Special Session 28: Analysis and Numerics of Differential Equations and Dynamical Systems in Mathematical Fluid Mechanics

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In the last decades great progresses have been made in the field of mathematical fluid mechanics both theoretically and numerically by using theory of differential equations and dynamical systems. The aim of this special session is to bring together researchers in this area to present most recent results, ideas and discuss on future research directions. The topics in this special session will range from mathematical modeling, theoretical analysis including well-posedness and asymptotic behavior with respect to small parameters and large time, and computational methods.

Dynamics of particle settling and resuspension in viscous liquids

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We derive and study a dynamic model for suspensions of negatively buoyant particles on an incline. Our theoretical model includes the settling/sedimentation due to gravity as well as the resuspension of particles induced by shear-induced migration,leading to different regimes observed in the experiments. Using an approach relying on asymptotics, we systematically connect our dynamic model with the previously developed equilibrium theory for particle-laden flows. We show that the resulting transport equations for the liquid and the particles are of hyperbolic type, and study the dilute limit, for which we compute exact solutions. We also carry out asystematic experimental study of the settled regime, focusing on the motion ofthe liquid and the particle fronts. Finally, we carry out numerical simulations of our transport equations. We show that the model predictions agree well with the experimental data.

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Gevrey regularity for dissipative equations with applications to decay.

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The regular solutions of a large class of dissipative equations are well-known to be analytic in both space and time variables. Moreover, the space analyticity radius has an important physical interpretation. It demarcates the length scale below which the viscous effect dominates the (nonlinear) inertial effect. Foias and Temam introduced an effective approach to estimate space analyticity radius via the use of Gevrey norms which, since then, has become a standard tool for studying analyticity properties for dissipative equations. We extend this approach to a class of (possibly nonlocal) equations with analytic nonlinearity in critical invariant spaces. As application, we obtain large time decay of higher derivatives. Applications include the Navier-Stokes, surface quasi-geostrophic, Burger's and Cahn-Hilliard equations among others. This is a joint work with Hantaek Bae.

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On a 1D alpha-patch model

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We consider a one-dimensional α-patch model. A dichotomy result between the finite time blowup and the global in time regularity is obtained. The result is sharp in terms of the range of α.

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Parameter estimation for nonlinear stochastic partial differential equations

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While the general form of a model is commonly derived from the fundamental properties of a physical process under study, frequently parameters arise in the formulation which need to be specified or determined on the basis of empirical observation. Given in particular the growing significance of nonlinear stochastic partial differential equations (SPDE) in applications there is a clear need to develop the theory of parameter estimation for such systems. Under the assumption that a phenomenon of interest follows the dynamics of such an SPDE, and given that some realizations of this process are measured, we wish to find these unknown parameters appearing in the model, such that the equations fit or predict as much as possible this observed data. In this work we discuss some recent results concerning the estimation of the "drift" parameter for a general class of nonlinear SPDE, based on the first N Fourier modes of a single sample path observed on a finite time interval. In particular, we exhibit specific estimators for the viscosity coefficient for the 2D stochastic Navier-Stokes equations, and study asymptotic properties of these estimators.

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Alternating direction second order method for the Navier-Stokes equations

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A fully discretized projection method is studied. It contains a parameter operator. Depending on this operator, we can obtain a first-order scheme, which is appropriate for theoretical analysis, and a second-order scheme, which is more suitable for actual computations. In this method, the boundary conditions of the intermediate velocity field and pressure are not needed. For the comparison, we apply the alternating direction method to the discretized projection method for the Navier-Stokes equations.

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Some mathematical theory of viscous Camassa-Holm equations

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Viscous Camassa-Holm equations, or Navier-Stokes-alpha (NS-α) equations were introduced as a natural mathematical generalization of the integrable inviscid 1D Camassa-Holm equation through a variation formulation. Mathematical theory, including well-posedness, and the existence of global attractors and their Hausdorff and fractal dimension, of viscous Camassa-Holm equations has been established by Foias, Holm and Titi. In this talk we address some further topics regarding the viscous Camassa-Holm equations, including the Gevrey regularity of the solutions, some bifurcation results and lower bounds of the global attractor.

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Bounded vorticity, bounded velocity (Serfati) solutions to 2D Euler equations

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In a short note in 1995, Philippe Serfati established the existence and uniqueness of solutions to the 2D Euler equations in the whole plane when the initial vorticity and initial velocity are bounded. We describe an extension of this result to an external domain in the plane, and discuss related issues of stability.

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On the arrow of time

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We propose a Theory to resolve the problem of the Arrow of Time. The Theory is composed of three ingredients: 1. The equations of the dynamics of gas molecules, 2. Chaotic instability of the equations of the dynamics, 3. unavoidable perturbations to the gas molecules. Numerical simulations on the Theory are conducted.

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Determination of viscosity in an incompressible fluid

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In this talk we study the inverse boundary value problems in fluid mechanics. We present the unique determination of the viscosity function in an incompressible fluid by boundary measurements of the velocity and the force.

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Invariant manifolds of Euler equations

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We consider a linearly unstable steady flow v_0 of the Euler equation in a fixed bounded domain in $\mathbf{R}^n$. By rewriting the Euler equation as an ODE on an infinite dimensional manifold of volume preserving mapping in H^k $(k > 1 + \frac{n}{2})$, the unstable and stable

manifolds of v_0 is constructed under certain spectral gap condition which is verified for both 2D and 3D examples. This implies the nonlinear instability of v_0 in the sense that arbitrarilly small H^k perturbations can lead to L^2 derivation of the solutions.

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Effective viscosity in dilute suspensions

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We give a rigorous derivation of Einstein's formula for the effective viscosity of a dilute suspension of spheres in a finite-size container. The spheres are arranged in a cubic lattice . We employ boundary integral equations for the Stokes operator and include boundary effects. Our proof admits a generalization to other particle shapes and the inclusion of point forces to model self-propelled particles.

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On the exponential decay of the power spectrum and the finite dimensionality for the solutions of the three dimensional primitive equations

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In this talk we study the three dimensional primitive equations, describing the motion of the oceans and of the atmosphere. More exactly, we prove the exponential decay of the spatial power spectrum for the solutions, as well as the finite dimensionality of the flow described by our model.

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On the long-time stability of the implicit Euler scheme for the 2d thermohydraulics equations

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In this article we consider the two-dimensional thermohydraulics equations, we discretize these equations in time using the implicit Euler scheme and with the aid of the classical and uniform discrete Gronwall lemmas, we prove that the scheme is H1-uniformly stable in time.

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A high order WENO scheme for detonation waves

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Chi-Wang Shu, Helen Yee, Bjorn Sjogreen

We propose a high order finite difference WENO method with Harten's ENO subcell resolution idea for the chemical reactive flows. In the reaction problems, when the reaction time scale is very small, e.g., orders of magnitude smaller than the fluid dynamics time scales, the problems will become very stiff. Wrong propagation of discontinuity occurs due to the underresolved numerical solutions in both the space and time. The proposed method is a modified fractional step method which solves the convection step and reaction step separately. A fifth-order WENO is used in convection step. In the reaction step, a modified ODE solver is applied but with the flow variables in the discontinuity region modified by the subcell resolution idea.

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A level set approach for dilute non-collisional fluid-particle flows

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Gas-particle and other dispersed-phase flows can be described by a kinetic equation containing terms for spatial transport, acceleration, and particle processes (such as evaporation or collisions). However, computing the dispersed velocity is a challenging task due to the large number of independent variables. A level set approach for computing dilute non-collisional fluid-particle flows is presented. We will consider the sprays governed by the Williams kinetic equation subject to initial distributions away from equilibrium of the form $\sum_{i=1}^{N} \rho_i(x)\delta(\xi - u_i(x))$. The dispersed velocity is described as the zero level set of a smooth function, which satisfies a transport equation. This together with the density weight recovers the particle distribution at any time. Moments of any desired order can be evaluated by a quadrature formula involving the level set function and the density weight. It is shown that the method can successfully handle highly non-equilibrium flows (e.g. impinging particle jets, jet crossing, particle rebound off walls, finite Stokes number flows).

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Global regularity results for 2D generalized MHD equations

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Chuong V. Tran, Zhichun Zhai

In this talk I will present several recent results on the global regularity for the 2D generalized MHD equations, where the dissipation terms in the usual MHD system are replaced by fractional powers of the Laplacian. I will also discuss some new criteria for global regularity of 2D and 3D generalized MHD system. This is joint work with Prof. Chuong V. Tran of University of St. Andrews, and Dr. Zhichun Zhai.

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