Special Session 48: Nonlinear Evolution Equations

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The theme of this session is nonlinear evolution equations including the NLS equation, the KdV equation, the Camassa-Holm equation, and the Euler equations of hydrodynamics. It will focus on questions of local and global well-posedness, dependence of solutions on the initial data, unique continuation, regularity, and integrability. Such questions are fundamental in both theory and applications.

Linear instability of periodic traveling waves for nonlinear dispersive models

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In this talk we present a study on the linear and nonlinear instability of periodic traveling wave associated with some general one-dimensional dispersive models. By using analytic and asymptotic perturbation theory, we establish sufficient conditions for the existence of exponentially growing solutions to the linearized problem and so the linear instability of periodic profiles is obtained. Applications of this approach are concerning with the linear/nonlinear instability of cnoidal wave solutions for the modified Benjamin-Bona-Mahony and the modified Kortewegde Vriesequations. The arguments presented in this investigation has prospects for the study of the instability of periodic traveling wave of other nonlinear evolution equations.

Ovsiannikov's theorem for autonomous equations and applications

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Using a refined version of Ovsiannikov's theorem for autonomous equations we prove local well-posedness of an abstract Cauchy problem in a space of periodic functions that extend analytically in a strip of the complex plane around the x-axis. Then, we apply this result to some important weakly dispersive equations.

Nonlinear evolution equation for magneto-convective flow in an active mushy layer

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Here we consider the solidification problem of a binary alloy which is cooled from below. It has been well established by experimentalists that a horizontal mushy layer is formed during the solidification of binary alloys. This study investigates nonlinear behavior of the convective flow in the mushy layer in presence of a magnetic field. The mushy layer is treated as an active porous media with variable permeability. We derive the linear, adjoint and first-order systems and use these solutions to obtain the Landau coefficients which appear in the resulting evolution equation. Numerical results obtained from our computations suggest that there is a slow transition of the flow to a steady state.

Illposedness of a weakly dispersive Boussinesq system

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In this talk we discuss some illposedness issues associated to the Cauchy problem of a long-wave water wave system. Using an abstract wellposedness theory developed by Bejenaru-Tao for semilinear dispersive equations, we find the critical Sobolev index below which the solution map fails to be continuous with respect to initial data. We also derive a criterion for the blow-up of strong solutions. This is a joint work with Yue Liu.

Generalized wave maps on the sphere

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Wave maps are nonlinear generalizations of the wave equation which have been studied for decades. In this talk, we will consider a generalization of wave maps based on the Adkins-Nappi model of nuclear physics. This model yields nonlinear hyperbolic partial differential equations, for which we consider the question of regularity of solutions. In particular, we will discuss the non-concentration of energy, a preliminary step in establishing a global regularity theory.

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Continuity properties of the solution map for the generalized reduced Ostrovsky equation

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It is shown that the data-to-solution map for the generalized reduced Ostrovsky (gRO) equation is not uniformly continuous on bounded sets in Sobolev spaces. Considering that for this range of exponents the gRO equation is well-posed with continuous dependence on initial data, this result makes the continuity of the solution map an optimal property.

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Solutions of Björling problem for timelike surface and the homogeneous wave equation

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We solve the Björling problem for timelike surfaces in $\mathbb{R}^3_1$ and $\mathbb{R}^4_2$ by constructing a split-complex representation formula for those surfaces. Our approach includes the construction of split-holomorphic extensions in a natural way using the point of view of solutions to the homogeneous wave equation. Then we also establish Schwarz reflection to obtain split-complex Börling representations in symmetric domains of the split-complex plane.

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Cauchy problem for some hyperbolic equations of mathematical cosmology

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The talk is concerned with the waves propagating in the universe modeled by the cosmological models. We will present fundamental solutions of the wave equation in the Einstein-de Sitter spacetime. In this talk we investigate initial value problem for this equation and give the explicit representation formulas for the solutions. The equation is strictly hyperbolic in the domain with positive time. On the initial hypersurface its coefficients have singularities that make difficulties in studying of the initial value problem. In particular, one cannot anticipate the well-posedness in the Cauchy problem for the wave equation in the Einstein-de Sitter spacetime. The initial conditions must be modified to so-called weighted initial conditions in order to adjust them to the equation. We will present also the Lp-Lq estimates for solutions. This is a joint work with Tamotu Kinoshita (Japan) and Karen Yagdjian(U.S.A.).

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Analysis of the b-family equation

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We shall consider the Cauchy problem for the b-family of equations with initial data in Sobolev spaces in both the periodic and non-periodic case. In particular, we shall discuss continuity properties of the data-to-solution map. Members of this family are intriguing since they all have peak on solutions. Furthermore, the b-family contains the Camassa-Holm and the Degasperis-Procesi equations, which are integrable.

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The initial value problem of a periodic KdV equation

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We shall discuss the periodic Cauchy problem for a KdV equation whose dispersion is of order $2j+1$, where j is a positive integer. The initial data considered range from Sobolev to analytic Gevrey spaces. For $s \geq -j/2$ well-posedness in H^s is proved by deriving the bilinear estimates corresponding to the linear part of the KdVm equation and using appropriate Bourgain spaces. When the initial data belong to an analytic Gevrey space of order σ then well-posedness is proved by using Foias-Temam-Bourgain type analytic Gevrey spaces.

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Next generation sequencing and differential gene expression

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We give an overview of the bioinformatics associated with next generation sequencing. In particular, we focus on the application of the RNASeq pipeline and consider an experiment involving differentially expressed genes.

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Well-posedness of the generalized Burgers equation in analytic Gevrey spaces

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We consider the initial value problem for the generalized Burgers equation and study its well-posedness. In particular, we show that the Cauchy problem is well posed for initial data in a class of Soblev-Gevrey spaces with index r greater than or equal to one.. This implies that the solution is Gevrey-r in the spacial variable and Gevrey-$2r$ in the time variable.

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Enhanced lifespan of smooth solutions of a Burgers-Hilbert equation

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We consider an initial value problem for a quadratically nonlinear inviscid Burgers-Hilbert equation that models the motion of vorticity discontinuities. We use a normal form transformation, which is implemented by means of a near-identity coordinate change of the independent spatial variable, to prove the existence of small, smooth solutions over cubically nonlinear time-scales. For vorticity discontinuities, this result means that there is a cubically nonlinear time-scale before the onset of filamentation.

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New ill-posedness results for the boussinesq equation

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We present new ill-posedness results for the nonlinear "good" Boussinesq equation, which improve upon the ones previously obtained in the literature. In particular, it is proved that the solution map is not continuous in Sobolev spaces H^s, for all s

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Local existence of solutions of self gravitating relativistic perfect fluids

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The talk will be devoted to the evolution counterpart of Einstein equations which are coupled to the Euler equations. The classical result of Hughes, Kato and Marsden yields a local in time existence and uniqueness for the vacuum Einstein equations in the Bessel potential spaces withregularity index $s > \frac{5}{2}$. Their results rely on the technique of quasilinear waves equations, however, this technique is not available when the gravitational fields are coupled withthe Euler equations. We have obtained the well-posedness of the coupled system, with the same regularity as in the classic case, by means of a modification of standard approach to the energy estimates for a certain form of first order symmetric hyperbolic systems.

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Smoothing results for Korteweg de-Vries equations on R and T

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We discuss smoothing results for KdV under low-regularity setting. The main technique involved in this discussion is the normal form approach. Let u be the solution of KdV with the initial data u_0. In $\mathbf{R}$, we obtain the smoothing of $1/2$-derivatives for $u - e^{-t\partial_x^3}u_0$ when the initial data lies in $H^{-1/2} \cap \dot{H}^{-1}$. In $\mathbf{T}$, we also obtain the smoothing of $1/2$-derivatives for $u - R^*[u_0]$ with $u_0 \in H^{-1/2}$ where $R^*[u_0]$ describes the explicit solution to the resonance equation arising from the normal form transformation.

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Well-posedness and regularity of the periodic gKdV equation

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In the periodic case it is proved that the generalized Korteweg-de Vries equation (gKdV) is locally well-posed in a class of analytic spaces that are similar to the ones used by Grujić and Kalisch in the non-periodic case. Thus, the uniform analyticity radius of the solution in the space variable does not change as time progresses. We also study regularity in the time variable.

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Nonuniform continuity of the solution map for CH type equations

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We consider the Cauchy problem for Camassa-Holm type equations and prove that the data-to-solution map is not uniformly continuous in Sobolev spaces. The main tools to be used in the proof of this result include, but are not limited to, approximate solutions and well-posedness estimates.

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Generic non self-adjoint Zakharov-Shabat operators

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I will discuss the generic properties of the spectrum of the Zakharov-Shabat operatorwith periodic boundary conditions.

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Global existence of the scalar field in de Sitter spacetime

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In this talk we present global existence of the small data solutions of the Cauchy problem for the semilinear Klein-Gordon equation in the de Sitter spacetime. The Klein-Gordon equation arising in relativistic physics and, in particular, general relativity and cosmology, as well as, in more recent quantum field theories, is a covariant equation that is considered in the curved pseudo-Riemannian manifolds.

The latest astronomical observational discovery that the expansion of the universe is speeding supports the model of the expanding universe that is mathematically described by the manifold with metric tensor depending on time and spatial variables. In this talk we restrict ourselves to the manifold arising in the so called de Sitter model of the universe, which is the curved manifold due to the cosmological constant. Unlike the same problem in the Minkowski spacetime, we have no restriction on the order of nonlinearity and structure of the nonlinear term, provided that a physical mass of the field is outside of some interval.

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