Special Session 26: Qualitative Aspects of Nonlinear Boundary Value Problems

Marta Garcia-Huidobro, Catholic University of Chile, Chile
Raul Manasevich, University of Chile, Chile
James Ward, University of Alabama at Birmingham, U.S.A.

In this session we will deal with qualitative aspects of nonlinear boundary value problems such as existence and multiplicity of solutions, dependence upon data and parameters, bifurcation, and topological properties of solutions and global structures. In the case of the periodic boundary value problem, qualitative aspects include Lyapunov stability or instability of solutions. We will consider scalar as well as systems of boundary value problems

Variational methods for nonlinear perturbations of the mean curvature operator in Minkowski space

Cristian Bereanu
Institute of Mathematics Simion Stoilow, Romania
cristian.bereanu@imar.ro
P. Jebelean, J. Mawhin

In this talk we present various existence and multiplicity result for Neumann boundary value problems associated to nonlinear perturbations of the mean curvature operator in Minkowski space. The main tool is Szulkin's critical point theory for smooth perturbations of convex and lower semi-continuous functionals.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On bound state solutions with a prescribed number of sign changes

Carmen Cortazar
P. Universidad Catolica de Chile, Chile
ccortaza@mat.puc.cl
Marta Garcia Huidobro, Cecilia Yarur

We consider radial solutions of the problem

$$\Delta u + f(u) = 0$$
$$\lim_{r\to\infty} u(r) = 0$$

with a prescribed number of sign changes. Conditions on f are established that guarantee the existence and the uniqueness of such solutions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On positive solutions for a class of Caffarelli-Kohn-Nirenberg type equations

David Costa
University of Nevada Las Vegas, USA
costa@unlv.nevada.edu
J. Chabrowski

We investigate the solvability of singular equations of Caffarelli-Kohn-Nirenberg type having a critical-like nonlinearity with a sign-changing weight function. We examine how the properties of the Nehari manifold and the fibering maps affect the question of existence of positive solutions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nonlocal maximum principles and applications

Ricardo Enguiça
Centro de Matematica e Aplicacoes Fundamentais, Portugal
rroque@adm.isel.pt
Luis Sanchez

We will make some remarks about the equation

$$-u''(t) - \frac{k}{t}u'(t) + \lambda^2 u(t) = h(t), \quad t \in (0,1],$$

considering some weighted Sobolev spaces and analyse special features of the associated Green's function. This will be used to derive some nonlocal maximum principles relevant in the search for solutions between lower and upper solutions of the nonlocal boundary value problem

$$\begin{cases} -u''(t) - \frac{n-1}{t}u'(t) = f\left(u(t), \omega_n \int_0^1 s^{n-1} g\left(u(s)\right)\, ds\right) \\ u'(0) = 0 = u(1). \end{cases}$$

$\longrightarrow \infty \diamond \infty \longleftarrow$

Boundary blow up of nonnegative solutions of some elliptic systems

Marta Garcia Huidobro
Pontificia Universidad Catolica de Chile, Chile
mgarcia@mat.puc.cl

We study the nonnegative solutions of the elliptic system

$$\Delta u = |x|^a v^\delta, \qquad \Delta v = |x|^b u^\mu,$$

where a, b are real numbers such that$a, b > \max\{-2, -N\}$, in the superlinear case $\mu\delta > 1$, which blow up near the boundary of adomain of $\mathbb{R}^N$, or at one isolated point. In the radial case wegive the precise behavior of the large solutions near the boundary in anydimension N. We also show the existence of infinitely many solutions blowingup at 0. Furthermore, we show that there exists a global positive solutionin $\mathbb{R}^N \setminus \{0\}$, large at 0, and wedescribe its behavior. We apply the results to the sign changing solutions ofthe biharmonic equation

$$\Delta^2 u = |x|^b \, |u|^\mu \, .$$

Our results are based on a new dynamical approach of the radial system bymeans of a quadratic system of order 4. This talk is part of a joint work with Prof. M. Véron and Prof. C. Yarur.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Solutions for a semilinear elliptic equation in dimension two with supercritical growth.

Ignacio Guerra
Universidad de Santiago de Chile, Chile
ignacio.guerra@usach.cl
Manuel del Pino, Monica Musso

We consider the problem

$$\begin{aligned} -\Delta u = \lambda u e^{u^p}, \quad u > 0, \quad \text{in} \quad \Omega, \\ u = 0 \quad \text{on} \quad \partial\Omega, \end{aligned}$$

where $\Omega \subset \mathbb{R}^2$ and $p > 2$. Let λ_1 be the first eigenvalue of the Laplacian. For each $\lambda \in (0, \lambda_1)$, we prove the existence of solutions for p sufficiently close to 2. In the case of Ω a ball, we also describe numerically the bifurcation diagram (λ, u) for $p > 2$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Convergence versus periodicity in a dynamical system arising in the study of a higher-order elliptic PDE

Monica Lazzo
University of Bari, Italy
lazzo@dm.uniba.it
Paul G. Schmidt

Trying to understand the blow-up behavior of large radial solutions of a polyharmonic PDE with power nonlinearity, we are led to analyze the dynamics of a parameter-dependent single-loop positive-feedback system; the dimension of the system corresponds to the order of the PDE. In low dimensions, we observe convergence to equilibrium; in high dimensions, multiple periodic orbits arise via successive Hopf bifurcations. We discuss the dynamics of the system and consequences for the underlying PDE.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A mean curvature type of geometric parabolic equation

Junfang Li
University of Alabama at Birmingham, USA
jfli@uab.edu
Pengfei Guan

In this talk, we will present a parabolic PDE defined on hypersurfaces and its fully non-linear analogue. For any closed star-shaped smooth hypersurface, this flow exists for all time t¿0 and exponentially converges to a round sphere. Moreover, we will show that all the quermass integrals evolve monotonically along this flow. Consequently, we prove a class of isoperimetric type of inequalities including the classical isoperimetric inequality on star-shaped domains. We will also present a fully non-linear analogue of this flow. More specifically, we study a fully non-linear parabolic equation of a function on the standard sphere and discuss its long-time existence and exponential convergence. As applications, we recover the well-known Alexandrov-Fenchel inequalities on bounded convex domains in Euclidean space.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Sign changing solutions with compact support for a nonlinear equation with a p-Laplace operator

Raul Manasevich
University of Chile, Chile
manasevi@dim.uchile.cl
Jean Dolbeault

In this talk we will consider radial solutions for a nonlinear equation with a p-Laplace operator. By a shooting method we prove the existence of solutionswith compact support that have any prescribed number of zeros.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On the energy of the current vector of a complex valued function in $\mathbb{R}^3$

Alberto Montero
P. Universidad Católica de Chile, Chile
amontero@mat.puc.cl

For a complex valued function $u : \Omega \subset \mathbb{R}^3 \to \mathbb{C}$, the current vector is usually defined as $j(u) = \mathrm{Im}(\bar{u}\nabla u)$. In this talk I will present an estimate for $\int_\Omega |j(u)|^2$ in terms of quantities determined by a given curve Γ in Ω. These estimates are valid when $\nabla \times j(u)$ and Γ are sufficiently close as functionals on vector fields.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Eigenvalue-curves and nonlinear second order elliptic equations with nonlinear boundary conditions

M. Nkashama
University of Alabama at Birmingham, USA
nkashama@math.uab.edu
N. Mavinga

We are concerned with the existence of eigenvalue-curves connecting the Steklov and Neumann-Robin spectra of linear second order elliptic equations. We then discuss nonlinear elliptic problems with nonlinear boundary conditions when the reaction and boundary nonlinearities stay in some sense between two consecutive eigenvalue-curves.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nonlinear periodic boundary value problems via initial value problems: generalized quasilinear techniques

Sudhakar Pandit
Winston-Salem State University, USA
pandits@wssu.edu
J. O. Adeyeye, D. H. Dezern

It is well known that the Monotone Iterative (MIT) and the Generalized Quasilinear Technique (GQT) are two important techniques for obtaining solutions, in a constructive way, of a variety of nonlinear Initial value Problems (IVPs) and Periodic Boundary Value Problems (PBVPs). Whereas the rate of convergence of the iterates in the former technique is linear, that in the latter is quadratic, and hence more rapid. In this talk, we present a new approach to solve nonlinear PBVPs by employing the GQT. The fact that the iterates in our approach are solutions of (linear) IVPs (as opposed to PBVPs, in the conventional approach), and converge to the multiple solutions (or the unique solution, under additional conditions) of the given nonlinear PBVP, allows us to dispense with many a hypothesis required in the conventional approach. We provide graphical and numerical examples to support the results in our new approach.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Hyperbolic fractional Laplacian

Mariel Saez
Pontificia Universidad Catolica, Chile
mariel@mat.puc.cl
V. Banica, M.d.M. González, M. Sáez

In this talk I will discuss an appropriate definition for the fractional Laplacian on Hyperbolic Space. In the spirit of the work of L. Caffarelli and L.Silvestre, we show that we can associate an extension problem to our definition.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Oscillatory entire solutions of polyharmonic equations with subcritical growth

Paul Schmidt
Auburn University, USA
pgs@auburn.edu
Monica Lazzo

Much of the literature on entire solutions for semilinear elliptic equations with power nonlinearities focuses on positive solutions, whose existence requires critical or supercritical growth. Even assuming radial symmetry, the subcritical case is completely understood only for second-order problems. We recently established the existence and uniqueness (up to scaling) of oscillatory entire radial solutions for subcritical fourth-order problems. We discuss the asymptotic behavior of these solutions and possible generalizations for higher-order problems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On the nonuniqueness of positive solutions for a class of superlinear problems

Satoshi Tanaka
Okayama University of Science, Japan
tanaka@xmath.ous.ac.jp

The following two-point boundary value problem is considered:$u''+h(x)f(u)=0$ on $(-1,1)$; $u(-1)=u(1)=0$.Here, $h\in C[-1,1]\cup C^1([-1,1]\backslash\{0\})$, $h(x)>0$ on $[-1,1]\backslash\{0\}$, $f\in C^1[0,\infty)$, $f(s)>0$ for $s>0$ and $f(0)=0$. In addition, the superlinear condition $sf'(s)>f(s)$ on $(0,\infty)$ issatisfied. In many cases, the positive solution of this problem is unique. Indeed, if $h(x)=1$, then the problem has at most one positive solution. However, for example, if $f(s)=s^p$, $p>1$, $h(x)=|x|^l$ and $l>0$ issufficiently large, then there exist three positive solutions. In this talk, a sufficient condition is derived for the existence of three positive solutions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On some connecting orbits for a class of singular second order Hamiltonian systems

Hossein Tehrani
UNLV, USA
tehranih@unlv.nevada.edu

We present some existence results for connecting orbits emanating from 0 of a class of singular second order Hamiltonian equations of the form

$$u''+V_u(t,u)=0, \qquad t\in\mathbb{R}$$

where the singular potential $V(t,u)$ is T periodic and satisfies Strong-Force condition at the singularity.

$\longrightarrow \infty \diamond \infty \longleftarrow$