Special Session 22: Topological and Variational Methods for Boundary Value Problems

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Topological methods have proved to be an important technique in the study of boundary value problems and related topics for ordinary and partial differential equations. Recently that has been a rapidly growing interest in applying variational methods and critical point theory to such problems. This session is devoted to the use of these methods in the study of boundary value problems including singular problems and those with multipoint conditions.

Existence of multiple positive solutions for p-Laplacian multipoint boundary value problem on time scales

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In this paper, we consider p-Laplacian multipoint boundary value problem on time scales. By using fixed point theorems, we prove the existence of at least three positive solutions to the boundary value problem. The interesting point is that the nonlinear term f depends on the first order derivative explicity. As an application, an example is given to illustrate the result.

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A note on a third-order multi-point boundary value problem at resonance

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Based on the coincidence degree theory of Mawhin, we prove some existence results for a third-order multi-point boundary value problem at resonance. In this talk, the dimension of the linear space Ker L is equal to 2. Since all the existence results for third-order differential equations obtained in previous papers are for the case dim Ker L = 1, our work is new.

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A class of decomposable nonlinear operators and its applications in BVP

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We will introduce a class of nonlinear operators that can be decomposed into a linear operator and a nonlinear map. Some properties for the class are proved. Applications to existence of solutions for some boundary value problems are given.

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Generalized upper and lower solutions on fourth order Lidstone problems

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In this work it is considered the nonlinear fully equation

$$u^{(iv)}(x) + f\left(x, u(x), u'(x), u''(x), u'''(x)\right) = sp(x) \tag{1}$$

for $x \in [0,1]$, where $f : [0,1] \times \mathbb{R}^4 \to \mathbb{R}$ and $p : [0,1] \to \mathbb{R}^+$ are continuous functions and s a real parameter, coupled with the Lidstone boundary conditions,

$$u(0) = u(1) = u''(0) = u''(1) = 0, \tag{2}$$

These types of problems are known as Ambrosetti-Prodi problems, and they provide the discussion of existence, nonexistence and multiplicity results on the parameter s. More precisely, sufficient conditions, for the existence of s_0 and s_1, are obtained, such that:

- if $s < s_0$, the problem has no solution.
- if $s = s_0$, the problem has a solution.
- if $s \in \left]s_0, s_1\right]$, the problem has at least two solutions.

In this work it is discussed how conditions in the lower and upper definitions influence the main results and vice-versa. This "power shift" between the Definition and Theorem makes it possible to extend some results to a functional version of (1)-(2). In addition we replace the usual bilateral Nagumo condition by a one-sided condition, allowing the nonlinearity to be unbounded.

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On a notion of category depending on a functional and an application to Hamiltonian systems

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A notion of category depending on a functional is introduced. This notion permits to obtain a better lower bound on the number of critical points of a functional than the classical Lusternik-Schnirelman category. A relation between this notion and the linking of type splitting spheres is discussed. An application to Hamiltonian systems is also presented.

References: [1] N. Beauchemin and M. Frigon, On a notion of category depending on a functional. Part II: An application to Hamiltonian systems, Nonlinear Anal. 72 (2010) 3376–3387. [2] N. Beauchemin and M. Frigon, On a notion of category depending on a functional. Part I: Theory and Application to critical point theory, Nonlinear Anal. 72 (2010) 3356–3375. [3] C.C. Conley and E. Zehnder, The Birkhoff-Lewis fixed point theorem and a conjecture of V.I. Arnol'd, Invent. Math. 73 (1983), 33–49.[4] G. Fournier, D. Lupo, M. Ramos and M. Willem, Limit relative category and critical point theory, in Dynamics reported. Expositions in Dynamical Systems, vol. 3, 1–24, Springer, Berlin, 1994.[5] M. Frigon, On a new notion of linking and application to elliptic problems at resonance, J. Differential Equations, 153 (1999), 96–120.[6] A. Szulkin, A relative category and applications to critical point theory for strongly indefinite functionals, Nonlinear Anal. 15 (1990), 725-739.

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Existence of nontrivial solutions to systems of multi-point boundary value problems

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The authors consider the system of n multi-point boundary value problems

$$\begin{cases} -(\phi_{p_i}(u_i'))' = \lambda F_{u_i}(x, u_1, ..., u_n), \quad x \in (0,1), \\ u_i(0) = \sum_{j=1}^m a_j u_i(x_j), \quad u_i(1) = \sum_{j=1}^m b_j u_i(x_j), \end{cases}$$

for $i = 1, \dots, n$. The existence of at least one nontrivial solution is proved using variational methods and critical point theory.

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Existence and multiplicity for positive solutions of a system of higher-order multi-point boundary value problems

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We investigate the existence and multiplicity of positive solutions of multi-point boundary value problems for systems of nonlinear higher-order ordinary differential equations.

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Solvability of second order three-point boundary value problem at resonance

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We are interested in existence theorems for second order three-point boundary value problems at resonance. The usual method of proof is based upon coincidence degree theory which differs from the fixed point theorem approach for problems subject to non-resonant boundary conditions. We introduce an alternate method by reducing the original problem to a two-point problem with non-homogeneous boundary condition containing a parameter μ. We then apply fixed point theorem and shooting method to determine the proper μ which gives rise to a solution of the original three-point problem.

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On a discrete fourth order periodic boundary value problem

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John R. Graef, Min Wang

By using the variational method and critical point theory, we obtain criteria for the existence of multiple solutions of the discrete fourth order periodic boundary value problem

$$\Delta^4 u(t-2) - \alpha\Delta^2 u(t-1) + \beta u(t) = f(t, u(t)),\ t \in [1,T]_{\mathbb{Z}},$$
$$\Delta^i u(-1) = \Delta^i u(T-1), \quad i = 0, 1, 2, 3,$$

where $T \geq 2$ is an integer, $[1,T]_{\mathbb{Z}} = \{1, 2, \dots, T\}$, $\alpha, \beta \geq 0$ are parameters, and $f : [1,T]_{\mathbb{Z}} \times \mathbb{R} \to \mathbb{R}$ is a continuous function. Examples are included to illustrate the results.

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A fourth-order functional problem at resonance

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We study the differential equation

$$u^{(4)}(t) - \omega^4 u(t) = f(t, u(t), u'(t), u''(t), u'''(t)),$$

satisfying linear functional conditions

$$B_i u = 0, \quad i = 1, ..., 4.$$

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Boundary data smoothness for solutions of nth order nonlocal boundary value problems

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In this talk, we investigate boundary data smoothness for solutions of the nonlocal boundary value problem, $y^{(n)} = f(x, y, y', \ldots, y^{(n-1)}), y^{(i)}(x_j) = y_{ij}$ and $y^{(i)}(x_k) - \sum_{p=1}^{m} r_{ip} y(\eta_{ip}) = y_{ik}$. Essentially, we show under certain conditions that partial derivatives of the solution to the problem above exist with respect to boundary conditions and solve the associated variational equation. Lastly, there will be a corollary and nontrivial example.

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Two-point boundary value problems with impulses.

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The authors look at nonlinear boundary value problems of the form

$$x'(t) = A(t)x(t) + f(t, x(t)), \quad t \in [0,1] \setminus \{t_1, \cdots, t_k\}$$

$$x(t_i^+) - x(t_i) = J_i(x(t_i)), \quad i = 1, ..., k$$

subject to

$$Bx(0) + Dx(1) = 0.$$

We focus on the resonant case, that is, the case in which the solution space of the linear homogeneous problem is nontrivial. In particular, we use degree theory to prove the existence of solutions when the solution space has dimension greater than 1.

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Existence and multiplicity of solutions in fourth order BVPs with unbounded nonlinearities

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This work studies the Ambrosetti-Prodi fourth order nonlinear fully equation

$$u^{(4)}(x) + f\left(x, u(x), u'(x), u''(x), u'''(x)\right) = s\, p(x)$$

for $x \in [a,b]$, $f : [a,b] \times \mathbb{R}^4 \to \mathbb{R}$, $p : [a,b] \to \mathbb{R}^+$ continuous functions and $s \in \mathbb{R}$, with the boundary conditions

$$u(a) = A, \ u'(b) = B, \ u'''(a) = C, \ u'''(b) = D.$$

In this work it will be presented an Ambrosetti-Prodi type discussion on s, with some new features: the existence part is obtained in presence of nonlinearities not necessarily bounded, and in the multiplicity result it is not assumed a speed growth condition or an asymptotic condition, as it is usual in the literature for these type of higher order problems.

The arguments used apply lower and upper solutions technique and topological degree theory.

An application to a continuous model of the human spine, used in aircraft ejections, vehicle crash situations and some forms of scoliosis, will be referred.

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Extremal points for an nth order three point boundary value problem

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We characterize first extremal points of an nth order three point boundary value by using a substitution method and working with the 4th order problem. These results are then used to find positive solutions of the nth order nonlinear problem.

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Oscillation results for fourth order nonlinear mixed neutral differential equations

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Oscillatory and asymptotic behaviour of a class of nonlinear fourth order neutral differential equations with positive and negative coefficients of the form

$$(r(t)(y(t)+p(t)y(t-\tau))'')'' \\ +q(t)G(y(t-\alpha))-h(t)H(y(t-\beta))=0,$$

and

$$(E)\ (r(t)(y(t)+p(t)y(t-\tau))'')'' \\ +q(t)G(y(t-\alpha))-h(t)H(y(t-\beta)) \quad = f(t)$$

are investigated under the assumption

$$\int_0^\infty \frac{t}{r(t)}dt=\infty$$

for various ranges of $p(t)$. Using Scauder's fixed point theorem, sufficient conditions are obtained for the existence of positive bounded solutions of (E).

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Boundary value problems governing fluid flow and heat transfer over an unsteady stretching sheet

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This talk will consider two situations involving unsteady laminar boundary layer flow due to a stretching surface in a quiescent viscous incompressible fluid. In one configuration, the surface is impermeable with prescribed heat flux, and in the other, the surface is permeable with prescribed temperature. The boundary value problems governing a similarity reduction for each of these situations are investigated and existence of a solution is proved for all relevant values of the physical parameters. Uniqueness of the solution is also proved for some (but not all) values of the parameters. Finally, a priori bounds are obtained for the skin friction coefficient and local Nusselt number.

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Existence analysis for nonlocal Sturm-Liouville boundary value problems

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We establish conditions for the existence of solutions to nonlinear differential equations subject to nonlocal boundary conditions. The problems considered are of the form

$$(p(t)x'(t))'+q(t)x(t)+\psi(x(t))=G(x(t))$$

subject to the global boundary condition

$$\alpha x(0)+\beta x'(0)+\eta_1(x)=\phi_1(x), \\ \gamma x(1)+\delta x'(1)+\eta_2(x)=\phi_2(x). \tag{1}$$

We assume $\psi:\mathbb{R}\to\mathbb{R}$ is continuously differentiable, $p(t)>0$ and $q(t)$ is real valued on $[0,1], p, p', q$ are continuous on $(0,1)$, and the boundary conditions (1) are such that $\alpha^2+\beta^2\neq 0$, $\gamma^2+\delta^2\neq 0$. Further, G,η_1,η_2,ϕ_1 , and ϕ_2 shall be nonlinear operators defined on a function space.

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Applications of variational methods to anti-periodic boundary value problem for second-order differential equations

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We discuss the existence of multiple solutions to a second-order anti-periodic boundary value problem by using variational methods and critical point theory. Furthermore, we get the existence of periodic solutions for corresponding second-order differential equations. In constructing variational structure, we prove a fundamental lemma, which plays an important role in prove the critical point of functional is just the solution of original problem.

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Fractional boundary value problems with integral boundary conditions

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The authors study a type of nonlinear fractional boundary value problem with integral boundary conditions. By constructing an associated Green's function, applying spectral theory, and using fixed point theory on cones, they obtain criteria for the existence, multiplicity, and nonexistence of positive solutions.

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Existence, location and approximation results for some nonlinear boundary value problems

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The significance and importance of boundary value problems (BVPs) is well-known. The aim of this talk is to present some results of existence, location and approximation for some nonlinear BVPs. In our study, the theory of differential inequalities plays a very important role. More precisely, in the first part, we will present the existence and location criteria of solutions for the general nonlinear system with the general nonlinear boundary conditions. Here, we introduce a new concept of bounding function pair and a method, which may be called simultaneous modification. In the second part, using the generalized quasilinearization method, we will study two classes of second-order nonlinear BVPs with nonlocal boundary conditions. We will establish some sufficient conditions under which corresponding monotone sequences converge uniformly and quadratically to the unique solution of the problem. The interesting point is that our boundary condition is nonlinear and nonlocal.

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New periodic solutions for N-body-type problems with prescribed energies

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In this paper, we use a variant of the Benci-Rabinowitz Theorem to study the solutions of N-body-type problems with prescribed energies.

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