Special Session 42: Global or/and Blowup Solutions for Nonlinear Evolution Equations and Their Applications

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This session is devoted to the recent developments in global or/and blowup solutions for nonlinear evolution equations and their applications, include fluid dynamics, delay, localized, non-local, degenerate evolution equations, steady states and their properties.

Global and blowup solutions for general quasilinear parabolic systems

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This talk discusses global and blowup solutions of the general quasilinear parabolic system $u_t = \alpha(u,v)\Delta u + f(u,v,Du)$ and $v_t = \beta(u,v)\Delta v + g(u,v,Dv)$ with homogeneous Dirichlet boundary conditions. We will give sufficient conditions such that the solutions either exist globally or blow up in a finite time. In special cases, a necessary and sufficient condition for global existence is given. We also discuss a degenerate case.

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Structure of principal eigenvectors and genetic diversity

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The main concern of this paper is long-term genotypic diversity. Genotypes are represented as finite sequences $(s_1, s_2, \ldots, s_n)$, where the entries $\{s_i\}$ are drawn from a finite alphabet. The mutation matrix is given in terms of Hamming distances. It is proved that the long time behavior of solutions for a class of genotype evolution models is governed by the principal eigenvectors of the sum of the mutation and fitness matrices. It is proved that the components of principal eigenvectors are symmetric and monotonely decreasing in terms of Hamming distances whenever the fitness matrix has those properties.

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On the behavior of certain nonlinear parabolic equations with periodic boundary conditions

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Let $\Omega = [0, L_1] \times \cdots \times [0, L_k]$. In this talk we will consider the problem

$$\begin{cases} \frac{\partial u}{\partial t} = u^n \Delta u + u^{n+1} & \text{on} \quad \Omega \times (0,T) \\ u(x,0) = \psi(x) & \text{in} \quad \Omega \end{cases} \tag{1}$$

n a positive integer, under periodic boundary conditions. It is known that if $\psi > 0$, then the classical solution to (1) blows up in finite time. We will show that under some assumptions on the Fourier expansion of the initial condition, and on the first nontrivial Laplacian eigenvalue of Ω, there are constants $C_k > 0$ and $\beta > 0$ such that

$$\left\| u(\cdot,t) - \frac{1}{\mathrm{Vol}(\Omega)} \int_\Omega u(x,t)\, dx \right\|_{C^k(\Omega)} \le C_k (T-t)^\beta$$

where $T > 0$ is the blow-up time of the solution to (1). Part of this talk is joint work with Alexander Murcia.

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Decay property of regualrity-loss type for quasi-linear hyperbolic systems of viscoelasticity

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In this talk, we consider a quasi-linear hyperbolic systems of viscoelasticity. This system has dissipative properties of the memory type and the friction type. The decay property of this system is of the regularity-loss type. To overcome the difficulty caused by the regularity-loss property, we employ a special time-weighted energy method. Moreover, we combine this time-weighted energy method with the semigroup argument to obtain the global existence and sharp decay estimate of solutions under the smallness conditions and enough regularity assumptions on the initial data.

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Everywhere regularity for cross diffusion systems involving p-Laplacian: the degenerate case

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Using nonlinear heat approximation and homotopy arguments, we study Hölder regularity of bounded weak solutions to strongly coupled and degenerate parabolic systems.

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Existence and blow-up results for fast diffusion equations with nonlinear lower order terms

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We deal with blow-up and existence results for solutions of equations with principal part of fast diffusion type and presenting a reaction term $g(u)$. More precisely, as far as the blow-up phenomenon is concerned, we consider the following problem

$$\begin{cases} u_t - \triangle(u^m) = \lambda g(u) & \text{in} \quad \Omega \times (0,\infty), \\ u = 0 & \text{on} \quad \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x) & \text{on} \quad \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is an open bounded set, 00, g nondecreasing and convex and such that

$$\int_0^\infty \frac{d\sigma}{g(\sigma)} < +\infty,$$

and we prove that for λ not too small Fujita property occurs i.e. whatever is the initial datum u_0, all the solutions blow-up in finite time in the sense that

$$\mathrm{limsup}_{t\to T^-} \|u(x,t)\|_{L^\infty(\Omega)} = +\infty.$$

On the other hand, as far as the existence is concerned, we will prove existence of global solutions to

$$\begin{cases} u_t - \triangle(|u|^{m-1}u) = g(u) + \mu & \text{in} \quad \Omega \times (0,\infty), \\ u = 0 & \text{on} \quad \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x) & \text{on} \quad \Omega \end{cases}$$

essentially covering the complementary situation , i.e.

$$\int_0^\infty \frac{d\sigma}{g(\sigma)} = +\infty,$$

for general data μ and u_0, i.e. μ finite Radon measure and $u_0 \in L^1(\Omega)$, and for $\frac{N-1}{N} < m < 1$. Here we mean $|u|^{m-1}u = 0$ in the set where $u = 0$.

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An approximation scheme for area-constrained curvature-driven multiphase motions

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Seiro Omata, Karel Svadlenka

We develop a numerical method for investigating area-constrained interfacial motions. We show how to reformulate the BMO algorithm in a vector-valued setting, which we then treat by means of a minimizing movement; hence the variational nature of our method permits the inclusion of area constraints, via penalization. As the constrained motions tend to be much slower than motion by mean curvature, the well-known time and grid resolution restrictions of the BMO algorithm become particularly relevant. We thus discuss how to alleviate these restrictions, and we show the numerical results of our method for investigating the viscous motion of bubble clusters.

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Two-dimensional curved fronts in a periodic shear flow

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M. El Smaily, F. Hamel

In this talk, we consider the traveling fronts of reaction-diffusion equations with periodic advection in the whole plane R^2. We are interested in curved fronts satisfying some “conical” conditions at infinity. We prove that there is a minimal speed c^* such that curved fronts with speed c exist if and only if $c \geq c^*$. Moreover, we show that such curved fronts are decreasing in the direction of propagation, that is, they are increasing in time. We also give some results about the asymptotic behaviors of the speed with respect to the advection, diffusion and reaction coefficients. This is a joint work with M. El Smaily and F. Hamel.

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On solvability for a class of quasilinear elliptic equations with superlinear growth in weighted spaces

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This talk will focus on the existence of the solutions for a class of quasilinear elliptic equations.

We are mainly interested in which the nonlinearity possesses the superlinear growth conditions. Our method is based on the Galerkin method, the generalized Brouwer's theorem and a weighted compact Sobolev-type embedding theorem established by V. L. Shapiro.

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An analysis in the space of BV functions for the equation of motion of a vibrating membrane with a "viscosity" term

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Let Ω be a bounded domain in $\boldsymbol{R}^n$ with Lipschitz continuous boundary $\partial\Omega$. In this talk we investigate the following equation in the space of BV functions:

$$u_{tt} - \operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) - \left(\operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right)\right)_t = 0, \tag{1}$$

$$u(0,x) = u_0(x), \quad u_t(0,x) = v_0(x), \quad x \in \Omega, \tag{2}$$

$$u(t,x) = 0, \quad x \in \partial\Omega. \tag{3}$$

A function u is said to be a *BV function* in Ω if the distributional derivative Du is an $\boldsymbol{R}^n$ valued finite Radon measure in Ω. The vector space of all BV functions in Ω is denoted by $BV(\Omega)$. It is a Banach space equipped with the norm $\|u\|_{BV} = \|u\|_{L^1(\Omega)} + |Du|(\Omega)$. The difficult point is that, for $u \in BV(\Omega)$, the operater $\operatorname{div}(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}})$ is multivalued. It is usually defined by the use of subdifferential of the area functional. Namely, supposing that $u \in BV(\Omega) \cap L^2(\Omega)$, we regard $-\operatorname{div}(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}})$ as

$$\begin{aligned}\partial J(u) = \{&f \in (BV(\Omega) \cap L^2(\Omega))'; \\ &J(u+\phi) - J(u) \geq (f, \phi) \\ &\text{for each } \phi \in BV(\Omega) \cap L^2(\Omega)\},\end{aligned}$$

where $J(u) = \sqrt{1+|Du|^2}(\overline{\Omega})$. We report that, if u_0 is slightly smooth, then there exists a unique solution to (1)–(3).

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Global solutions of a diffuse interface model for the two-phase flow of compressible viscous fluids in 1-D

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In this talk, we consider a coupled Navier-Stokes/Cahn-Hilliard system which describes a diffuse interface model for the two-phase flow of compressible viscous fluids in a bounded domain in one dimension. We prove the existence and uniqueness of global classical solution for $\rho_0 \in C^{3,\alpha}(I)$ with $\rho_0 \geq c_0 > 0$. Moreover, we also discuss the existence of weak solutions and the existence of unique strong solution for $\rho_0 \in H^1(I)$ and $\rho_0 \in H^2(I)$, respectively, satisfying $\rho_0 \geq c_0 > 0$.

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Numerical study for long-time solutions for some hyperbolic conservation laws with nonlinear term

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In this talk, we study solution behavior for two hyperbolic conservation laws with nonlinear force terms at large time via central-upwind schemes. The most advantage of central-typed scheme is simplicity because no approximate Riemann solver is needed. Central-upwind scheme employs this advantage with less numerical viscosity so that it can be applied to study solution behavior at large time. We start with simulation for an initial-boundary value problem of a2x2 p-system with nonlinear force term via an central-upwind scheme introduced by Tadmor and Korganov. We confirm that the solution globally exists and converges to its corresponding diffusion wave, or the solution blows up at a finite time under suitable condition. For convergence case, convergence rates are calculated. We then turn to study solution behavior of an initial-value problem of a 1D Euler-Poisson equation defined on bounded domain. With the help of an improved Kurganov-Tadmorscheme introduced by Kurganov, Noelle and Petrova in 2001, we shall demonstrate that the solution converges to its corresponding boundary value problem.

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Traveling waves for nonlocal dispersion equation

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Rui Huang, Yong Wang

In this talk, we study a class of nonlocal dispersion equation with monostable nonlinearity in n-dimensional space

$$\begin{cases} u_t - J * u + u + d(u(t,x)) = \\ \qquad\qquad \displaystyle\int_{\mathbb{R}^n} f_\beta(y) b(u(t-\tau, x-y))dy, \\ u(s,x) = u_0(s,x), \quad s \in [-\tau, 0], \ x \in \mathbb{R}^n, \end{cases}$$

where the nonlinear functions $d(u)$ and $b(u)$ possess the monostable characters like Fisher-KPP type, $f_\beta(x)$ is the heat kernel, and the kernel $J(x)$ satisfies $\hat{J}(\xi) = 1 - \mathcal{K}|\xi|^\alpha + o(|\xi|^\alpha)$ for 00. After establishing the existence for both the planar traveling waves $\phi(x \cdot \mathbf{e} + ct)$ for $c \geq c_*$ (c_* is the critical wave speed) and the solution $u(t,x)$ for the Cauchy problem, as well as the comparison principles, we prove that, all noncritical planar wavefronts $\phi(x \cdot \mathbf{e} + ct)$ are globally stable with the exponential convergence rate $t^{-n/\alpha} e^{-\mu_\tau t}$ for $\mu_\tau > 0$, and the critical wavefronts $\phi(x \cdot \mathbf{e} + c_* t)$ are globally stable in the algebraic form $t^{-n/\alpha}$, and these rates are optimal. As application, we also automatically obtain the stability of traveling wavefronts to the classical Fisher-KPP dispersion equations. The adopted approach is Fourier transform and the weighted energy method with a suitably selected weight function. This is a joint work with Rui Huang and Yong Wang.

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Mathematical and computational aspects of problems involving adhesion, detachment, and collision

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We examine the behavior of solutions to free boundary problems expressing the motion of oil droplets and soap bubbles over flat surfaces, as well as bounce-type collision dynamics. Such phenomena are difficult to treat, both analytically and numerically, due to the presence of free boundaries and global constraints (which arise from the presence of volume constraints). We will discuss the problem settings and focus on the development of numerical methods for investigating such phenomena. We will also show the computational results obtained by our methods.

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On decay estimates for solutions of some parabolic equations

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It is well known that the solution of the heat equationwith summable initial datum u_0 satisfies the decay estimate

$$\|u(\cdot,t)\|_{L^\infty} \leq C \frac{\|u_0\|_{L^1}}{t^{\frac{N}{2}}}, \quad t > 0.$$

We show here that decay estimates can be derived simply by integral inequalities. This result allows us to prove this kind of estimates, with an unified proof, for different nonlinear problems, thus obtaining both well known results (for example for the p-Laplacian equation and the porous medium equation) and new decay estimates.

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Viscosity solutions of a class of degenerate quasilinear parabolic equations

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We study a class of degenerate quasilinear parabolic equations in a bounded domain with a Dirichlet or nonlinear Neumann type boundary condition. The equation under consideration arises from a number of practical model problems including reaction-diffusion processes in a porous medium. Our goal is to establish some comparison properties between viscosity upper and lower solutions and to show the existence of a continuous viscosity solution between them. Applications are given to a porous-medium type of reaction-diffusion model whose global existence and blow up property are sharply different from that of the nondegenerate one.

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On a class of doubly nonlinear parabolic equations with nonstandard growth: existence, blow-up and vanishing

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The talk addresses the questions of existence and the qualitative behavior of solutions of the homogeneous Dirichlet problem for the doubly nonlinear anisotropic parabolic equation

$$u_t = \sum_{i=1}^{n} D_i \left(|D_i(|u|^{m(x)-1}u)|^{p_i(z)-2} D_i(|u|^{m(x)-1}u) \right) + c(z)|u|^{\sigma(z)-2}u.$$

The exponents of nonlinearity $m(x) > 0$, $p_i(x,t) \in (1,\infty)$, $\sigma(x,t) \in (1,\infty)$ are given functions of their arguments. We prove that the problem admits a strong energy solution in a suitable Orlicz-Sobolev space prompted by the equation and establish sufficient conditions of the finite time blow-up or vanishing.

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Cauchy problem for the damped singularly perturbed Boussinesq-type equation

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We are concerned with the Cauchy problem for the damped singularly perturbed Boussinesq-type equation

$$u_{tt} - u_{xx} - \alpha u_{xxxx} + 2bu_{xxxxt} - \beta u_{xxxxxx} = (u^n)_{xx} \quad \text{in } \mathbf{R} \times (0,\infty),$$
$$u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \qquad x \in \mathbf{R},$$

where $\alpha, \beta > 0$ and $b > 0$ are real numbers, $n \geq 2$ is an integer, $u_0(x)$ and $u_1(x)$ are the given functions. Under suitable assumptions, we prove that for any $T > 0$, the Cauchy problem admits a unique global smooth solution $u(x,t) \in C^\infty((0,T]; H^\infty(\mathbf{R})) \cap C([0,T]; H^5(\mathbf{R})) \cap C^1([0,T]; H^1(\mathbf{R}))$.

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Refined asymptotics for the infinite heat equation with homogeneous Dirichlet boundary conditions

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We show that the nonnegative viscosity solutions to the infinite heat equation $\partial_t u = \Delta_\infty u$ with homogeneous Dirichlet boundary conditions converge as $t \to \infty$ to a uniquely determined limit after a suitable time rescaling. The proof relies on the half-relaxed limits technique as well as interior positivity estimates and boundary estimates. Moreover, we also study the expansion of the support.

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Finite-time blow-up in the higher-dimensional Keller-Segel system

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We study the Neumann initial-boundary value problem for the fully parabolic Keller-Segel system

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \nabla v), \; x \in \Omega, \; t > 0, \\ v_t = \Delta v - v + u, \; x \in \Omega, \; t > 0, \end{cases} \qquad (\star)$$

in a ball $\Omega \subset \mathbb{R}^n$ with $n \geq 3$. This system forms the core of numerous models used in mathematical biology to describe the spatio-temporal evolution of cell populations governed by both diffusive migration and chemotactic movement towards increasing gradients of a chemical that they produce themselves. We demonstrate that for any prescribed $m > 0$ there exist radially symmetric positive initial data $(u_0, v_0) \in C^0(\bar{\Omega}) \times W^{1,\infty}(\Omega)$ with $\int_\Omega u_0 = m$ such that the corresponding solution blows up in finite time. Moreover, by providing an essentially explicit blow-up criterion it is shown that within the space of all radial functions, the set of such blow-up enforcing initial data indeed is large in an appropriate sense; in particular, this set is dense with respect to the topology of $L^p(\Omega) \times W^{1,2}(\Omega)$ for any $p \in (1, \frac{2n}{n+2})$. One focus of the presentation is on the method through which this result is obtained. In contrast to previous approaches, it is based on a more elaborate use of the natural energy inequality associated with $(\star)$, involving an estimate of the form

$$\int_\Omega uv \leq C \cdot \left(\left\| \Delta v - v + u \right\|_{L^2(\Omega)}^{2\theta} + \left\| \frac{\nabla u}{\sqrt{u}} - \sqrt{u} \nabla v \right\|_{L^2(\Omega)} + 1 \right),$$

valid with certain $C > 0$ and $\theta \in (0,1)$ for a wide class of smooth positive radial functions $(u,v) = (u(x), v(x))$.

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Global existence and asymptotic behavior of the solutions to the three dimensional bipolar Euler-Poisson systems

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In this talk, I will present the Green function of the linerized Euler-Poisson with electric field and frictional damping added to the momentum equations. This method was used to study the optimal decay rate of the evolution PDEs. It was applied to consider the existence of the smooth solution to the three-dimensional bipolar hydrodynamic model when the initial data are close to a constant state.

We found that the electric field affects the dispersion of fluids and reduces the time decay rate of solutions.

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Existence of monotone traveling waves for a delayed non-monotone population model on 1-D lattice

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In this talk, the existence of monotone traveling waves for general lattice equations with delays will be obtained by a new monotone iteration technique based on a lower solution. The results can be well applied to a delayed non-monotone population model on 1-D lattice and thus the monotone traveling wave will be obtained by choosing a pair of suitable upper-lower solutions, which was left open in some recent works.

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Longtime dynamics for an elastic waveguide model

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The paper studies the longtime dynamics for a nonlinear wave equation arising in elastic waveguide model

$$u_{tt} - \Delta u - \Delta u_{tt} + \Delta^2 u - \Delta u_t - \Delta g(u) = f(x).$$

Under the assumption that g is of the polynomial growth order, say p, it proves that (i) when $1 \le p \le \frac{N+2}{(N-2)^+}$, the above mentioned model has a global solution in phase space with low regularity E_0; (ii) when $2 \le p \le \frac{N}{(N-2)^+}$, the related solution semigroup $S(t)$ possesses in E_0 a finite dimensional global attractor $\mathcal{A}$, which has E_1-regularity, and an exponential attractor; (iii) when $1 \le p \le \frac{N+2}{(N-2)^+}$, the above model possesses a global trajectory attractor.

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