Special Session 24: Geometric Mechanics

Tom Mestdag, Ghent University, Belgium
Manuel de Leon, Instituto de Ciencias Matematicas (CSIC-UAM-UC3M-UCM), Spain
Frans Cantrijn, Ghent University, Belgium
Aziz Hamdouni, University of La Rochelle, France
Dina Razafindralandy, LEPTIAB, France

Mathematical modelling of dynamical systems plays an important role in many branches of science. Since the second half of last century, differential geometry has developed into a mathematical discipline with an ever growing impact on the construction of such models. In particular, "geometric mechanics" has become the common name that is given to those research activities that are devoted to the application of differential geometry in various fields of theoretical physics such as classical mechanics (Newtonian, Lagrangian and Hamiltonian mechanics), continuum mechanics, dynamical systems theory, control theory and quantum mechanics.

Using the powerful tools and techniques of Riemann geometry, contact geometry, symplectic and Poisson geometry, and exploiting the properties of Lie groups, fibre bundles, jet bundles, connections, distributions, etc., geometric mechanics has contributed a lot to the description and analysis of the structure and properties of mechanical systems. In addition, many of these geometrical ideas have found an extension to field theories, classical and quantum, such as general relativity, classical and quantum gauge theories.

This session has a double goal: to promote the AIMS Journal of Geometric Mechanics by bringing together excellent researchers in the field and to provide a scientific platform for the international partners within the 'Geometric Mechanics' network (a network within Marie Curie's International Research Staff Exchange Scheme (IRSES) in the 7th European Framework Program).

Generalized Navier-Stokes flows

Marc Arnaudon
University of Poitiers, France
marc.arnaudon@math.univ-poitiers.fr
Ana Bela Cruzeiro

We introduce a notion of generalized Navier-Stokes flows on manifolds, that extends to the viscous case the one defined by Brenier. Their kinetic energy extends the kinetic energy for classical Brownian flows, defined as the L^2 norm of their drift. We prove that there exists a generalized flow which realizes the infimum of kinetic energies among all generalized flows with prescribed initial and final configuration. Following a method of Ocone and Pardoux we construct generalized flows from solutions to finite variation transport equations. They are flows with prescribed drift, and their kinetic energy is smaller than the L^2 norm of the solutions to the transport equation.

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On the geometry of nonholonomic systems

Paula Balseiro
Universidade Federal Fluminense, Brazil
pbalseiro@vm.uff.br

As it is known, nonholonomic systems are characterized by the failure of the Jacobi identity of the bracket describing the dynamics. In this talk I will present different (geometric) technics to deal with the failure of the Jacobi identity and we will see how twisted Poisson structures might appear once we reduce the system by a group of symmetries.

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The geometry of integrable and gradient flows and dissipation

Anthony Bloch
University of Michgan, USA
abloch@umich.edu

In this talk I will discuss the dynamics and geometry of various integrable systems that exhibit asymptotic stability and dissipative behavior, as well as dissipative perturbations of integrable systems. Examples include the finite Toda lattice, the dispersionless Toda partial differential equation and certain nonholonomic systems I will describe the geometric structures, including metric and complex structures, that give rise to some of these flows and determine their behavior. This includes work with P. Morrison and T. Ratiu.

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Some applications of some geometric integrators

Marx Chhay
LOCIE, University of Savoie, France
marx.chhay@univ-savoie.fr

PDE's geometric structure contains the physical informations traduced by the mechanical model. One can expect from numerical methods to preserve this structure. We present different constructions of some popular geometric methods. Comparisons and performances are performed on illustrative examples.

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On the geometry of mechanical control systems on Lie groups

Leonardo Colombo
ICMAT, Spain
leo.colombo@icmat.es
David Martín de Diego

In this talk we will describe a geometric setting for higher-order lagrangian problems on Lie groups. Using left-trivialization of the higher-order tangent bundle of a Lie group and an adaptation of the classical Skinner-Rusk formalism, we will deduce an intrinsic framework for this type of dynamical systems. Interesting applications as, for instance, a geometric derivation of the higher-order Euler-Poincaré equations, optimal control of underactuated control systems with symmetries, etc, will be considered.

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Stochastic Euler-Poincaré reduction on Lie groups

Ana Bela Cruzeiro
Dep.Mathematics IST and GFMUL, Portugal
abcruz@math.ist.utl.pt
Marc Arnaudon, Xin Chen

A Euler-Poincaré reduction theorem for stochastic processes taking values in a Lie group is presented, as well as some examples of its application to $SO(3)$ and to the group of diffeomorphisms.

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Hamilton-Jacobi theory for classical field theories

Manuel de Leon
Instituto de Ciencias Matematicas, ICMAT, Spain
mdeleon@icmat.es

We will present a Hamilton-Jacobi theory for classical field theories based on the multisymplectic formalism. In addition, we will develop the corresponding theory on the space of Cauchy data.

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Variational integrators for hamiltonizable nonholonomic systems

Oscar Fernandez
Wellesley College, USA
fernandez.um@gmail.com
Anthony M. Bloch, Peter J. Olver

I will discuss some new applications of the Poincaré and Sundman time-transformations to the simulation of nonholonomic mechanical systems. We will see how the application of these transformations permits the usage of variational integrators for these non-variational mechanical systems. Two new geometric integrators for nonholonomic systems known to be Hamiltonizable (briefly, nonholonomic systems whose constrained mechanics are Hamiltonian after a suitable reparameterization of time) will be discussed, along with examples and numerical results comparing the results of the new integrators to those obtained by applying a standard nonholonomic integrator.

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Invariant higher-order variational problems and computational anatomy

Francois Gay Balmaz
Ecole Normale Superieure de Paris, France
gaybalma@lmd.ens.fr
D. Holm, D, Meier, T. Ratiu, F.-X. Vialard

Motivated by applications in computational anatomy, we consider a second-order problem in the calculus of variations on object manifolds (or shapes) that are acted upon by Lie groups of smooth invertible transformations. This problem leads to solution curves known as Riemannian cubics on object manifolds that are endowed with normal metrics. The prime examples of such object manifolds are the symmetric spaces. We characterize the class of cubics on object manifolds that can be lifted horizontally to cubics on the group of transformations. Conversely, we show that certain types of non-horizontal geodesics on the group of transformations project to cubics. Finally, we apply second order Lagrange-Poincaré reduction to the problem of Riemannian cubics on the group of transformations. This leads to a reduced form of the equations that reveals the obstruction for the projection of a cubic on a transformation group to again be a cubicon its object manifold.

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Lagrangian submanifolds and classical field theories of first order on Lie algebroids

Elisa Lavinia Guzman Alonso
University of La Laguna, Spain
eguzman@ull.es
Juan Carlos Marrero, Joris Vankerschaver

A description of classical field theories in the context of Lie Algebroids in terms of Lagrangian submanifolds of premultisymplectic manifolds is presented. For this purpose, a Tulczyjew's triple associated with a fibration is discussed. The triple is adapted to the extended Hamiltonian formalism. Using this triple, we prove that Euler-Lagrange and Hamilton equations are the local equations defining Lagrangian submanifolds of a premultisymplectic manifold.

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Geometric control of electromagnetic docking

Marin Kobilarov
California Institute of Technology, USA
marin@cds.caltech.edu
Gwendolyn Johnson

The talk considers geometric control method for stabilization and trajectory tracking of electromagnetic propulsion systems. The work is motivated by the development of robust randezvous and docking systems for small satellites. The main difficulty lies in dealing with complex pose-dependent control vector fields arising from the magnetic field interaction. We will address the issue of underactuation and discuss stability results.

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Schroedinger problem and Ricci curvature of graphs

Christian Leonard
Universite Paris Ouest, France
christian.leonard@u-paris10.fr

The Schroedinger problem is a statistical physics analogue of the Monge-Kantorovich optimal transport problem. Its solution is a stochastic process with prescribed initial and final marginal probability measures. The time-marginal flow of this optimal process is an entropic interpolation between the endpoint marginals which is a stochastic deformation of McCann's interpolation. It allows both recovering the basic results of the Bakry-Emery theory on a Riemannian manifold and extending it to a graph structure, suggesting a natural definition of Ricci curvature on a graph. We also recover modified logarithmic inequalities and derive transport inequalities under the assumption that the Ricci curvature is bounded below. This approach can be viewed as entering a stochastic deformation of the Lott-Sturm-Villani theory of curvature of metric measure length spaces.

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Hamilton-Jacobi theory and hamiltonian systems with respect to fiber-wise linear Poisson structures

Juan Carlos Marrero
University of La Laguna, Spain
jcmarrer@ull.es

It is well-known that the existence of a fiber-wise linear Poisson structure on a vector bundle is equivalent to the existence of a Lie algebroid structure on the dual bundle. Assume that this Lie algebroid is integrable, that is, it is the Lie algebroid AG of a Lie groupoid G and that $h : A^*G \to R$ is a hamiltonian function on the dual bundle A^*G. Then, in this talk, we will see that it is possible to reconstruct the flow of the corresponding hamiltonian vector field from a complete solution of the time-dependent Hamilton-Jacobi equation associated with a suitable hamiltonian function on the cotangent bundle T^*G of the Lie groupoid G.

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On discrete mechanics for optimal control theory

David Martin de Diego
ICMAT, Spain
david.martin@icmat.es
F. Jimenez, M. Kobilarov

During this talk we will discuss numerical methods for optimal control of mechanical systems in the Lagrangian setting. It extends the theory of discrete mechanics to enable the solutions of optimal control problems through the discretization of variational principles. The key point is to solve the optimal control problem as a variational integrator of a specially constructed higher-dimensional system. The developed framework applies to systems on general manifolds, Lie groups, underactuated and nonholonomic systems, and can approximate either smooth or discontinuous control inputs. The resulting methods inherit the preservation properties of variational integrators and result in numerically robust and easily implementable algorithms. The control of an underwater vehicle, will illustrate the application of the proposed approach.

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Involutive distributions and dynamical systems of second-order type

Tom Mestdag
Ghent University, Belgium
tom.mestdag@ugent.be

We investigate the existence of coordinate transformations which bring a given vector field on a manifold equipped with an involutive distribution into the form of a second-order differential equation field with parameters. We define associated connections and we give a coordinate-independent criterion for determining whether the vector field is of quadratic type. Further, we investigate the underlying global bundle structure of the manifold under consideration, induced by the vector field and the involutive distribution. We illustrate the results in the context of so-called mechanical control systems, and we apply them to Routh reduction of mechanical systems with an Abelian symmetry group.

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An extension of the Marsden-Weinstein reduction process to the symplectic algebroid setting

Edith Padron
University of La Laguna, Spain
mepadron@ull.es
J.C. Marrero, M. Rodríguez-Olmos

In this talk we will show a reduction theorem for Lie algebroids with respect to a Lie group action by complete lifts. This result allows to obtain a Lie algebroid version of the classic Marsden-Weinstein reduction theorem for symplectic manifolds. Additionally, we will apply it to the particular case of the canonical cover of a fiberwise Poisson structure.

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Projective symmetry in Randers spaces

Mehdi Rafie-Rad
University of Mazandaran, Iran
rafie-rad@umz.ac.ir

A significant extent of researches in the contexts is devoted to study symmetries in Mathematical Physics to be applied in dynamical systems. In each case, characterizing the maximum symmetry is of interests and includes considerations on the dimension of the transformation groups and the Lie algebra of vector fields. A class of important symmetries in Physics are the projective symmetries. On the other hand, the Randers metrics are the most original Finsler metrics in Physics. This work is to study the projective symmetry in Randers spaces. In particular, we prove that the Randers spaces with maximum projective symmetry are locally projectively flat. This extends an analogue result in Riemannian geometry. Some non-Riemannian invariants and specifically important projective symmetry are also studied.

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Lie goup theory in turbulence

Dina Razafindralandy
LEPTIAB, France
drazafin@univ-lr.fr
Aziz Hamdouni

Lie group theory is used to analyse turbulent non-isothermal fluid flows. Using the symmetry properties of the correlation equations, new scaling laws for velocity and for temperature are computed. Next, a symmetry-preserving turbulence model for the subgrid stress tensor and the subgrid heat flux is developped and numerically tested.

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Newtonoids vector fields and conservation laws on the Lagrangian k-symplectic formalism

Modesto Salgado
Universidad de Santiago de Compostela, Spain
modesto.salgado@usc.es

In this talk we discuss symmetries, Newtonoid vector fields, conservation laws, Noether Theorem and its converse, in the framework of the k-symplectic formalism.For the k=1 case it is well known that Cartan symmetries induce and are induced by conservation laws, and these results are known as Noether Theorem and its converse. For k¿1, we provide a new proof that Noether Theorem is true, and hence each Cartan symmetry induces a conservation law. We show that under some assumptions, the converse of Noether Theorem is true and provide examples when this is not. We also study the relation between dynamical symmetries, Newtonoid vector fields, Cartan symmetries and conservation laws, showing when one will imply the others. We use several examples of partial differential equations to illustrate when these concepts are related and when they are not.

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Invariant metrics on Lie groups

Gerard Thompson
University of Toledo, USA
gerard.thompson@utoledo.edu

We will investigate integrability properties of left-invariant metrics on low-dimensional Lie groups. We will consider Killing's equations of degree one and higher and also Hamiltonian-Jacobi separable systems.

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Stochastic methods for Navier-Stokes equations

Gazanfer Unal
Yeditepe University, Turkey
gunal@yeditepe.edu.tr

Using a stochastic transformation we obtain stochastic Navier-Stokes equations in the sense of Ito-Skorokhod. We seek for exact solutions to the latter by utilizing both anticipative calculus and symmetry methods.

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The geometry of multi-Dirac structures

Joris Vankerschaver
University of California, San Diego, USA
joris.vankerschaver@gmail.com
Hiroaki Yoshimura, Melvin Leok

In this talk, I will introduce the concept of a multi-Dirac structure, which is a graded analogue of the concept of a usual Dirac structure. After discussing some aspects of the geometry of multi-Dirac structures, I will point out how these structures can be used for the description of classical field theories, and I will show how a multi-Dirac structure on a jet bundle gives rise to an infinite-dimensional Dirac structure on the space of fields when a 3+1 decomposition has been selected.

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Hamilton-Jacobi theory for Singular Lagrangians

Miguel Vaquero
ICMAT, Spain
miguel.vaquero@icmat.es
Manuel de León, Juan Carlos Marrero, David Martín de Diego

In classical mechanics regular lagrangians leads to canonical hamiltonian formalism. For singular lagrangians Dirac developed a theory of constrainsts that describes the dynamics for such lagrangians. It is well known the important role played by the Hamilton-Jacobi theory in integrating Hamilton's equation when the lagrangian is regular. In this talk, we introduce a Hamilton-Jacobi theory for almost regular lagrangians in the Skinner-Rusk setting. Some examples and future developments will be also discussed.

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A very general Hamilton-Jacobi theorem

Luca Vitagliano
DipMat, University of Salerno, Italy
lvitagliano@unisa.it

The classical Hamilton-Jacobi theorem for an Hamiltonian system with Hamiltonian $H = H(x,p)$ states that $S = S(x)$ is a solution of the Hamilton-Jacobi equation iff for any solution $x(t)$ of the ODE $\frac{dx}{dt} = \frac{\partial H}{\partial p}(x, dS(x))$, $(x(t), dS(x(t)))$ is a solution of the Hamilton equations. Thus it may be understood as a way of finding (some) solutions of a certain ODE by lifting solutions of a simpler ODE. I will present a wide, field theoretic generalization of the Hamilton-Jacobi theorem within the geometric theory of PDEs on jet spaces.

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Twisted angles for central configurations formed by two twisted regular polygons

Xiang Yu
Sichuan University, Peoples Rep of China
xiang.zhiy@gmail.com
Zhang Shiqing

In this paper, we study the necessary conditions and sufficient conditions for the twisted angles of the central configurations formed by two twisted regular polygons, in particular, we prove that for the 2N-body problem, the twisted angles must be $\theta = 0$ or $\theta = \pi/N$. And we study also the necessary conditions and sufficient conditions for the existence of the central configurations formed by two twisted regular polygons.

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Stochastic geometric mechanics

Jean-Claude Zambrini
GFMUL, Lisbon University, Portugal
jczambrini@gmail.com

We shall describe a way to deform stochastically the main tools of Geometric Mechanics. And, in particular, consider various aspects of integrability in this context.

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Variational principles for Hamel's equations

Dmitry Zenkov
North Carolina State University, USA
dvzenkov@ncsu.edu
Kenneth Ball, Anthony Bloch

Hamels' equations are a generalization of the Euler-Lagrange equations of Lagrangian mechanics obtained by measuring the velocity components relative to a frame that is not associated with system's configuration coordinates. These equations often simplify the representation of system's dynamics. This talk will elucidate the variational nature of Hamel's equations.

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