Special Session 41: New Developments in Qualitative Behavior of Evolutionary PDEs

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New developments in the theory of evolutionary PDEs will be reported. The topics discussed will include well-posedness, regularity, formation of singularities, stability/instability and asymptotic behavior of solutions to important evolutionary PDE systems. Applications of these problems will also be addressed.

Uniform decay rate estimates for Schrödinger and Plate equations with nonlinear locally distributed damping

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On a compact $n-$dimensional Riemannian manifold $(\mathcal{M}, \mathbf{g})$, we establish uniform decay rate estimates for the linear Schrödinger and plate equations subject to an internal nonlinear damping locally distributed on the manifold. Our approach can be also employed for other equations provided that inverse inequality for the linear model occurs. In the particular case of the wave equation, where the well known geometric control condition (GCC) is equivalent to the observability inequality, our method generalizes the results due to Cavalcanti et. al. (Arch. Ration. Mech. Anal. (2010), TRANSACTIONS AMS (2009)) regarding the optimal choice of dissipative regions.

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Energy decay of a magnetoelastic system in an exterior 3-D domain

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In this talk we present results of existence and uniform decay of the total energy of solutions for a system of magneto-elasticity with localized damping "near" infinity in an exterior 3-D domain. The model which we consider is motivated by a phenomenon which appears frequently in nature: The interaction between the strain and electromagnetic fields in an elastic body. For example, this model can be used to investigate the propagation of elastic waves in the presence of Earth's magnetic field. The system under consideration in this work may be viewed as a coupling between the hyperbolic system of elastic waves and a parabolic system for the magnetic field. Somehow, the coupled system under consideration has a similar structure as the classical isotropic thermoelastic system . When we try to study the asymptotic behavior of the total energy for the model we are considering in this work, similar difficulties as for the thermoelastic system (in $n = 2$ or 3 dimensions) will appear.

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Global existence and sharp decay estimates for the semilinear wave equation with time-dependent damping

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We recently studied the Cauchy problem for the wave equation with time-dependent effective damping. We first derive Matsumura-type estimates for the solution to the linear problem, then we prove the same decay rate with no loss for the semilinear problem. We find the critical exponent $1 + 2/n$ for the global existence of small data solution, where n is the space dimension. This exponent is the same obtained in the constant coefficient case by Todorova and Yordanov, and Ikehata and his collaborators. In space dimension $n > 2$ we use data from weighted energy space. We prove the sharpness of the critical exponent by using a modified test function method.

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Regularity of solutions for a third order differential equation

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In this talk, which is part of a joint work with V. Poblete and C. Lizama (Chile), we shall discuss well posedness in Lebesgue spaces for the abstract model,

$$\begin{cases} \alpha u'''(t) + u''(t) + \beta Au(t) + \gamma Au'(t) = f(t), t \geq 0; \\ u(0) = u_0, \\ u'(0) = u_1, \\ u''(0) = u_2. \end{cases}$$

that corresponds to a flexible space structure under appropriate initial and boundary conditions. We also study regularity of mild and strong solutions and apply our results showing qualitative properties of the

trajectories in the case of the negative Laplacian operator.

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Finite time blow-up for damped wave equations with nonlinear memory and space-dependent potential

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In this paper, we consider the Cauchy problem in $\mathbb{R}^n$, $n \geq 1$, for a semilinear damped wave equation with space-dependent potential and nonlinear memory term. A blow-up result under some positive data in any dimensional space is obtained.

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On the stabilization of Timoshenko systems with finite memory

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In this work, we consider a one-dimensional Timoshenko systems with different speeds of wave propagation and with only one control given by a viscoelastic term on the angular rotation. For a wide class of relaxation functions and for sufficiently regular initial data, we establish a general decay result for the energy of solution.

This is a joint work with Salim Messaoudi (KFUPM, Dhahran, Saudi Arabia).

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Energy decay estimates for wave equations with a fractional damping

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Masato Natsume

We consider the Cauchy problem in the whole space for wave equations with a fractional damping. We will introduce our recent results concerning the energy decay estimates. The proof is based on the energy method in the Fourier space combined with a device which was introduced by the speaker.

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Global existence for critical nonlinear massless Dirac equations with null structure in 3D

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In this talk I wish to present a result on the Cauchy problem for the nonlinear massless Dirac equation in 3D. It is known that the quadratic nonlinearity is on the critical level for global solvability of the Cauchy problem. We shall show that if the nonlinearity takes some special forms, then the problem has a global solution for small initial data. Moreover, the solution is asymptotically free. The idea of the proof is to exploit the null structure from the special quadratic forms. We remark that this result could not be deduced from the global existence result for the nonlinear wave equations with the null condition.

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Attractors for a class of Kirchhoff-Boussinesq models with memory

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This talk is concerned with the existence of finite dimensional attractors for a class of Kirchhoff-Boussinesq models with memory. Roughly speaking one considers an Euler-Bernoulli plate equation with a perturbation of p-Laplacian type and a memory term with past history.

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Best asymptotic profile for bipolar hydrodynamic model of semiconductors

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D. Donatelli, B. Rubino, R. Sampalmieri

In this talk, we consider the bipolar hydrodynamic system of semiconductors which is presented by the coupled Euler-Poisson equations. Traditionally, the asymptotic profiles of the solutions are regarded as the diffusion waves (the solutions to the corresponding (parabolic) porous media equations), and the convergence rates are showed to be algebraic. However, by a deep observation and a heuristic analysis, we recognize that the best asymptotic profiles are the stationary waves (the corresponding steady-state solutions). We further prove that the original solutions converge time-asymptotically to these best asymptotic profile with exponential rates. The adopted approach for proof is the energy method.

This is a joint work with D. Donatelli, B. Rubino and R. Sampalmieri.

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Uniform resolvent estimates for Helmholtz equation in an exterior domain and their application to scattering problems

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Uniform resolvent estimates for Helmholtz equations in an exterior domain in R^N with $N \geq 2$ is derived by using Hardy type inequalities related to radiation conditions, which are hold for $N \geq 1$. From this resolvent estimate, the result by Mizohata and Mochizuki (1966) on the principle of limiting amplitude for dissipative wave equation is improved. Moreover, the smoothing estimate for the related evolution equations (Schroedinger, relativistic Schroedinger or wave (Klein-Gordon) equations) in an exterior domain in R^2 is also proved.

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Orbital stability of periodic waves

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Results of existence as well as orbital stability of periodic waves associated with evolutionary partial differential equations will be treated in this talk. Our method establishes such results without knowing an explicit periodic solution and we use a numerical approach. Applications of the theory allow us to determine the first proof of the orbital stability of a family of periodic waves for the 3-Korteweg-de Vries and logarithmic Schrodinger equations.

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Existence and blow-up of solutions for nonlinear wave equations

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Several results on wellposedness of solutions for nonlinear wave equations use the structure of the problems that include damping terms to establish local existence in time of solutions in the presence of energy accretive source terms. We will discuss the limitations of these methods and the intricate balance between local existence and regularity of solutions .

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Global existence and asymptotic behavior of solutions of thermoelsaticity of second sound

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Reinhard Racke and Aslan Kasimov

We consider the one dimensional Cauchy problem in thermoelsaticity of second sound, where the heat conduction is given by the so-called Cattaneo law. First, for the linear problem, and based on Fourier's analysis and Lyapunov's functional method, we show that the L^2-norm of solutions decays with the rate $t^{-1/4}$. In addition, by means of a careful spectral analysis, we give sharp description on the decay rates of solutions. In other words, all the decay rates can be improved by $t^{-1/2}$ for initial data in some weighted spaces. For the nonlinear model, we show a global existence result and obtain polynomial decay rates of the global small solution when the time goes to infinity by employing the large time decay estimates of solutions to the linearized Cauchy problem. Our decay estimates improve some early results.

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The final problem on the optimality of the general theory for nonlinear wave equations and related topics.

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Kyouhei Wakasa

The general theory of the initial value problem for fully nonlinear wave equations is to clarify lower bounds of the lifespan, the maximal existence time, of classical solutions in terms of the amplitude of small initial data according to the order of smooth nonlinear terms and space dimensions. All the results had been obtained till 1995. So we have been interested in the optimality of the lower bounds. This can be obtained by blow-up results for model equations. Among such several results, only the case of the quadratic semilinear term in 4 space dimensions has been remained open for more than 20 years. This final problem on the optimality has been known to be the critical case of Strauss' conjecture on semilinear wave equations. In this talk, I will present the final answer for the problem. Also I will discuss its application to systems and introduce another equation in high dimensions which has blowing-up solutions.

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Higher order expansion of solutions to damped wave equations

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In this talk, we consider the Cauchy problem for damped wave equations. Based on the precise analysis in the Fourier space, we obtained the higher order expansion of the solution of damped wave equations. Our theorem states that the large time behavior of the solution to damped wave equation is different from that of the heat equation.

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Generalized diffusion phenomenon in Hilbert space

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We show a generalized diffusion phenomenon in Hilbert space. The results have important applications regarding the asymptotic behavior of damped wave equations with variable coefficients in exterior domains. We establish sharp decay estimates which apply to second order hyperbolic equations, fourth order plate equations and hyperbolic systems.

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Asymptotic profiles for the isothermal Falk-Konopka system of shape memory alloys with weak damping

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Hiroshi Takeda

We will talk about the results for the weakly damped isothermal Falk-Konopka system which describes the martensitic phase transitions on shape memory alloys. We prove the unique global existence of solution for the Cauchy problem of the system and several asymptotic profiles of the solution. The asymptotic profiles enable us to clarify the relation between higher and lower order terms.

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