Special Session 23: Topological and Combinatorial Dynamics

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Francisco Balibrea Gallego, Universidad de Murcia, Spain
Piotr Oprocha, AGH University, Poland

The session will be focused on the topological and combinatorial aspects of low dimensional discrete dynamical systems including entropy, chaos, limit sets, sets of periods, rotation theory and the like. Applications of this theory to Economics, Physics, Engineering, Biology and others are highly welcome. Related areas of dynamical systems, combinatorial aspects of dynamical systems and ergodic theory are not excluded.

Simple permutations with order $4n+2$

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The problem of genealogy of permutations has been solved partially by Stefan (odd order) and Acosta-Humánez & Bernhardt (power of two). It is well known that Sharkovskii's theorem shows the relationship between the cardinal of the set of periodic points of a continuous map, but simple permutations will show the behavior of those periodic points. This paper studies the structure of permutations of mixed order $4n+2$, its properties and a way to describe its genealogy by using Pasting and Reversing as in the case of simple permutations with order a power of two.

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Topological and algebraic reducibility for patterns on trees

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We extend the classical notion of block structure for periodic patterns of interval maps to the setting of tree maps and study the algebraic properties of the Markov matrix of a periodic tree pattern having a block structure. We also prove a formula which relates the topological entropy of a pattern having a block structure with that of the underlying periodic pattern obtained by collapsing each block to a point, and characterize the structure of the zero entropy patterns in terms of block structures. Finally, we prove that an n-periodic pattern has zero (positive) entropy if and only if all n-periodic patterns obtained by considering the k-th iterate of the map on the invariant set have zero (respectively, positive) entropy, for each k relatively prime to n.

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Hofbauer towers and inverse limit spaces

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We use Hofbauer towers for unimodal maps to study the collection of endpoints of the associated inverse limit spaces. We show that if f is a unimodal map for which the kneading map $Q_f(k)$ tends to infinity and $f|_\omega(c)$ is one-to-one, then the collection of endpoints of (I,f) is precisely the set $\mathcal{E}_f = \{(x_0, x_1, \ldots) \in (I,f) \mid x_i \in \omega(c) \text{ for all } i \in \mathbb{N}\}$.

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Li-Yorke chaos in rational difference equations

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Given the difference equation

$$x_{k+1} = f(x_k)$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ is differentiable, a result of Marotto (1978) stated that if f has a *snap-back repeller*, then the equation is chaotic in Li-Yorke sense. When every component of f is a quotient of n-variables polynomials, we have a *rational diference equation.* In such case f can have discontinuity points. The union of such points is the *forbidden set of the equation.* In the case of rational difference equations with non-empty forbidden set, the existence of snap-back repellers does not guarantee Li-Yorke chaos. We will present examples on it and talk on the general validity of Marotto's result. In particular we will use the additional condition on f of having the property of *compact preimage* as a tool to obtain Li-Yorke chaos. Additionally, we will explain the obtained results using as a model the discontinuous rational equation $x_{k+1} = \frac{1}{x_k^2 - 1}$

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Omega-limit sets of quadratic maps on their Julia sets

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We provide a characterization of *omega*-limit sets for dendritic Julia sets of quadratic maps, using Baldwin's symbolic representation of these spaces as a non-Hausdorff itinerary space. The property of shadowing tells us that a map is stable under small perturbations of orbits, and internal chain transitivity tells us that pairs of points can always be linked using arbitrarily small perturbations of orbits. We show that the quadratic maps with dendritic Julia sets have shadowing, and that for all such maps, a closed set is an *omega*-limit set if, and only if, it has internal chain transitivity.

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Periods of periodic orbits for vertex maps on graphs

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Given a graph G with n vertices, a *vertex map* f from G to itself is a continuous map that permutes the vertices of G. We only consider examples for which the n vertices form one periodic orbit. The basic question is to classify the set of the periods of the periodic orbits that f must have. In this setting, Sharkovsky's Theorem gives the answer when the underlying graph is simply a path. The set of periods that must occur in this case is the set of positive integers forced in the Sharkovsky order by n.

This talk will look at Sharkovsky-type orderings for vertex maps of general graphs.

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Aperiodic Cantor dynamics

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A homeomorphism T of a Cantor set X is called aperiodic if for every point $x \in X$ the orbit $(T^n(x))$ is infinite. The pair (X, T) represents a Cantor aperiodic dynamical system. In my talk I will first discuss the topological properties of the set of all aperiodic homeomorphisms considered as a subset of $Homeo(X)$. Every aperiodic homeomorphism admits a realization as a Vershik map acting on a path space of a Bratteli diagram. The second part of my talk will be focused on aperiodic homeomorphisms whose Bratteli-Vershik models are represented by Bratteli diagrams of the simplest form: stationary and finite rank diagrams. For such diagrams we can explicitly describe the set of ergodic invariant measures.

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Patterns, topological transitivity, and entropy

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Let f denote a continuous map of the compact interval I to itself. We use the term pattern to denote an ordered pair (P, π) where $P = \{x_1 < x_2 < \cdots < x_n\}$ is an ordered finite subset of the real line, and $\pi : P \to P$. We say that f exhibits the pattern (P, π) if and only if there exists a finite subset $Q = \{y_1 < y_2 < \cdots < y_n\}$ of I such that $f(y_i) = y_j$ if and only if $\pi(x_i) = x_j$. Given a pattern (P, π) we have an associated piecewise linear $L_{(P,\pi)}$ sometimes called the linearization or connect the dots map. Suppose that f exhibits a pattern (P, π). Suppose that f is topologically transitive. We prove that the topological entropy of f is equal to the topological entropy of $L_{(P,\pi)}$ if and only if f is topologically conjugate to $L_{(P,\pi)}$. It is known that the topological entropy of f is the supremum of the topological entropies of the maps $L_{(P,\pi)}$, where the supremum is taken over all patterns exhibited by f. In fact, it is sufficient to take only those patterns (P, π) such that π is a cyclic permutation. Our theorem implies that for topologically transitive maps the supremum is actually a maximum if and only if f is topologically conjugate to $L_{(P,\pi)}$, for some pattern (P, π) exhibited by f.

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On dynamics of surface homeomorphisms with invariant continua

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Examples of homeomorphisms of $\mathbb{S}^2$ with circle-like continua as minimal sets, or attractors, are well known. For example, there is the C^∞ area-preserving diffeomorphism of Handel that is minimal on the pseudo-circle. On the other end of the spectrum a hereditarily decomposable example was given by Walker, by an extension of a minimal homeomorphism similar to the one constructed by Gottschalk and Hedlund on a union of infinitely many copies of the Warsaw circle. We shall discuss the dynamics of self-homeomorphisms of hereditarily decomposable circle-like continua, including the existence of homeomorphisms of type 2^∞. As a tool, a characterization of covering spaces of these continua will be given. The proofs are built upon related results obtained by Barge&Gilette, Bellamy&Lewis, Heath, Ingram, Minc&Transue, Mouron, and Ye. Time permitting we shall also discuss some new fixed point results for torus homeomorphisms.

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Analysis of an infinite dynamical system using substitution systems

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We consider an infinite dynamical system obtained by iteration of functions of a class of differentiable functions, $\mathcal{A}$, under m-modal maps f. If we consider functions in the class $\mathcal{A}$ whose critical values are periodic points for f then we define and characterize a substitution system associated with the dynamical system. With these substitution systems, we analyze the behavior of the whole system. Moreover, we consider a new subclass of functions where the substitution rules are independent of the initial conditions.

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Dynamics on tiling spaces, invariant measures and generalized Thurston semi-norm

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The problem of deciding whether a given finite set of tiles can tile the euclidean plane is known to be an undecidable problem. An aim of this work is to translate this undecidability in a purely topological and geometrical way. When non-empty the set of tilings of the euclidean plane constructed from a given finite set of tiles $\mathcal{T}$ inherits a natural structure of compact metric space: this is a compact laminated space with transverse structure a Cantor set, equipped with an action of R^2 on the leaves of the lamination. There is then a non-empty set of invariant measures: each one defines a certain homology-class in the second homology group of a branched surface constructed from $\mathcal{T}$. The aim of the current work is to characterize, among all the homology classes, those coming from invariant measures on the laminated space. This is done by the introduction of a kind of Thurston semi-norm: the homology-classes one is looking for are exactly those on which this semi-norm vanishes.

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Ergodic theory and topological intersections as a tool to solve geometrical problems

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This talk presents a method to reduce certain (very difficult) geometrical problems to (less difficult) topological intersection problems, and reduce these again to (even less difficult) problems in ergodic theory. While the resulting problems are still not easy, they can actually be solved. The talk illustrates these methods to solve some old problems concerning dynamical systems arising from Riemannian geometry.

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Maximally transitive semigroups of matrices

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We discuss the problem of finding a pair of $n \times n$ matrices that generates a topologically n-transitive semigroup action on $\mathbb{K}^n$, where $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. Equivalently, we construct dense 2-generator subsemigroups of $GL(n, \mathbb{K})$ as well as $SL(n, \mathbb{K})$ for all $n \geq 1$.

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Devaney chaos and singularities of invertible piecewise isometric dynamics

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It is known that piecewise isometric dynamical systems can exhibit Devaney-chaos in an appropriate invariant set. Clearly, the chaotic behavior comes from singularity, because the piecewise isometric dynamics is innocuous besides the singularity. However, the clear connection between the afore-mentioned Devaney-chaos and the properties of singularity has never been established. This talk aims to discuss and investigate such connection.

In general, the singularities of bounded invertible piecewise isometric dynamical systems in Euclidean plane can be classified as, removable, sliding and shuffling singularities according to their geometrical aspects. Furthermore, it is proved that the removable singularities and the shuffling singularities do not generate the Devaney-chaos. Thus, the presence of the sliding singularities is necessary in order to have the chaotic dynamics. Through this talk, the speaker will survey these results and discuss some new results on the unsolved half, the sufficiency. If time permits, the speaker will talk about a possible connection between the shuffling singularity and the ergodicity as well.

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Infinite-dimensional topology and the Hilbert-Smith conjecture

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The Hilbert-Smith Conjecture states that if G is a compact topological group acting effectively on a connected manifold M^n, then G is a Lie group. The conjecture is false if and only if it is not possible for any p-adic group Δ_p to act effectively on some manifold. There are classic results showing that the conjecture is true for $n = 1$ and $n = 2$. It has also been recently shown for $n = 3$ by John Pardon.

We present a new approach to this classic problem using the theory of compactifications. We prove several theorems using this approach and give several examples from infinite-dimensional topology illustrating their application.

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Bizarre topology is natural in complex dynamical systems

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A continuum is a closed bounded connected set. When a dynamical system is complex the presence of complicated continua can be expected. A much-studied example is the Poincare map on the cylinder determined by the forced dampled pendulum for some parameters. In this example there are at least three disjoint basins of attraction with the boundary of the basins being a Lakes of Wada continuum - a thin fractal set that has the property that each point of the continuum is in the boundary of all three basins. We discuss this example and a number of others. Some of the work is joint with Miguel Sanjuan and James Yorke and some is due to other authors.

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A lower bound for the maximum topological entropy of $4k + 2$-cycles

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Lluis Alseda, David Juher

In recent work, we have started an investigation of the outstanding cases for the problem of classifying cycles with maximum topological entropy. We will present an overview of the problem, discuss the state of known results, and introduce our recent work. For continuous interval maps we formulate a conjecture on the shape of the cycles of maximum topological entropy of period $4k + 2$. We also present numerical support for the conjecture.

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On almost specification and average shadowing properties

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I am going to present our results describing the properties of systems exhibiting average shadowing and/or almost specification property. We have explored recurrence properties of dynamical systems with the average shadowing property and proved that every dynamical system with the average shadowing property and full invariant measure is topologically weakly mixing. Moreover, I will present examples showing that without the assumption on the invariant measure, there is no recurrence property that is implied by the average shadowing property. Another theorem states that the almost specification implies average shadowing property and that f has the average shadowing property if f restricted to the closure of the union of all supports of invariant measures has the average shadowing property.

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Shadowable chain transitive sets of C^1-vector fields

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Let X be a C^1-vector field on a closed smooth manifold M. We show that for C^1-generic X, if a locally maximal chain transitive set is shadowable for X, then the chain transitive set is hyperbolic.

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Robust dynamics of C^1-generic diffeomorphisms

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A basic problem in differentiable dynamical system is to understand how a robust dynamical property (meaning a property that holds for a system as well as all C^1 nearby systems) on the underlying manifold would influence the behavior of the tangent map on the tangent bundle. In this talk we discuss some recent and some ongoing works on the robust dynamics of C^1 generic diffeomorphisms.

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Central strips of sibling leaves in laminations of the unit disk

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Quadratic laminations of the unit disk were introduced by W. Thurston as a vehicle for understanding the (connected) Julia sets of quadratic polynomials and the parameter space of quadratic polynomials. The "Central Strip Lemma" plays a key role in Thurston'sclassification of gaps in quadratic laminations, and in describing the corresponding parameter space. We generalize the notion of *Central Strip* to laminations of degree $d \geq 2$ and prove a Central Strip Lemma for degree $d \geq 2$ that may play a similar role for higher degree laminations. We provide an example of an application to cubic laminations.

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Tiles in convex dynamics; error diffusion on simplices: invariant regions, tessalations and acuteness

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The error diffusion algorithm can be represented as a time dependent dynamical system, generated by piecewise isometries, in fact by translations acting on a partition of the phase space. Long term behaviour of the algorithm can be deduced from the asymptotic properties of invariant sets, especially from the absorbing ones. We study a special case of the translations generated by vectors from the vertices of a simplex to its internal point. Each translation acts on an element of partition, which in a special case is a Voronoi partition relative to the vertices of this simplex. Such systems arise from the error diffusion algorithm with constant input.

We study the properties of the minimal absorbing invariant set and prove that typically those sets are bounded fundamental sets for a discrete lattice generated by the simplex and that the partition parts of those sets are fundamental sets for specific derived lattices.

Theorem A [Ergodic Inputs] For acute simplices the minimal absorbing set for the error diffusion with an ergodic constant input is a fundamental set for the lattice generated by the simplex. Theorem B [SubTiles] If a bounded forward invariant set of a generalized (arbitrary partition) error diffusion on a simplex is fundamental for the simplex lattice, then each part of this invariant set (intersection with the partition) is a fundamental for a derived lattice.

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Weak product recurrence and related properties

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A point x is F-product recurrent, if it is recurrent in pair with any point whose recurrent times belong to family F.In this talk we will survey recent advances on classification of product recurrence and remaining open problems.

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Memory loss for time-dependent dynamical systems

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We discuss recent results on memory loss for time-dependent dynamical systems.

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Dstributional chaos – recent progress and open problems

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In 1994, the notion of distributional chaos was introduced for continuous maps of the interval. This notion, now denoted as DC1, is applicable for continuous maps of a compact metric space. Later two weaker notions, DC2 and DC3, were introduced. Currently, there are many versions of this type of chaos, and hundreds paper has been written on this subject. It is known that positive topological entropy does not imply the strongest version of distributional chaos. The most recent, and very nice result by Tomasz Downarowicz, that for a continuous map of a compact metric space, positive topological entropy implies DC2, has several consequences and yields interesting open problems. We point out some of them.

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Strange chaotic triangular maps

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We show that in the class $\mathcal{T}$ of the triangular maps $(x, y) \mapsto (f(x), g_x(y))$ of the square there is a family of maps of type 2^∞ which are Li-Yorke chaotic on a minimal set, but not distributionally chaotic in the weaker sense, *DC2*. This result makes possible to answer an open question concerning classification of maps in $\mathcal{T}$ with zero topological entropy, and contributes to an old problem formulated by A. N. Sharkovsky.

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Disjunctive cuts, lattice-free sets and mixed-integer programming

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In the talk we describe the topic of mixed-integer programming. The talk is addressed to a wide audience, no specific knowledge of MIP algorithms is assumed. The results are an outcome of combining forces of domain expert computer scientists and mathematicians not working on MIP problems before. We discuss how this subject is related to geometry of convex sets by means of disjunctive cuts introduced by Li and Richard in 2008. We present new results describing the complexity of such algorithms which lead to interesting geometric questions about lattice-free convex bodies. By analyzing n-dimensional lattice-free sets, we prove that every facet-defining inequality of the convex hull of a mixed-integer polyhedral set with n integer variables is a t-branch split cut for some positive integer t. Moreover, this number t does not depend on the data defining the polyhedral set and is bounded by a function of the dimension n only. We use this result to give a finitely convergent cutting-plane algorithm to solve mixed-integer programs. We also show that the minimum value t, for which all facets of polyhedral mixed-integer sets with n integer variables can be expressed as t-branch split cuts, grows exponentially with n. In particular, when $n = 3$, we observe that not all facet-defining inequalities are 6-branch split cuts. We analyze the cases when $n = 2$ and $n = 3$ in detail, and show that an explicit classification of maximal lattice-free sets is not necessary to express facet-defining inequalities as branching disjunctions with a small number of atoms.

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Nonlinearly perturbed heat equation: a symbolic approach

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We consider the linear heat equation with appropriate boundary conditions in order to model the temperature on a wire with adiabatic endpoints. We assume there is some perturbation which provokes a change in the temperature of the wire. This perturbation, which is modelled by an iterated nonlinear map of the interval f, occurs periodically and in this case we observe a stabilization on the number of new critical points of heat function. We study the parameters in order to characterize these behaviours and its dependence on the topological invariants of f.

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