Special Session 45: Stochastic and Deterministic Dynamical Systems, and Applications

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The aim of this session is to offer an overview on recent results concerning the asymptotic behaviour of solutions of stochastic and deterministic partial and ordinary differential equations. The main topics of the session are: existence and properties of pullback attractors for stochastic and non-autonomous equations, stability, stabilization, attractors for equations without uniqueness, dynamics of equations with delay and finite-dimensional dynamics for dynamical systems.

Lyapunov exponents for autonomous and non-autonomous systems

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It is an extended idea to associate positive Lyapunov exponents to instability of orbits in both autonomous and non-autonomous systems and stability to negative values of such exponents. But such issue is not true in general. We will consider separately the two cases stating under what conditions the former statement is true. We will construct one and two dimensional systems to show the differences between the autonomous and non-autonomous situations. Obviously the non-autonomous case is too much more complicated. We will extend the results to n-dimensional systems of both types.

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Analytical and numerical results on escape of brownian particles

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A particle moves with Brownian motion in a material, e.g. a unit disc, with reflection from the boundaries except for a portion (called a "window" or "gate") in which it is absorbed. A key question involves the differences between the small, finite step size and the limiting Brownian motion. The stochastic problem of Brownian motion can be transformed into an elliptic PDE with mixed boundary conditions. Our work confirms an asymptotic formula that had been obtained by Chen and Friedman. Furthermore, we obtain an exact solution for a gate of any size.

The main problems are to determine the first hitting time and spatial distribution. Also given is the probability density of the location where a particle hits if initially the particle is at the center or uniformly distributed. Numerical simulations of the stochastic process with finite step size and sufficient amount of sample paths are compared with the exact solution to the Brownian motion (the limit of zero step size), providing an empirical formula for the divergence. Histograms of first hitting times are also generated. These problems have applications to cell biology and materials science.

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Asymptotic behavior of linear elliptic problems with Dirichlet conditions on random varying domains

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Given $(\Omega, \mathcal{F}, P)$ a probability space, we study the asymptotic behavior of the solutions of linear elliptic problems posed in a fixed domain $O \subset \mathbb{R}^N$, $N \geq 3$, satisfying Dirichlet boundary conditions on a random sequence of "holes" $\Gamma_n(\omega) \subset O$, P-a.e. ω in Ω. Under assumptions about the size and the distribution of the holes, analogously to the classical Cioranescu-Murat "strange term", we show the existence of a new term of order zero in the limit equation. The proof of this result is based on the ergodic theory and Nguetseng and Allaire's two scale convergence method.

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Asymptotic behaviour of lattice systems perturbed by additive noise

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In this talk we study the asymptotic behaviour of solutions of a first-order stochastic lattice dynamical system perturbed by noise. We do not assume any Lipschitz condition on the nonlinear term, just a continuity assumption together with growth and dissipative conditions, so that uniqueness of the Cauchy problem fails to be true. Using the theory of multi-valued random dynamical systems we prove the existence of a random compact global attractor.

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Fractional stochastic porous media equations

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The aim of this talk is to study the existence and uniqueness of solutions of stochastic porous media equations driven by fractional Brownian motion, when the Hurst parameter $H > 1/2$. Using the theory of random dynamical systems we also discuss some results on the asymptotic behavior of these solutions.

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Cahn-Hilliard equations with memory effects

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We introduce a modified Cahn-Hilliard equation proposed by P. Galenko et al. in order to account for rapid spinodal decomposition in deep supercooling glasses or in binary alloys. In this equation the dynamics of the order parameter depends on past history of the chemical potential. Such dependence is expressed through a time convolution integral characterized by a smooth nonnegative exponentially decreasing memory kernel. We intend to present some results on this non-standard equation with special regard to dynamic boundary conditions and long-time behavior of solutions. In particular, we discuss the convergence of solutions when the memory kernel approaches the Dirac mass.

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A fractional stochastic Schrödinger equation

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We consider $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$ to be a filtered complete probability space.Let $(V, (\cdot,\cdot)_V)$ and $(H, (\cdot,\cdot))$ be separable complex Hilbert spaces,such that (V, H, V^*) forms a triplet of rigged Hilbert spaces. Let K be a separable real Hilbert space.We assume $\Big(W(t)\Big)_{t\geq 0}$ to be a K-valued cylindrical Wiener processadapted to the filtration $(\mathcal{F}_t)_{t\geq 0}$ and $(B^h(t))_{t\geq 0}$ to be a K-valuedcylindrical fractional Brownian motion with Hurst index $h \in (\frac{1}{2}, 1)$ adapted to the filtration $(\mathcal{F}_t)_{t\geq 0}$.
We study the properties of the *variational solution* X of the following stochastic nonlinear evolution equation of Schrödinger type

$$\begin{aligned}(X(t), v) &= (X_0, v)\\ &- i\int_0^t \langle AX(s), v\rangle ds\\ &+ i\int_0^t (f(s, X(s)), v)ds\\ &+ i(\int_0^t g(s, X(s))dW(s), v)\\ &+ i(\int_0^t b(s)dB^h(s), v)\end{aligned}$$

for a.e. $\omega \in \Omega$ and all $t \in [0, T], v \in V$.
Especially, conditions are given such that the Malliavin derivative of the solution process exists.

References

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[2] Grecksch, W.; Lisei, H. Stochastic Schrödinger Equation Driven by Cylindrical Wiener Process and Fractional Brownian Motion.Stud. Univ. Babes-Bolyai Math. 56(2011), No.2, 381-391.

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On the global attractor for the Kazhikhov-Smagulov equations

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In this talk we will study the existence of the global attractor for the Kazhikhov-Smagulov equation which govern the dynamics of the two fluids having different densities subject to Fick's law:

$$\left\{\begin{array}{rcll} \partial_t\rho + \boldsymbol{u}\cdot\nabla\rho - \lambda\Delta\rho &=& \boldsymbol{0} & \text{in } \Omega\times(0,\infty),\\ \rho\partial_t\boldsymbol{u} + (\rho\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \mu\Delta\boldsymbol{u} + \nabla p & & & \\ -\lambda(\nabla\rho\cdot\nabla)\boldsymbol{u} - \lambda(\boldsymbol{u}\cdot\nabla)\nabla\rho &=& \rho\boldsymbol{f} & \text{in } \Omega\times(0,\infty),\\ \nabla\cdot\boldsymbol{u} &=& 0 & \text{in } \Omega\times(0,\infty),\end{array}\right.$$

for a bounded two-dimensional domain Ω, with initial conditions

$$\boldsymbol{u}(\boldsymbol{x}, t) = \boldsymbol{0}, \quad \partial_{\boldsymbol{n}}\rho(\boldsymbol{x}, t) = 0 \quad (\boldsymbol{x}, t) \in \partial\Omega\times(0,\infty),$$

and the boundary conditions

$$\rho(\boldsymbol{x}, 0) = \rho_0(\boldsymbol{x}), \quad \boldsymbol{u}(\boldsymbol{x}, 0) = \boldsymbol{u}_0(\boldsymbol{x}) \quad \boldsymbol{x} \in \Omega.$$

Here $\boldsymbol{u} : \Omega\times(0,\infty) \to \mathbb{R}^2$ is the velocity vector field, $p : \Omega\times(0,\infty) \to \mathbb{R}$ is the pressure scalar field, and $\rho : \Omega\times(0,\infty) \to \mathbb{R}$ is the density scalar field. The parameters μ and λ are assumed to be constant and represent the kinematic viscosity and a mass diffusion coefficient, respectively.

To simplify the discussion, we will only focus on the evolution of the velocity. For this, we will take a fixed initial density data ρ_0. Then we will prove that the velocity trajectories of the weak solutions of the Khazhikov-Smagulov equations are absorbed by a connected, compact invariant set $\mathcal{A}$ in the natural phase space for the Navier-Stokes problem.

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Features of fast living: on the weak selection for longevity in degenerate birth-death processes.

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Deterministic descriptions of dynamics of competing species with identical carrying capacities but distinct birth, death, and reproduction rates predict steady state coexistence with population ratios depending on initial conditions. Demographic fluctuations described by a Markovian birth-death model break this degeneracy. A novel large carrying capacity asymptotic theory confirmed by conventional analysis and simulations reveals a weak preference for longevity in the deterministic limit with finite-time extinction of one of the competitors on a time scale proportional to the total carrying capacity.

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Some generalizations of the Cahn-Hilliard equation

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Our aim in this talk is to discuss a Cahn-Hilliard model with a proliferation term which has, e.g., applications in biology. In particular, we will discuss the asymptotic behavior of the system in terms of finite-dimensional attractors.

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SDEs driven by fractional Brownian motion: continuous dependence on the Hurst parameter

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In this talk, we study the stochastic differential equation

$$dX_t^H = a(X_t^H)dt + \sigma(X_t^H)dB_t^H,$$

$t \in [0,T]$, $\quad X_0^H = x_0 \in \mathbb{R}^d$, where $a : \mathbb{R}^d \to \mathbb{R}^d$ and $\sigma : \mathbb{R}^d \to \mathbb{R}^{d,m}$ satisfy standard smoothness assumptions and $B^H = (B_t^H)_{t\in[0,T]}$ is an m-dimensional fractional Brownian motion with Hurst parameter $H \in (0,1)$ defined on a probability space $(\Omega, \mathcal{A}, \mathbf{P})$. Using tools from rough path theory we show that the law of the solution $X^H = (X_t^H)_{t\in[0,T]}$ depends continuously on $H \in [1/2, 1)$, i.e. we have

$$\mathbf{P}^{X^H} \longrightarrow \mathbf{P}^{X^{H_0}} \quad \text{for} \quad H \to H_0$$

with $H, H_0 \in [1/2, 1)$. Moreover, in the case of additive noise we give a stronger pathwise continuous dependence result for $H \in (0,1)$ and discuss applications to random attractors of such equations.

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Localization of solutions to stochastic porous media equations: finite speed of propagation

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We present a localization result for stochastic porous media equations with linear multiplicative noise. More precisely, we prove that the solution process P-a.s. has the property of "finite speed propagation of disturbances" in the sense of [Antontsev/Shmarev, Nonlinear Analysis 2005].

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The effects of noise on sliding motion

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Vector fields that are discontinuous along codimension-one surfaces are used as mathematical models of a wide range of physical systems involving a discontinuity or switch such as relay control systems and vibro-impacting systems. Trajectories evolve on discontinuity surfaces whenever the vector field on both sides of the surface points towards the surface. This is known as sliding motion and is formulated by the method of Filippov. To investigate the possible role of background vibrations, parametric uncertainty and model error, we analyze the effect of small, additive, white Gaussian noise on sliding trajectories in a simple two-dimensional, discontinuous vector field. We find that the noise pushes orbits slightly off the discontinuity surface and that this may induce a significant change in the observed dynamics. In addition, for this system we show that the mean of the stochastic solution limits on Filippov's definition as the noise amplitude is taken to zero.

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Sufficient and necessary criteria for existence of pullback attractors for non-compact random dynamical systems

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We study pullback attractors of non-autonomous non-compact dynamical systems generated by differential equations with non-autonomous deterministic as well as stochastic forcing terms. We first introduce the concepts of pullback attractors and asymptotic compactness for such systems. We then prove a sufficient and necessary condition for existence of pullback attractors. We also introduce the concept of complete orbits for this sort of systems and use these special solutions to characterize the structures of pullback attractors. For random systems containing periodic deterministic forcing terms, we show the pullback attractors are also periodic. As an application, we prove the existence of a unique pullback attractor for Reaction-Diffusion equations on unbounded domains with both deterministic and random external terms.

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