their velocity. This adaptation is modelled by a correction velocity which is chosen minimal in a least-square sense. We are especially interested in two situations: a single active species moves in a passive matrix (cell migration) with a given desired velocity, and a closed-loop Keller-Segel type model, where the desired velocity is the gradient of a self-emitted chemoattractant. We propose a numerical strategy to discretize the model, and illustrate its behaviour in the case of a prescribed velocity, and for the saturated Keller-Segel model.

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Long-term analysis of strongly damped nonlinear wave equations

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We consider the strongly damped nonlinear wave equation

$$u_{tt} - \Delta u_t - \Delta u + f(u_t) + g(u) = h$$

with Dirichlet boundary conditions, which serves as a model in the description of thermal evolution within the theory of type III heat conduction. In particular, the nonlinearity f acting on u_t is allowed to be nonmonotone and to exhibit a critical growth of polynomial order 5. The main focus is the longterm analysis of the related solution semigroup, which is shown to possess the global attractor in the natural weak energy space.

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Attractors for a Caginalp phase-field model type on the whole space R^3

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Brice Doumbé

We consider in this paper a generalization of Caginalp phase-field system derived from a generalization of the Maxwell-Cattaneo law in an unbounded domain namely R^3 in our case; which make the analysis challenging. We prove the well-posedness of the problem and the dissipativity of the associated semigroup. Finally, we study the long time behavior of solutionsin terms of attractors.

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Vlasov-Poisson system with diffuse boundary conditions

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Kinetic models play an important role in many areas of mathematical physics. One of the branches of the theory deals with systems whose elements interact with each other in a conservative way through a self-consistent field. A typical example of such systems is the hot plasma which is governed by a collisonless kinetic equation (Vlasov) coupled with electrostatic equation (Poisson).

In the last decades many important results have been obtained for the Vlasov-Poisson system. In most of the works the theories were developed in the whole space. Much less is known on systems with boundary conditions. In this talk we will present results for the nonlinear Vlasov-Poisson system with the absorbing and non-absorbing diffuse boundary conditions. Existence and uniqueness of solutions will be addressed in one dimension for the finite interval and the half line and in higher dimensions for particular geometries. Some exact solutions will be discussed for simple cases.

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Global solutions for the 2D NS-CH model for a two-phase flow of viscous , incompressible fluids with mixed partial viscosity and mobility

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Whether or not global solutions of the 2DNavier-Stokes-Cahn-Hilliard system without full viscosity and mobility can develop finite time singularities is a difficult issue. A major part of this talk deals with global regularity of strong solutions for the NS-CH system with mixed partial viscosity and mobility. In addition, we will also discuss the 2D NS-CH system without viscosity but with full mobility. We prove the global existence and uniqueness of classical solutions.

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Stability analysis and travelling fronts in the phase field crystal model

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The phase field crystal (PFC) model was proposed to incorporate the physics naturally embedded on atomic length scales for processes that occur on diffusive time scales. The model describes a field that is spatially periodic in a solid and constant in metastable or unstable (liquid) phases. Using the PFC-model, we provide a stability analysis to predict the velocity and periodicity of a crystal front invading a uniform unstable state. Amplitude expansions of the hyperbolic and parabolic PFC models are used to examine a periodic solid invading a metastable liquid in both slow and rapid front propagation regimes. The analysis is applicable to systems with long-range interactions, such as block copolymers (second order transformation) as well as the solidification of undercooled liquids (first order phase transformation).

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Attractors for weakly damped wave equations

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This talk is devoted to the issue of the existence and regularity for low regular solutions to weakly damped forced equations, as Korteweg de Vries and nonlinear Schrodinger equations.

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On thermodynamically consistent schemes for two-phase flow with mass density contrast and species transport

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Recently, Abels, Garcke, and Grün proposed a class of diffuse interface models for two-phase flow with different mass densities which allow both for energy estimates and for solenoidal velocity fields. It differs from earlier approaches – apparently not consistent with thermodynamics or not frame-indifferent – by a convective coupling of velocity field and Cahn-Hilliard flux. We present a subtle discretization of these terms which entails energy estimates and existence results in the discrete setting. Numerical simulations in various settings – Rayleigh-Taylor instability, rising droplets, species transport and Henry's law – are compared with those obtained by other models.Finally, we suggest extensions of the model to electrowetting with electrolyte solutions in the case of general mass densities.

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A fourth order Cahn-Hilliard type equation

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Our aim in this talk is to discuss the well-posedness (existence and uniqueness of solutions) and the asymptotic behavior, in terms of finite-dimensional attractors (global and exponential attractors) for an equation of Cahn-Hilliard type.

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Well-posedness of finite energy solutions ton supersonic flow structure interactions

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We shall consider a model of flow-structure interaction which consists of perturbed wave equation coupled with a nonlinear plate. The interaction between two media takes place on the edge of the plate . We shall consider supersonic case where it is known that the static problem looses ellipticity. Existence and uniqueness of finite energy solutions will be addressed. By using methods of microlocal analysis Hadamard wellposednesss of finite energy solutions will be shown.

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The thin film equation with backwards second order diffusion

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We focus on the thin film equation with lower order "backwards" diffusion which can describe, for example, structure formation in biofilms and the evolution of thin viscous films in the presence of gravity and thermo-capillary effects. We treat in detail the equation

$$u_t + \{u^n(u_{xxx} + \neq u^{m-n}u_x - Au^{M-n}u_x)\}_x = 0,$$

where $\neq = \pm 1, 0 < n, m < M$, and $0 \leq A$.

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A thermodynamic format for phase-field modeling

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We discuss the thermodynamic issues underlying the phase-field modeling of phase segregation and phase transitions by atom rearrangement in crystalline materials, both in the absence and in the presence of diffusion. In particular, we contrast the standard derivation of the classic mathematical models of Allen-Cahn (or Ginzburg-Landau, according to [2]) and Cahn-Hilliard with the versions given by Friedand Gurtin [1], Gurtin [2], and Podio-Guidugli [3].

[1] E. Fried and M.E. Gurtin, Continuum theory of thermally induced phase transition based on an order parameter. Physica D 68(1993).
[2] M.E. Gurtin, Generalized Ginzburg-Landau and Cahn-Hilliard equations based on a microforce balance. Physica D 92 (1996).
[3] P. Podio-Guidugli, Models of phase segregation and diffusion ofatomic species on a lattice. Ric. Mat., 55 (1), 105-118 (2006).

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Time regularity and uniqueness of non-Newtonian binary fluid mixtures

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We consider binary mixture of incompressible non-Newtonian fluid with diffused interfaces. More precisely, the fluid is governed by Ladyzhenskaya rate-type model, where stress tensor is a polynomial function of $|Du|$ and also a smooth function of the order parameter, the latter being driven by convected Cahn-Hilliard equation. We consider the 3D setting with Dirichlet/Neumann boundary conditions. Assuming p-coercivity of stress tensor, with $p > 11/5$, we employ fractional Nikolskii spaces to establish higher time integrability of (arbitrary) weak solution. This implies uniqueness, but also existence of finite-dimensional attractor.

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Degenerating PDE system for phase transitions and damage.

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In the talk we propose a suitable weak notion of solution for a PDE system coupling a weak heat and momentum equation with a generalized principle of virtual powers for the phase/damage parameter, partially mutuated for the theory of rate-independent damage process. In order to prove an existence result for this weak formulation, an approximating problem is introduced, where the elliptic degeneracy is ruled out. First, a global-in-time existence result for the full system consisting of the heat, displacement, and phase/damage parameter equations is established and well-posedness is proved. Then, the passage to the limit to the degenerate system is performed via suitable variational techniques.

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Efficient numerical methods for the Cahn-Hilliard-Brinkman equation

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I will describe some first and second-order (in time) convex splitting schemes for the Cahn-Hilliard-Brinkman equation, which models two-phase flows in porous media and related phenomena. I will present some simulation results in 2 and 3D and discuss some applications in modeling tumor growth, biofilms, and lava lamps.

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Global dynamics of a diffuse interface model for solid tumor growth

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In this talk I will report recent progress on a diffuse interface model (referred as the Cahn-Hilliard-Hele-Shaw system) which arises in modeling of spinodal decomposition in binary fluid in a Hele-Shaw cell, tumor growth and cell sorting, and two phase flows in porous media. Previous numerical simulations showed that the model is capable of modeling all the stages of a solid tumor growth - avascular, vascular and metastasis. In this work, wellposedness, regularity and long-time asymptotic behavior of solutions to an initial-boundary value problem of the model are rigorously justified.

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