Special Session 32: Existence and Multiplicity Results in Elliptic Variational Problems

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S. Carl, University of Halle, Germany
S. A. Marano, University of Catania, Italy
D. Motreanu, University of Perpignan, France

This session is considered as a platform for the presentation of very recent topics and results in the qualitative study of nonlinear elliptic problems such as, e.g., existence, multiplicity, and comparison principles. Emphasis is put on variational techniques, combined with topological arguments and sub-super-solution methods, in both the smooth and non-smooth framework. Thus, a wide range of elliptic problems in bounded or unbounded domains, with or without constraints will be covered by the speakers of this session.

Multiple solutions for a class of superlinear Neumann problems

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We prove the existence of multiple nontrivial smooth solutions, with precise sign information, for a Neumann boundary value problem governed by the p-Laplacian, with a superlinear perturbation.

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Multiplicity results to elliptic problems in $\mathcal{R}$

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We present some existence and multiplicity results for the following elliptic problem:
Find $u \in W^{1,p}(\mathcal{R})$ satisfying

$$(P_\lambda) \quad -(|u'(x)|^{p-2}u'(x))' + B|u(x)|^{p-2}u(x) = \lambda\alpha(x)g(u(x)) \ \textit{a.e. in } \mathcal{R},$$

where λ is a real positive parameter, B is a real positive number α and g are nonsmooth functions.
We give an existence result for (P_λ), when the parameter λ varies in a suitable interval. We also show that under additional assumptions on the behavior of g at infinity or at zero the interval is unbounded from above or its lower bound is zero. Finally, we present some multiplicity results for the problem.

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A characterization of the mountain pass geometry and applications to nonlinear differential problems

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A characterization of the mountain pass geometry is presented. Relations between the mountain pass theorem and local minima are pointed out. As consequences, multiplicity results for nonlinear elliptic Dirichlet problems are obtained.

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Some existence results for a perturbed asymptotically linear problem

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Let us consider the following semilinear elliptic problem

$$(P_\varepsilon) \qquad \begin{cases} -\Delta u - \lambda u = f(x,u) + \varepsilon g(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is an open bounded domain of $\mathbb{R}^N$ $(N \geq 3)$ with smooth boundary $\partial\Omega$, $\varepsilon \in \mathbb{R}$ and p, g are given real functions on $\Omega \times \mathbb{R}$. If the problem (P_ε) perturbs an asymptotically linear problem, i.e.

$$\lim_{|t| \to +\infty} \frac{f(x,t)}{t} = 0$$

uniformly with respect to a.e. $x \in \Omega$, we prove that the number of distinct critical levels of the functional associated to the unperturbed problem is "stable" under small perturbations also in lack of symmetry, both in the non–resonant and in the resonant case, even when the perturbation term g is continuous but satisfies no growth assumption.

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Three solutions for a quasilinear elliptic problem via critical points in open level sets and truncation principle

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The aim of this talk is to present a novel approach, jointly developed with S. Carl and R. Livrea, to investigate the existence of at least three solutions for the following quasilinear elliptic problem depending on a parameter λ, with homogeneous Dirichlet boundary conditions in a smooth bounded domain Ω,

$$-\Delta_p u = \lambda f(u) \quad \text{in} \ \ \Omega, \quad u = 0 \ \ \text{on} \ \ \partial\Omega.$$

More precisely, the existence of at least two constant sign solutions is established owing to an abstract localization principle of critical points for functionals of the form $\mathbb{E} = \Phi - \lambda\Psi$ on open sublevels $\Phi^{-1}(]-\infty, r[)$ recently obtained in collaboration with G. Bonanno. The existence of a sign-changing solution is pointed out adapting the sub-supersolution method recently developed by S. Carl and D. Motreanu, where, variational and topological arguments, such as the Mountain Pass Theorem, in conjunction with comparison principles and truncation techniques are the main tools.

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Elliptic variational inequalities with discontinuous multifunctions

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We consider multi-valued elliptic variational inequalities for operators of the form

$$u \mapsto Au + \Theta(u),$$

where A is a second order elliptic operator of Leray-Lions type, and $u \mapsto \Theta(u)$ is a multi-valued lower order term that may neither be lower nor upper semicontinuous. Our main goal is to provide an analytical framework for this new class of multi-valued variational inequalities that includes variational-hemivariational inequalities as special case.

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Multiple solutions for Dirichlet problems involving the p(x)-Laplace operator

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We investigate the existence and multiplicity of weak solutions for Dirichlet problems involving the $p(x)$-Laplace operator where p is a continuous function defined on an open bounded domain with smooth boundary $\Omega \subset R^N$. In particular, in the case $N < p^-$, under an appropriate oscillating behaviour of the nonlinearity, the existence of infinitely many weak solutions is established while, in the more general case $1 < p^-$ and under a suitable growth condition of the nonlinear term, we obtain the existence of at least three weak solutions. The approach is based on variational methods. Bonanno G. and Chinni' A., Existence results of infinitely many solutions for p(x)-Laplacian elliptic Dirichlet problems, Complex Variables and Elliptic Equations,(2012)

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Multiplicity results for elliptic Neumann problems

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In this talk we present some multiplicity results for an elliptic Neumann problem. Precisely, recent critical point results for differentiable functionals are exploited in order to prove the existence of a determined open interval of positive eigenvalues for which the problem admits multiple weak solutions.

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Variational methods for differential equation with small impulsive effects

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In this talk we will deal with the following nonlinear Dirichlet value problem with small perturbations of impulse:

$$(1)\begin{cases} -u''(t) + q(t)u(t) = \lambda f(t, u(t)) \ \ t \in [0, T], \ \ t \neq t_j \\ u(0) = u(T) = 0 \\ \Delta u'(t_j) = u'(t_j^+) - u'(t_j^-) = \mu I_j(u(t_j)), \ \ 1 \leq j \leq p \end{cases}$$

By using the critical points theorems obtained in [1] and [2] we will prove the existence and multiplicity of solutions for problem (1) when the parameters λ and μ lie in precise intervals (see [3]).

[1] Bonanno G. and Candito P., Non-differentiable functionals and applications to elliptic problems with discontinuous nonlinearities, J. Differential Equations,244 (2008), 3031–3059.

[2] Bonanno G. and Marano S.A, On the structure of the critical set of non-differentiable functionals with a weak compactness condition, Appl. Anal, 89 (2010), 1–10.

[3] Bonanno G. and Di Bella B., A Dirichlet boundary value problem with small pertubations of impulse, preprint.

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Multiple critical orbits for a class of lower semicontinuous functionals

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We deal with a class of functionals I on a Banach space X, having the structure $I = \Psi + \mathcal{G}$, with $\Psi : X \to (-\infty, +\infty]$ proper, convex, lower semicontinuous and $\mathcal{G} : X \to \mathbb{R}$ of class C^1. Also, I is G-invariant with respect to a discrete subgroup G of X,with dim(span G) $= N$. Under some appropriate additional assumptions we prove that I has at least $N + 1$ critical orbits. As a consequence, we obtain that the periodically perturbed N-dimensional relativistic pendulum equation has at least $N + 1$ geometrically distinct periodic solutions. Based on joint work with Cristian Bereanu.

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Standing waves of nonlinear Schrodinger equation

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We consider the standing wave solutions for a wide class of superlinear Schrodinger equations with periodic potential and asymptotically linear Schrodinger equations whose nonlinearity is not sublinear. In both cases the linear part of the eqautions may be indefinite.

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Some recent existence and multiplicity results for second order Hamiltonian systems

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In this talk we will show an overview on some existence and multiplicity results of periodic solutions for wide classes of second order Hamiltonian systems that are included in the following general case

$$(\mathrm{P})_\lambda \quad \begin{cases} -\ddot{u}(t) = \nabla_u F(t, u(t), \lambda) \text{ a.e. in } [0,\mathrm{T}] \\ u(T) - u(0) = \dot{u}(T) - \dot{u}(0) = 0, \end{cases}$$

where $T > 0$, $F : [0, T] \times \mathbf{R}^N \times]0, +\infty[\to \mathbf{R}$ is a smooth function. In particular, exploiting some abstract critical points theorems, there will be analyzed several situations when F satisfies different conditions at zero and/or at infinity with respect to u.

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Multiple solutions to Dirichlet eigenvalue problems with p-Laplacian

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The existence of both constant -sign and nodal solutions to homogeneous Dirichlet problems with p-Laplacian and reaction term depending on a positive parameter is investigated via variational as well as topological methods, besides truncation techniques.

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Elliptic variational problems with nonlocal operators

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We study the nonlocal and nonlinear problem

$$\begin{aligned} L\phi + V\phi - |\phi|^2 * W\phi &= -\lambda\phi, \\ \|\phi\|_{L^2} &= 1, \end{aligned}$$

for a large class of potentials V and W on R^3. The operator $L = \sqrt{-\alpha^{-2}\Delta + \alpha^{-4}} - \alpha^{-2}$ (the quasirelativistic Laplacian), with α being Sommerfeld's fine structure constant, is a nonlocal, pseudo differential operator of order one. We prove the existence of multiple solutions for two separate cases: (1) unconstrained problem; (2) constrained problem.

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Existence and non-existence of solutions for p-laplacian equations with decaying cylindrical

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In this paper we deal with a class of quasilinear elliptic problem in whole space, involving p-laplacian operator and the Hardy-Sobolev critical exponent. The potential has a unbounded singular set. Combining a version of the concentration compactness result by Solimini, Hardy-Sobolev type inequality with the Mountain Pass Theorem, existence of nontrivial solutions are obtained. Decay properties of these solutions are showed by applying Vassilev results. Pohozaev type identities are established in order to get non-existence results.

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Variational problems in geometrical analysis

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We present some existence results, obtained in collaboration with G. Bonanno and V. Rădulescu, for a parametric elliptic problem on a Riemannian manifold without boundary. In particular, for a precise location of the parameter, is established the existence of a nontrivial solution, without requiring any asymptotic condition at zero or at infinity on the nonlinearity. In the case of sublinear terms at the origin, we deduce the existence of solutions for small positive values of the parameter and we obtain that the corresponding solutions have smaller and smaller energies as the parameter goes to zero. It is worth noticing that these results also hold in presence of nonlinearities with critical growth. Finally, a multiplicity result is obtained and concrete examples of applications are provided. A basic ingredient in our arguments is a recent local minimum theorem for differentiable functionals due to G. Bonanno.

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Count and symmetry of global and local minimizers of the Cahn-Hilliard energy functional over cylindrical domains.

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We address the problem of minimization of the Cahn-Hilliard energy functional under a mass constraint over two and three-dimensional cylindrical domains. Although existence is presented for a general case the focus is mainly on rectangles, parallelepipeds and circular cylinders. According to the symmetry of the domain the exact number of global and local minimizers are given as well as their geometric profile and interface location; all are one-dimensional increasing/decreasing and odd functions for domains with lateral symmetry in all axes and also for circular cylinders. The selection of global minimizers by the energy functional is made via the smallest interface area criterion. The approach utilizes $\Gamma-$convergence techniques to prove existence of an one-parameter family of local minimizers of the energy functional for any cylindrical domain. The exact number of global and local minimizers as well as their geometric profiles are accomplished via suitable applications of the unique continuationprinciple while exploring the domain geometry in each case and also the preservation of global minimizers through the process of $\Gamma-$convergence.

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Bernstein-Nagumo conditions and solutions to nonlinear differential inequalities

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For Ω, an open bounded subset of $\mathbb{R}^N$, with smooth boundary, and $1 < p < \infty$, we establish $W^{1,p}(\Omega)$ *a priori* bounds and prove the compactness of solution sets to differential inequalities of the form

$$|\mathrm{div} A(x, \nabla u)| \leq F(x, u, \nabla u),$$

which are bounded in $L^\infty(\Omega)$. The main point in this work is that the nonlinear term F may depend on ∇u and may grow as fast as a power of order p in this variable. Such growth conditions have been used extensively in the study of boundary value problems for nonlinear ordinary differential equations and are known as Bernstein-Nagumo growth conditions. In addition, we use these results to establish a sub-supersolution theorem.

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Multiplicity theorems for (p,2) equations

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We consider a parametric nonlinear elliptic equation driven by the (p,2)-differential operator. We prove multiplicity theorems when the parameter is bigger than the second eigenvalue of the p-Laplacian. We establish three or four nontrivial smooth solutions

with sign information. Our approach combines variational methods with Morse theory.

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Some results for impulsive problems

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Impulsive problems arise naturally in processes that involve abrupt changes in the state of the system. In this talk we will present some existence and multiplicity results for asymptotically piecewise linear elliptic problems with superlinear impulses using variational methods.

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Symmetric problems in unbounded domains

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We present some existence and multiplicity results concerning a special class of radially symmetric elliptic systems in unbounded domains. Furthermore, in unbounded cylinders we study also partially radially symmetric problems.

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Mountain pass and linking solutions for fractional Laplacian equations

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Motivated by the interest shown in the literature for non-local operators of elliptic type, in some recent papers, joint with Enrico Valdinoci, we have studied problems modeled by

$$\begin{cases} (-\Delta)^s u - \lambda u = |u|^{q-2}u & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^n \setminus \Omega\,, \end{cases} \tag{1}$$

where $s \in (0,1)$ is fixed and $(-\Delta)^s$ is the fractional Laplace operator, $\Omega \subset \mathbb{R}^n$, $n > 2s$, is open, bounded and with Lipschitz boundary, $\lambda > 0$, $2^* = 2n/(n-2s)$ is the fractional critical Sobolev exponent and $2 < q \le 2^*$, that is problems with subcritical $(q < 2^*)$ and critical growth $(q = 2^*)$. In problem (1) the Dirichlet datum is given in $\mathbb{R}^n \setminus \Omega$ and not simply on $\partial\Omega$, consistently with the non-local character of the operator $(-\Delta)^s$.

Problem (1) represents the non-local counterpart of the following nonlinear elliptic equation

$$\begin{cases} -\Delta u - \lambda u = |u|^{q-2}u & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega\,, \end{cases}$$

with $2 < q \le 2_*$, where $2_* = 2n/(n-2)$ and $n > 2$.

Aim of this talk will be to present some results which extend the validity of some existence theorems known in the classical case of the Laplacian to the non-local framework. In particular, in the critical setting our theorems may be seen as the extension of the classical Brezis-Nirenberg result to the case of non-local fractional operators.

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A concentration phenomenon for a semilinear elliptic equation

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We consider the problem

$$-\Delta u + V(x)u = Q(x)|u|^{p-2}u, \quad x \in \Omega,\ u \in H_0^1(\Omega),$$

where $\Omega \subset \mathbb{R}^N$ is a domain containing the origin, $2 < p < 2^*$, V is bounded, $V \ge 0$ and $\sigma(-\Delta + V) \subset (0,\infty)$. Further, we assume that Q is bounded, positive on a small ball centered at the origin and negative outside a slightly larger ball. We show that the solutions of this problem concentrate at the origin as the size of the ball tends to 0. We also consider the same problem with Q positive on two spots of small size and show that ground state solutions concentrate at one of these spots.

This is joint work with Nils Ackermann.

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Three weak solutions for elliptic Dirichlet system

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Consider the following elliptic Dirichlet system

$$\begin{cases} -\Delta u = \lambda \nabla_u F(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ (with $N \ge 3$) is a non-empty bounded open set with a smooth boundary $\partial\Omega$, λ is a positive real parameter and $m \ge 1$, $F : \Omega \times \mathbb{R}^m \to \mathbb{R}$ is a C^1-Caratheodory function, $F(x,0) = 0$ for every $x \in \Omega$ and $\nabla_u F = (F_{u_i})_{i=1,\cdots,m}$ where F_{u_i} denotes the partial derivative of F respect on u_i $(i = 1,\cdots,m)$. Under suitable assumptions on F, the existence of three non zero weak solutions is obtained. The approach is based on critical points theorems.

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