Special Session 47: Dynamics and Games

Alberto Pinto, LIAAD UP, Portugal
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The session is devoted to the development and the diffusion of mathematical ideas and techniques that arise from the analysis and the modelling of systems where agents (whether they be rational players, markets, plants, animals, ecosystems, communication systems, etc) interact dynamically over time. Abstracts should either be motivated by challenging mathematical questions occurring in such systems or provide a rigorous mathematical analysis of models where tools from dynamics and games prove to be useful. Areas covered include differential games, evolutionary games, models of learning and evolution, repeated games, mean field models, etc. and their applications in social, life, physical and computer sciences.

Anosov and renormalized circle diffeomorphisms

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Alberto A. Pinto, David Rand

In this talk we prove a one-to-one correspondence between C^{1+} smooth conjugacy classes of circle diffeomorphisms that are C^{1+} fixed points of renormalization and C^{1+} conjugacy classes of Anosov diffeomorphisms whose Sinai-Ruelle-Bowen measure is absolutely continuous with respect to Lebesgue measure. Furthermore, we use ratio functions to parametrize the infinite dimensional space of C^{1+} smooth conjugacy classes of circle diffeomorphisms that are C^{1+} fixed points of renormalization.

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Anosov diffeomorphisms and golden tilings

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In this talk we present the definition of *golden sequence*. These golden sequences have the property of being Fibonacci quasi-periodic and determine a tiling in the real line. We prove a one-to-one correspondence between:(i) affine classes of golden tilings;(ii) smooth conjugacy classes of Anosov difeomorphisms, with an invariant measure absolutely continuous with respect to the Lebesgue measure, that are topologically conjugate to the Anosov automorphism GA(x; y) = (x + y; x)(iii) solenoid functions. A. Pinto and D. Sullivan developed a theory relating 2-adic sequences (Pinto-Sullivan tilings in the real line) with smooth conjugacy classes of doubling expanding circle maps. The solenoid functions give a parametrization of the infinite dimensional space consisting of the mathematical objects described in the above equivalences.

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Performance of investment strategies in the absence of correct beliefs

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We study an evolutionary market model with long-lived assets. We show that in the absence of correct beliefs, the strategy which is "closer" to the Kelly rule cannot be driven out of the market. This means that this strategy will either dominate or at least survive. Our techniques are borrowed from the theory of random dynamical systems (RDS).

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Bayesian-Nash equilibria in a cave psychological model

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We construct a cave psychological model for the theory of Planned Behavior or Reasoned Action. In this model the individuals taste and crowding types follow the shadows of the taste and crowding types of the platonic idealized psychological model, according to a given probability distribution and the individuals know only the expected value of their welfare function. We present sufficient conditions for an individual or group to adopt a certain behavior decision according to the Bayesian-Nash Equilibria. We demonstrate how saturation, boredom and frustration can lead to the adoption of a variety of different behavior decisions and how no saturation can lead to the adoption of a single consistent behavior decision.

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Nash equilibria in a platonic idealized psychological model

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We construct a platonic idealized psychological model, using Game Theory, for the Theory of Planned Behavior. This model consists of individuals with no uncertainties in their taste and crowding types and welfare function. We propose the Nash Equilibria as one of many possible mechanisms to transform human intentions into behavior decisions. We show that saturation, boredom and frustration can lead to the adoption of a variety of different behavior decisions, as opposed to no saturation, which leads to the adoption of a single consistent behavior decision.

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Application of queueing theory to emergency care

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Congestion and long waiting times are a serious problem in emergency care in the United States. We are applying Queueing Theory to analyze the activities in an emergency care unit to see what improvements and economies can be obtained.

The study is being done in cooperation with Dr. Joseph Adrian Tyndall, Head of Emergency Care at Shands Hospital at the University of Florida. We will present some preliminary results of this study.

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The $W^{2,p}$ regularity for solutions of the simplest Isaacs equations

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In this talk, we discuss the interior $W^{2,p}$ regularity of viscosity solutions of the simplest uniformly elliptic Isaacs equations of the form $F(u_{xx}) := \max_{y\in Y}\min_{z\in Z} tr[a(y,z)u_{xx}] = 0$ in a domain $D \subset \mathbb{R}^d$, where Y, Z are finite sets. Here, $\forall\, y \in Y$, $z \in Z$, $a(y,z)$ is a symmetric $d \times d$ matrix, satisfying $\lambda I_d \leq a(y,z) \leq \Lambda I_d$, for some constants $0 < \lambda \leq \Lambda$. The Isaacs equation originates from the theory of stochastic differential games, and is of interest to pde specialists, since it is the prototypical, fully nonlinear elliptic equation which is neither convex nor concave in the Hessian matrix u_{xx}. Consequently, many of the standard techniques used to obtain regularity results for solutions of concave equations cannot be used in an obvious way for solutions of Isaacs equations. It is well-known that if $u \in C(D)$ is a viscosity solution of a uniformly elliptic equation of the form $F(u_{xx}) = 0$ in D, then $u \in W^{2,\delta}_{loc}(D)$ for some $\delta \in (0,1)$. The question is, for solutions of the simplest Isaacs equations described above, will viscosity solutions be locally $W^{2,2}$? We explore this question for C^2 solutions, and give examples.

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A game theoretical approach to resort prices

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We introduce a resort pricing model, where different types of tourists choose between different resorts. We study the influence of the resort prices on the choices of the different types of tourists. We characterize the coherent strategies of the tourists that are Nash equilibria. We find the prices that lead to the bankruptcy of the resorts and, in particular, their dependence on the characteristics of the tourists.

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A game theoretical approach to human decision

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Renato Soeiro, Abdelrahim Mousa, Alberto A. Pinto

We study a dichotomous decision model, where individuals can make the decision yes or no and can influence the decisions of others. We characterize all decisions that form Nash equilibria. Taking into account the way individuals influence the decisions of others, we construct the decision tilings where the axes reflect the personal preferences of the individuals for making the decision yes or no. These tilings characterize geometrically all the pure and mixed Nash equilibria. We show, in these tilings, that Nash equilibria form degenerated hystereses with respect to the replicator dynamics, with the property that the pure Nash equilibria are asymptotically stable and the strict mixed equilibria are unstable. These hystereses can help to explain the sudden appearance of social, political and economic crises. We observe the existence of limit cycles for the replicator dynamics associated to situations where the individuals keep changing their decisions along time, but exhibiting a periodic repetition in their decisions. We introduce the notion of altruist and individualist leaders and study the way that the leader can affect the individuals to make the decision that the leader pretends.

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The replicator dynamics and human decisions

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We add uncertainty to the model previously introduced. We study how these changes in the setup influence the previous results. In particular, we characterize all decisions that form Bayesian Nash equilibria. We look at how this change in the settings affects the way individuals influence the decisions of others and we construct the decision tilings.

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Random matching Edgeworthian economies trading in the core via a prisoners dilemma

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We show that for a specific class of random matching Edgeworthian economies, the expectation of the limiting equilibrium price coincides with the equilibrium price of the related Walrasian economies. This result extends to the study of economies in the presence of uncertainty within the multi-period Arrow-Debreu model, allowing to understand the dynamics of how beliefs survive and propagate through the market.

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Strategic optimization in R&D investment with uncertainty in the investment

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We use d'Aspremont and Jacquemin's strategic optimal R&D investment in a duopoly Cournot competition model to construct myopic optimal discrete and continuous R&D dynamics. We consider the existence of uncertainty in the outcome of the R&D investment. We show that for some high initial production costs, the success or failure of a

firm is very sensitive to small variations in its initial R&D investment strategies.

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Asynchronous stochastic approximation for learning in stochastic games

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We develop tools to analyse the convergence properties of fictitious play like algorithms in discounted reward stochastic games. To examine the best-response dynamic in stochastic games when asynchronous updating is present the differential inclusions framework for stochastic approximation is required. The asymptotic pseudo-trajectory approach to stochastic approximation of Benaïm, Hofbauer and Sorin is extended for asynchronous stochastic approximations with a set-valued mean field. The asynchronicity of the process is incorporated into the mean field to produce convergence results which remain similar to those of an equivalent synchronised process. In addition, this allows many of the restrictive assumptions previously associated with asynchronous stochastic approximation to be removed. By extending this approach to two timescales we are able to examine the convergence of an actor-critic algorithm for discounted reward stochastic games.

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Uncertainty effects in resort prices

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We add uncertainty to the resort pricing model previously introduced. We study how these changes in the setup influence the previous results. In particular, the coherent strategies of the tourists and the prices that lead to the bankruptcy of the resorts.

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A distance for belief spaces

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Given a finite set K, we denote by $X = \Delta(K)$ the set of probabilities on K and by $Z = \Delta_f(X)$ the set of Borel probabilities on X with finite support. Studying a Markov Decision Process with partial information on K naturally leads to a Markov Decision Process with full information on X. We introduce a new metric d_* on Z such that the transitions become 1-Lipschitz from $(X, \|.\|_1)$ to (Z, d_*). In this talk, we define and prove several properties of the metric d_*. Especially, d_* satisfies a Kantorovich-Rubinstein

type duality formula and can be characterized by using disintegrations. A second talk will contain applications to the existence and the characterization of "Long-term values in Markov Decision Processes and Repeated Games".

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Long-term values in Markov decision processes and repeated games

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We consider several types of Markov Decision Processes with or without partial observation, and Repeated Games. Each case will be expressed as a "compact non expansive" MDP, where the state space is (pre)compact and the transitions are non expansive for a well-chosen distance. In particular we use an appropriate metric for belief spaces to characterize the limit value in Partial Observation MDP with finitely many states and in repeated games with an informed controller with finite sets of states and actions. Moreover in each case we prove the existence of a generalized notion of uniform value where we consider not only the Cesaro mean when the number of stages is large enough but any evaluation function $\theta = (\theta_t)_{t\geq 1}$ (weights on stages) when the impatience $I(\theta) = \sum_{t\geq 1} |\theta_{t+1} - \theta_t|$ is small enough.

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Evolutionary stability in multiple-move games

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We consider a basic dynamic evolutionary model (or viewed as a rudimentary learning process) with rare mutation and a best-reply (or better-reply) selection mechanism. We call a state evolutionarily stable if its long-term relative frequency of occurrence is bounded away from zero as the mutation rate decreases to zero. For finite extensive-form games of perfect information where each player can only play at one node, the combined results of Hart and Gorodeisky show that the backward induction equilibrium becomes in the limit the only evolutionarily stable outcome as the mutation rate decreases to zero and the populations increase to infinity. We give three games where players may play at more than one node along some path. We show that, even when the populations increase to infinity, the backward induction equilibrium or even the whole backward induction equilibrium component is not always the only evolutionarily stable outcome in this model.

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Project evaluation and hedging in incomplete markets using historical prices

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The methodology of real options is by now a well accepted one by many corporations, especially for R&D projects. Yet, there are many discussions concerning the validity of applying mathematical finance techniques such as hedging and risk neutral pricing to situations where the decision maker cannot fully replicate his or her business decisions. In this talk we shall present a methodology for project evaluation that makes use of historical data and minimizes certain functionals that quantify risk. We shall also discuss the connection with games and with earlier work of M. Grasselli on the use of utility functions for real option evaluation.

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