Special Session 35: Qualitative Theory of Nonlinear ODEs and Applications

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The study of qualitative aspects of nonlinear ordinary differential equations is a classical and important topic which is widely studied for his interconnections with nonlinear analysis and its applications to mathematical sciences. The aim of this session is to put together different experts in this area in order to show the central role of the qualitative theory of ODEs both from the point of view of the applications and also as a source of new and interesting problems which may lead to the introduction and development of new abstract tools. Some of the main topics to be included in this section are: Autonomous and non-autonomous systems, Limit cycles, Oscillation theory and comparison theory, Boundedness of solutions, Nonlinear oscillations, Coupled oscillators, Transformation and reduction of equations and systems, Bifurcation, Periodic solutions, Complex behavior and chaotic systems, Dynamical systems, ODEs methods for particular solutions of PDEs (radially symmetric solutions, traveling front solutions), Homoclinic and heteroclinic solutions, Applications of variational and topological methods to nonlinear boundary value problems for ODEs, Equations and systems on manifolds, Asymptotic behavior of the solutions, Applications to ODE models in mathematical sciences.

A local minimum theorem and applications to nonlinear ordinary differential problems

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A local minimum theorem for a continuously Gâteaux differentiable function, possibly unbounded from below and without any weak continuity assumption, is presented. As a consequence, some classes of nonlinear ordinary differential problems are investigated.

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Positive solutions to second order ODEs with indefinite weight: multiplicity and complex dynamics

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We consider scalar second order differential equations of the type $u'' + a(t)g(u) = 0$, being $a(t)$ a function which changes its sign ("indefinite weight"). In the past few decades, this class of equations has been widely investigated, as a simple model exhibiting high multiplicity of sign-changing solutions (with different boundary conditions) and, eventually, chaotic dynamics. Here, we re-consider the problem from the point of view of positive solutions only, presenting some recent results which show that, again, multiplicity and complex dynamics can occur.

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Chaotic dynamics in some pendulum type equations

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We investigate the presence of chaotic-like dynamics for a class of second order ODEs (including some pendulum-type equations) using the concept of "stretching along paths" and the theory of "linked twist maps". The chaotic dynamics considered is of the coin-tossing type as in the Smale's Horseshoe. The proof relies on some recent results about chaotic planar maps combined with the study of geometric features which are typical of linked twist maps.

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Existence and multiplicity of solutions for a Dirichlet boundary value problem

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We present some recent results concerning existence, multiplicity and localization of solutions for a Dirichlet boundary value problem involving the $p-$Laplacian. The approach adopted is chiefly based on critical point theory.

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Existence results of solutions for p(x)-Laplacian elliptic Dirichlet problems

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We investigate the existence and multiplicity of weak solutions for Dirichlet problems that involve the $p(x)$-Laplace operator. The results are based on two multiple critical points theorems. The first theorem, established by G. Bonanno and P. Candito [Non-differentiable functionals and applications to elliptic problems with discontinuous non linearities, J. Differential Equations, 244, 3031-3059, (2008)], is applied to obtain three weak solutions for a Dirichlet problem in case of small perturbations of the nonlinear term and when $p^- > 1$. The second theorem, established by G. Bonanno and S.A. Marano [On the structure of the critical set of nondifferentiable functions with a weak compactness condition, Appl. Anal., 89,1-10 (2010)], is applied to obtain three weak solutions for a Dirichlet problem in absence of small perturbations and whenever $p^- > N$.

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Mixed boundary value problems with Sturm-Liouville equations

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We present some existence results of solutions for a mixed boundary value problem with Sturm-Liouville equation where no asymptotic condition on the nonlinear term either at zero or at infinity is requested. The approach is based on variational methods

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Radial solutions of Dirichlet problems with concave-convex nonlinearities

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We prove the existence of a double infinite sequence of radial solutions for a Dirichlet concave-convex problem associated with an elliptic equation in a ball. We are interested in relaxing the classical positivity condition on the weights, by allowing the weights to vanish. The idea is to develop a topological method and to use the concept of rotation number. The solutions are characterized by their nodal properties.

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Existence of solutions to second order boundary value problems.

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This talk examines the existence of solutions to nonlinear singular boundary value problems. This area of differential equations arises constantly in the modeling of dynamical phenomenon, for example; thermal explosions, electrohydronamics and radially symmetric nonlinear diffusion in the n-dimensional sphere. This naturally motivates the study for further understanding of the solutions of the qualitative nature. The main focus is generating novel a priori bounds on all possible solutions via differential inequalities. Then by using these new a priori bounds and existence theorems of D. O'Regan to produce existence of solution(s).

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Star flows and singular hyperbolicity

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A vector field is called star vector field if every singularity and periodic orbit of its small perturbations are hyperbolic. We will prove that every Lyapunov stable chain recurrent class of a star flow is singular hyperbolic. Especially, if dimension of phase space is less than 5, then every chain recurrent class is singular hyperbolic.

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Nonlinear first order systems in the plane with positively homogeneous principal term

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In the first part of the lecture, we will overview some basic features of positively homogeneous Hamiltonian systems in the plane, i.e., systems of the type

$$Ju' = \nabla H(u), \quad u \in \mathbb{R}^2,$$

where

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

is the standard symplectic matrix and H is a C^1-function, with locally Lipschitz continuous gradient, satisfying

$$0 < H(\lambda u) = \lambda^2 H(u), \quad u \in \mathbb{R}^2, \lambda > 0.$$

We will then focus on the T-periodic boundary value problem ($T > 0$) for a nonlinear planar system like

$$Ju' = \nabla H(u) + R(t, u),$$

with $H(u)$ as above and $R(t, u)$ having a sublinear growth in the variable u. Our attention will be devoted to the possible occurrence of resonance phenomena, depending on some minimal period associated with the autonomous system $Ju' = \nabla H(u)$. We will explore some recent results of existence and multiplicity of solutions for both the nonresonant and the resonant case (these last ones mainly relying on a suitable planar version of the Landesman-Lazer condition). With a similar approach, we can also deal with other kinds of boundary value problems, as it will be briefly shown at the end of the seminar.

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Existence and multiplicity results for second order differential problems

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Some results concerning the existence and the multiplicity of solutions for second order boundary value problems will be pointed out. The approach adopted is fully variational and it relies on recent general abstract critical points theorems.

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Existence results for parameterized Emden-Fowler equations

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We present some recent results, obtained in collaboration with G. Bonanno and V. Rădulescu on the existence of multiple solutions for a class of Emden-Fowler equations. Some comparisons with several results present in literature are pointed out.

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Periodic solutions of the prescribed curvature equation

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We discuss existence, multiplicity and regularity of periodic solutions of the one-dimensional curvature equation

$$-\left(u'/\sqrt{1+u'^2}\right)' = f(t, u)$$

in the space of bounded variation functions. We study the case where f interacts with the first and the second eigenvalue (or more generally with the first non-trivial branch of the Fucik spectrum) of the one-dimensional 1-Laplace operator with periodic boundary conditions. The problem of the existence of subharmonic solutions is considered as well.

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Liapunov-type integral inequalities for higher order dynamic equations on time scales

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In this paper, we obtain Liapunov-type integral inequalities for certain nonlinear, nonhomogeneous dynamic equations of higher order without any restriction on the generalized zeros of their higher-order delta derivatives of the solutions by using elementary time scale analysis. As an applications of our results, we show that oscillatory solutions of the equation converge to zero as $t \to \infty$. Using these inequalities, it is also shown that $(t_{m+k} - t_m) \to \infty$ as $m \to \infty$, where $1 \le k \le n-1$ and $\langle t_m \rangle$ is an increasing sequence of generalized zeros of an oscillatory solution of $D^n y + y(\sigma(t)) f(t, y(\sigma(t))) |y(\sigma(t))|^{p-2} = 0$, $t \ge 0$, provided that $W(., \lambda) \in L^\mu([0, \infty)_{\mathbb{T}}, \mathbb{R}^+)$, $1 \le \mu \le \infty$ and for all $\lambda > 0$. A criterion for disconjugacy of nonlinear homogeneous equation is obtained in an interval $[a, b]_{\mathbb{T}}$.

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Periodic solutions and chaotic dynamics in 3D equations with applications to Lotka Volterra systems

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We discuss a geometric configuration for a class of homeomorphisms in $\mathbb{R}^3$ producing the existence

of infinitely many periodic points as well as a complex dynamics due to the presence of a topological horseshoe. We also show that such a class of homeomorphisms appear in the classical Lotka- Volterra system.

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Symbolic dynamics for the N-centre problem at negative energies

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We consider the planar N-centre problem, namely the study of the ODE

$$\ddot{x}(t) = -\sum_{k=1}^{N} \frac{m_k}{|x(t)-c_k|^3}(x(t)-c_k),$$

where $x : I \subset \mathbb{R} \to \mathbb{R}^2$ and the positions of the centres c_k are fixed in $\mathbb{R}^2$. We prove the existence of infinitely many collisions-free periodic solutions with small (in absolute value) and negative energy; these solutions satisfy particular topological constraints, which are expressed in terms of the partitions of the centres in two different non-empty sets. The proof is based upon perturbative, variational and geometric techniques. As a consequence, for small and negative values of the energy, the dynamical system associated to the motion equation has a symbolic dynamics, where the symbols are the partitions of the previous type.

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An even solution to a fourth-order ODE

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We study a fourth-order ordinary differential equation of the form $u^{(4)} - cu'' + a(x) = f(u)$, where f is similar to a power function $|q|^{p-2}q(p > 1)$ and a is even with $a(x) \to l > 0$ as $|x| \to \infty$. Using variational mountain-pass and concentration-compactness ideas, it is shown that the equation has a nontrivial solution homoclinic to 0.

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Entire parabolic trajectories as minimal phase transitions

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For the class of anisotropic Kepler problems in $\mathbb{R}^d \setminus \{0\}$ with homogeneous potentials, we seek parabolic trajectories having prescribed asymptotic directions at infinity and which, in addition, are Morse minimizing geodesics for the Jacobi metric. Such trajectories correspond to saddle heteroclinics on the collision manifold, are structurally unstable and appear only for a codimension-one submanifold of such potentials. We give them a variational characterization in terms of the behavior of the parameter-free minimizers of an associated obstacle problem. We then give a full characterization of such a codimension-one manifold of potentials and we show how to parameterize it with respect to the degree of homogeneity.

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Unbounded solutions for a class of singular hamiltonian systems

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The existence of unbounded solutions is proved for a class of singular Hamiltonian systems $\ddot{u}(t) + \nabla V(u(t)) = 0$ by taking limit for a sequence of periodic solutions which are obtained by variational methods.

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The Poincaré-Birkhoff "twist theorem": some remarks and recent applications to ODEs

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After a brief historical survey of the topic, we present and discuss some variants of the Poincare'-Birkhoff fixed point theorem which look particularly suitable to deal with the study of periodic solutions to nonautonomous planar Hamiltonian system. We also propose some recent applications which, in our opinion, put in evidence how such a classical theorem may be useful in providing some sharp results for second order nonlinear ODEs.

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