Special Session 21: Dynamical Systems and Spectral Theory

David Damanik, Rice University, USA

Talks in this special session will address recent results and developments in dynamical systems and/or spectral theory.

Derivation of NLS from an interacting Bose gas in $d = 3$ via Klainerman-Machedon type spaces

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The Gross-Pitaevskii (GP) hierarchy is an infinite system of coupled linear non-homogeneous PDEs, which appear in the derivation of the nonlinear Schrodinger equation (NLS). In this talk we will discuss a new derivation of the defocusing cubic GP hierarchy in dimensions $d = 2, 3$, from an N-body Schrodinger equation describing a gas of interacting bosons in the GP scaling, in the limit $N \to \infty$. In particular, we prove convergence of the corresponding BBGKY hierarchy to a GP hierarchy in the spaces introduced in our previous work on the well-posedness of the Cauchy problem for GP hierarchies, which are inspired by solution spaces based on space-time norms introduced by Klainerman and Machedon. We note that in $d = 3$, this has been a well-known open problem in the field. While our results do not assume factorization of the solutions, consideration of factorized solutions yields a new derivation of the cubic, defocusing NLS in $d = 2, 3$.

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Subshifts and low regularity potentials

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We discuss the spectral theory of Schrödinger operators with potentials defined by low regularity sampling functions along a suitable base transformation and approximations by subshifts over finite alphabets.

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Inverse problems for Jacobi operators

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M. Kudryavtsev

We consider a linear finite spring mass system which is perturbed by modifying one mass and adding one spring. We study when masses and springs can be recovered from the natural frequencies of the original and the perturbed systems. This is a problem about rank two or rank three perturbations of finite Jacobi matrices where we are able to describe quite explicitly the associated Green's functions. We give necessary and sufficient conditions for two given sets of points to be eigenvalues of the original and modified systems respectively.

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Spectral properties for the quasi-periodic Schroedinger equation

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We consider the spectral properties of the quasi-periodic Schroedinger operator and the 2-dimensional Schroedinger equation using the relation between the rotation number and the uniform hyperbolicity of the corresponding differential systems. Using also some numerical computation (due to Cinzia Elia), we obtain information on the structure of the spectrum of the one-dimensional operator.

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Properties of the IDS of the Fibonacci Hamiltonian

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David Damanik

The Trace Map techniques not only allow to describe the spectrum of the discrete Schrodinger operator with Fibonacci potential as a set, but also provide an insight into the properties of the integrated density of states (IDS) that turn out to be related to the measure of maximal entropy for the Trace Map. In particular, we show the exact dimensionality of the IDS, and provide sharp estimates on its Holder exponent.

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Quasi-periodic Schrödinger operators beyond the almost Mathieu

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This talk is devoted to quasi-periodic Schrödinger operators beyond the Almost Mathieu, with more general potentials and interactions, considering the connections between the spectral properties of these operators and the dynamical properties of the associated quasi-periodic linear skew-products. In particular, we present a Thouless formula and some consequences of Aubry duality. We illustrate the results with numerical computations.

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Recent developments for skew-shift Schroedinger operators

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The skew-shift is given by $T : (x, y) \to (x+2\omega, y+x)$ where ω is an irrational number. Potentials given by evaluating a sampling function along the second coordinate of an orbit have many interesting properties. I will discuss properties related to the distribution of eigenvalues and the structure of the spectrum.

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Jacobi matrices with decaying oscillatory coefficients

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We investigate decaying oscillatory perturbations of the free Jacobi matrix. The perturbation can, for instance, be a quasiperiodic sequence multiplied by ℓ^p decay. Under mild conditions, we prove preservation of absolutely continuous spectrum and give bounds on the Hausdorff dimension of the singular part of spectral measures.

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Analytic quasi-perodic cocycles with singularities and the Lyapunov Exponent of Extended Harper's model

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We show how to extend (and with what limitations) Avila's global theory of analytic SL(2,C) cocycles to families of cocycles with singularities. This allows to develop a strategy to determine the Lyapunov exponent for extended Harper's model, for all values of parameters and all irrational frequencies. In particular, this includes the self-dual regime for which even heuristic results did not previously exist in physics literature. The extension of Avila's global theory is also shown to imply continuous behavior of the LE on the space of analytic M(2,C)-cocycles. This includes rational approximation of the frequency, which so far has not been available.

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Orthogonal polynomials on the unit circle with almost periodic recursion coefficients

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Given a probability measure on the unit circle, we perform a Gram-Schmidt orthogonalization process on $\{1, z, z^2, \ldots\}$ and obtain a sequence of orthogonal polynomials with respect to that measure. These polynomials obey a recurrence relation, and it is of natural interest to relate properties of the recurrence coefficients with properties of the probability measure. We present various results about the probability measure when the corresponding recurrence coefficients form an almost periodic sequence. We arrive at these conclusions by expressing the problem in terms of dynamically defined unitary operators, and by exploiting well-known connections between these unitary operators and the discrete Schrödinger Operator.

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Fractals and dynamic

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Let A be an alphabet over 3 letters and A^* is the set of finite words written with the alfabet A. A *substitution* σ is a map from A to A^*. It is known that to any substitution we can associate a shift symbolic dynamical system. It is known that for a large class of substitutions σ, the associated dynamical system is measure theoretically isomorphic to an exchange of pieces over a compact set $\mathcal{K}_\sigma$ of$\mathbb{R}^2$. This set is called Rauzy Fractal and has many beautiful properties. In particular $\mathcal{K}_\sigma$ induces a periodic tiling of the plane and moreover its boundary is fractal. In this work, we will present some geometrical and dynamical properties of the Rauzy fractal and its boundary.

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Absolutely continuous spectrum and ballistic behavior for the Anderson model on the Bethe strip

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Abel Klein

The Bethe strip is the cross product graph of the Bethe lattice with a finite graph. We consider random Schroedinger operators on such graphs such as the Anderson model. For low disorder we find almost surely absolutely continuous spectrum in a certain interval. Moreover, the quantum dynamical wave spreading is ballistic.

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Anderson localization for non-monotone Schroedinger operators

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We show how the fractional moment method of Aizenman and Molchanov can be applied to a class of Anderson-type models with non-monotone potentials, to prove (spectral and dynamical) localization. The main new feature of our argument is that it does not assume any a priori Wegner-type estimate: the (nearly optimal) regularity of the density of states is established as a byproduct of the proof. The argument is applicable to finite-range alloy-type models and a class of operators with matrix-valued potentials.

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Spectral applications of McMullen's Hausdorff dimension algorithm

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We consider singular continuous measures on the unit circle obtained as limit measures of groups generated by reflections in the hyperbolic plane. We extend McMullen's Hausdorff dimension algorithm to approximate the moments of these measures. This allows us to study the corresponding orthogonal polynomials on the unit circle and to investigate various spectral properties of the associated CMV matrices.

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Spectral analysis of tridiagonal Fibonacci Hamiltonians

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We consider a family of discrete Jacobi operators on the one-dimensional integer lattice, with the diagonal and the off-diagonal entries given by two sequences generated by the Fibonacci substitution on two letters. We show that the spectrum is a Cantor set of zero Lebesgue measure, and discuss its fractal structure and Hausdorff dimension. We also extend some known results on the diagonal and the off-diagonal Fibonacci Hamiltonians.

Our methods involve dynamical properties of the so-called Fibonacci trace map (a polynomial map of degree two on the three-dimensional Euclidean space).

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Positive Lyapunov exponents for quasiperiodic Szego cocycles

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I will talk about positivity of Lyapunov exponents for quasi-periodic Szĕgo cocycles. I will use different methods to consider potentials of different smooth categories: for C^0 ones, I will use Avila and Damanik's technique for genericity of singular spectrum for Schrödinger operators; for C^r case, $1 \leq r \leq \infty$, I will use Lai-Sang Young's induction method, which in spirit is Benedicks-Carleson's method for Hénon map; for C^ω ones, I will use subharmonicity and acceleration, which is recently introduced by Avila. In particular, new examples of analytic quasiperiodic Szĕgo cocycles with uniformly positive Lyapunov exponents (uniform in energy) will be constructed.

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