Special Session 38: Bifurcations and Asymptotic Analysis of Solutions of Nonlinear Models

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Yoshio Yamada, Waseda University, Japan
Shoji Yotsutani, Ryukoku University, Japan

The aim of this special session is to exchange recent results, ideas and techniques on bifurcation and asymptotic analysis described by nonlinear elliptic and parabolic PDE models from various fields. In particular, we are interested in the global bifurcation structure for such models. Combinations of numerical simulation and theoretical approach with asymptotic analysis will be very useful to understand the nonlinear phenomena together with underlying structure of solutions. We will give opportunity to both established and junior researchers working in the related area to present their recent results.

On positive solutions of semilinear elliptic equations with supercritical exponent

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I will discuss the existence and the asymptotic behavior of positive solutions of semilinear elliptic equations with supercritical exponent. When coefficient function has some monotonicity, solutions may possess stability property. Slowly decaying solutions appear naturally in semilinear equations with supercritical exponent. I will explain the existence of such solutions and present related questions.

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Asymptotic behavior of solutions of epidemic models with delays

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We consider a global dynamics of epidemic models with delays, in which an immunity loss of infectious diseases of recovered individuals is incorporated. By applying Lyapunov functional techniques in McCluskey (2010) and monotone iterative techniques, we establish new sufficient conditions under which the positive equilibrium is globally asymptotically stable for any delay. Some applications of our approach are also offered. Moreover, for a delayed SIR epidemic model governed by a logistic growth of susceptible individuals in the absence of diseases, we identify a threshold parameter with respect to the basic reproduction number such that the positive equilibrium loses its local stability as the length of the delay increases past a critical value.

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Dynamical bifurcation of the two dimensional Swift-Hohenberg equation with odd periodic condition

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In this paper, we study the stability and dynamic bifurcation for the two dimensional Swift-Hohenberg equation with an odd periodic condition. It is shown that an attractor bifurcates from the trivial solution as the control parameter crosses the critical value. The bifurcated attractor consists of finite number of singular points and their connecting orbits. Using the center manifold theory, we verify the nondegeneracy and the stability of the singular points.

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Global existence in sub-critical cases for 1-D quasilinear degenerate Keller-Segel systems

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We consider the one-dimensional quasilinear degenerate Keller-Segel systems of power type. When the spatial dimension is greater than or equal to 2, the global existence of weak solutions to the systems is shown by Ishida-Yokota (2012) via a slightly complicated approximation. In this talk we derive the same result on the global existence for "1-D" case by using a "simple" approximation.

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Structures of positive solutions to nonlinear elliptic equations on the hyperbolic space

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We consider nonlinear elliptic equations on the hyperbolic space and investigate the structures of positive solutions to them. We use a transformation to reduce the equations to the Matukuma type ones on an Euclidean space. This transformation gives us a simple view to the problem. A part of this talk is based on the joint work with C. Bandle (University of Basel).

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Free boundary problems modeling the spreading of species in symmetric domains

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We consider free boundary problems modeling the spreading of species, where unknown functions are population density and spreading front of the species. Moreover, the dynamical behavior of the free boundary is determined by Stefan-like condition. Such model was first proposed by Du and Lin (2010) and large time behaviors of solutions (spreading and vanishing of species) are completely understood. In this talk, we discuss free boundary problems for reaction-diffusion equations with general nonlinearity in symmetric domains.

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Non-symmetric low-index solutions for symmetric boundary value problems

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We consider a semilinear boundary value problem that is invariant under a nontrivial group of Euclidean symmetries. While earlier work has focused on conditions under which solutions inherit some of the symmetries (a common numerical observation), our goal was to find solutions that have no symmetries at all. In the case where the domain is a square, we give a computer-assisted proof for the existence of a non-symmetric index 2 solution. Some numerical results for the disk will be described as well.

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Bifurcation structure of steady-states to a reaction-diffusion-advection system in surface chemistry

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Hildebrand et al. (1999) have proposed a reaction-diffusion-advection model for pattern formations in the catalytic CO-oxidation on Pt-surface. We consider the stationary problem of the chemical model to give a universal bound for every solution and a sufficient condition for the existence of nonconstant solutions. Furthermore, we obtain the global bifurcation structure of steady-states for a related shadow system. This structure implies that nonconstant steady-states can form a (spontaneous) bifurcation curve connecting boundary-layer states with internal-layer states.

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On a dynamics of solution with a transition layer to some bistable reaction diffusion equation

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In this talk, we study a dynamics of solution with a sharp transition layer of a bistable reaction diffusion equation. In particular, we consider the case where bistable nonlinearity in the equation has spatial heterogeneity. If this heterogeneity degenerates on an interval, the dynamics of the transition layer on this interval becomes so called very slow dynamics.

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Structure and blow up phenomena for plane closed elastic curves

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Let Γ be plane closed elastic curve with length 2π. We denote arc-length and curvature by s and $\kappa(s)$, respectively. Let M be the signed area of the domain bounded by Γ. We consider the variational problem: Find a curve Γ such that minimize $\frac{1}{2}\int_0^{2\pi} \kappa(s)^2 ds$ subject to $M < \pi$, and $\omega M \neq \pi$, where ω is the rotation number.

In this talk, we will discuss the structure and the blow up phenomena of the solution $\kappa(s)$ to the Euler-Lagrange equation for the above variational problem.

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The jamming of camphor boats in a circular water channel

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The jamming of camphor boats similar to the traffic jam in a highway was observed in the experiment in a circular water channel. We reproduce the jamming of camphor boats using the mathematical model, and show the occurrence of jamming is Hopf bifurcation.

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Coexistence problem for a prey-predator model with a protection zone

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We consider a prey-predator model with a protection zone for the prey. We give a necessary and sufficient condition for the existence of positive stationary solutions by using the bifurcation theory. We also discuss the asymptotic behavior of positive stationary solutions as the intrinsic growth rate of the predator goes to infinity. Our results show that the environment inside the protection zone is important for the prey.

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Bifurcation analysis for the superconducting/normal phase transition of the Ginzburg-Landau system

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In the talk, I will shortly introduce the Ginzburg-Landau system of superconductivity. When lowering the temperature, superconductors become the superconducting state from the normal state. In order to study this phase transition phenomena, we present a bifurcation and stability analysis on the Ginzburg-Landau system of superconductivity with an applied magnetic field and the de Gennes boundary condition. It is proved there are two different kinds of phase transition from the normal state to the superconducting state: one is jump transition and the other is continuous transition. In particular, we analyse the behavior of solution when the domain is a cylinder and the applied field is parallel to the axis.

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On a generalized Jacobian elliptic function associated with p-Laplacian

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We generalize the Jacobian elliptic function $\mathrm{sn}(x,k)$ and apply it to a bifurcation problem associated with p-Laplacian. If the modulus k equals zero, then the novel function $\mathrm{sn}_{pq}(x,k)$ coincides with the p-trigonometric function $\sin_p x$ or the (p,q)-trigonometric function $\sin_{pq} x$ developed by P.Lindqvist, J.Peetre, P.Drabek and R.Manasevich.

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Structural analysis of solutions to nonlinear systems of elliptic partial differential equations

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In this talk, we deal with some nonlinear elliptic systems, including sublinear systems, a coupled system in Hamiltonian type, Liouville-type system and simplified Bennett-type system with singular data. Some qualitative properties and complete structures of solutions will be provided by applying the bifurcation theory and linearization approach.

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On the sharp constant for the weighted Trudinger-Moser type inequality of the scaling invariant form

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We establish the weighted Trudinger-Moser inequality of the scaling invariant form including its best constant and prove the existence of a maximizer for the associated variational problem. The non-singular case was treated by Adachi-Tanaka and the existence of the maximizer is a new result even for the non-singular case. We also discuss the relation between the best constants of the weighted Trudinger-Moser inequality and the Caffarelli-Kohn-Nirenberg inequality in the asymptotic sense.

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Exact solutions for bifurcation problems of some reaction diffusion systems

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For various nonlinear evolutional PDEs it is one of central problems to study the global bifurcation structure of stationary/periodic solutions. Various numerical results on global bifurcation diagrams usually shows us a difficulty of theoretical understanding for bifurcation phenomena. In this talk we will introduce some global bifurcation results on some reaction-diffusion systems with use of exact solutions, and will investigate, both theoreticall and numerically, the property of the corresponding linearized operators associated with nontrivial stationary solutions.

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Structure and stability of stationary solutions to a cross-diffusion equation

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We are interested in a cross-diffusion equation proposed by Shigesada-Kawasaki-Teramoto in 1979. Limiting equations and their analysis by Lou - Ni (JDE 1996, 1999) play crucial roles. One of the limiting equations is very important to obtain segregated solutions. Lou-Ni-Yotsutani (DCDS, 2004) almost revealed the structure of it. We show the complete structure of it, and also investigate stability problems.

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