Special Session 49: Growth Models and Interface Dynamics

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The aim of this session is to bring together mathematicians and physicists working on nonlinear growth processes as well as processes occurring on the interfaces. These processes include but are not limited by the Laplacian and elliptic growth, crystal growth and coarsening, growth of solid nuclei in the subdiffusive medium as well as front propagation into the unstable medium.

Driven free-standing foam films

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Thin liquid films or foam films are the basic constituents of emulsions, or foams and are important for a wealth of applications ranging from process engineering to everyday articles of use as shampoos or soft drinks. The interface processes are highly interesting and of fundamental interest for physics, physical chemistry, engineering and eventually mathematics. Thin film theory is worked out and understood to a reasonable degree for films on a horizontal or inclined surface. Free-standing,vertical films are different in that they have two free surfaces and consequently the physics of the surfactants play a dominant role. Whereas the equilibrium properties of such free-standing foam films are investigated experimentally and theoretically, transient processes and nonequilibrium dynamics in general are still an area to explore. We present a summary of experimental results for foam films driven by a thermal gradient or electromagnetic forces. Further, we give the basic equations using lubrication theory and explain the relevant dynamics in terms of partial diffrerential equations. A numerical approach using lattice Boltzmann method is presented with first quantitative results. In particular, we present experiments and discuss the propagation of a stable front of so-called balck film (on the nanometer scale) into the unstable thicker material.

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1D integrable systems and 2D hydrodynamics

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I will present relations between 2D hydrodynamics and a phase space representation of certain many particle 1D systems of interacting particles. The purpose of establishing such relations is to make use of hidden symmetries of 1D physics in the study of 2D hydrodynamics.

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Hollow vortices, capillary waves, and double quadrature domains

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This talk will describe several newly discovered equilibria for so-called hollow vortex solutions of the two-dimensional Euler equations. A hollow vortex is a finite-area region, with non-zero circulation, in an otherwise irrotational flow. The challenge of finding equilibrium configurations is a free boundary problem; here we employ conformal mapping and free streamline theory to find exact solutions in several cases. It turns out that the solutions have surprising mathematical connections with the physically distinct problem of free surface capillary waves as well as the notion of a "double quadrature domain".

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Motion of interfaces governed by the Cahn-Hilliard equation with highly disparate diffusion mobility

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We consider a two phase system governed by a Cahn-Hilliard type equation with a highly disparate diffusion mobility. It has been observed from recent numerical simulations that the microstructure evolution described by such a system displays a coarsening rate different from that associated with the Cahn-Hilliard equation having either a constant diffusion mobility or a mobility that degenerates in both phases. Using the asymptotic matching method, we derive sharp interface models of the system under consideration to theoretically analyze the interfacial motion with respect to different scales of time t. In a very short time regime, the transition layer stabilizes into the well-known hyperbolic tangent single-layer profile. On an intermediate time regime, due to the small mobility in one of the phases, the sharp interface limit is a one-sided Stefan problem. On a slower time scale, the leading order dynamics is a one-sided Hele-Shaw problem. The normal velocity of the interface has a small correction that is determined by

the surface diffusion, that is, the surface Laplacian of the mean curvature. As a result, scaling arguments suggest that there should be a crossover in the coarsening rate from $t^{1/3}$ to $t^{1/4}$.

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Diffusion-generated motion algorithms for multiphase curvature motion

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Many materials, including most metals and ceramics, are composed of crystallites (often called grains), which are differentiated by their crystallographic orientation. Classical models describing annealing-related phenomena for these materials involve multiphase curvature-driven motion. The distance function-based diffusion-generated motion (DFDGM) algorithm is introduced and demonstrated to be an accurate and efficient means for simulating the evolutions described by such models. The DFDGM algorithm makes use of implicit representations of the phases, allowing topological changes to occur naturally. Large-scale simulations of grain growth are presented and are shown to agree well with available theoretical predictions and empirical observations.

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Capillary-mediated pattern branching

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The Gibbs-Thomson temperature distribution can act as a weak thermal field from its tangential gradients along the interface. Provided that the interface conductivity is non-zero, energy conservation for non-equilibrium shapes shows varying rates of deposition and removal of small amounts of capillary-mediated energy. The local freezing rate is retarded slightly where the Stefan energy balance requires that capillary energy is released. Where energy is withdrawn, the local rate of freezing is enhanced. These opposing responses balance if the surface Laplacian of the interface temperature vanishes. The bias in the local freezing rates surrounding this balance point, which is 4th-order in the interface shape, induces deterministic rotation (tilting) of the interface. Rotation couples with the transport field in the melt by changing the local curvature, which eventually produce a side branch. A precision Greens function solver confirms that rotations and branches arise dynamically at locations predicted analytically for various 2-D shapes. Subsequent rotations develop episodically as the tip shape and rotation points coevolve. A synchronous limit cycle may develop under conditions not yet fully understood. Noise, per se, and stochastic stability, play no direct roles in this proposed mechanism of pattern morphogenesis.

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Non-uniqueness of quadrature domains

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The talk will be devoted to the construction of one perimeter family of unbounded quadrature domains with the same source. In the two dimensional plane it is related to the inverse problem of the logarithmic potential for a contact surface. We will consider the constructions both in the plane and in higher dimensions. We shall also discuss the applications to equilibrium measures associated with an admissible weight function.

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Propagation of singularities of solutions of linear PDE

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We shall discuss applications of Leray's theory of singularites of holomorphic PDEs to the problem of locating the singularities of Laplacian Growth processes.

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Topological transitions in interface dynamics of evaporating thin films

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Oded Agam

A thin water film on a cleaved mica substrate undergoes a first-order phase transition between two values of film thickness. By inducing a finite evaporation rate of the water, the interface between the two phases develops a lingering instability similar to that observed in the Saffman-Taylor problem. The dynamics of the droplet interface is dictated by an infinite number of conserved quantities: all harmonic moments decay exponentially at the same rate. One can link the interface dynamics to the evolution of a Riemann surface. A typical scenario is the nucleation of a dry patch within the droplet domain. This corresponds to a topological transition of the Riemann surface from genus zero to genus one. We shall construct solutions of this problem and highlight similarities and differences between them and solutions of Laplacian growth problems.

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Quadrature domains in interface dynamics

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We discuss the role of area-quadrature domains in the Laplacian growth or "Hele-Shaw" problem and the appearance of quadrature domains of various types in other problems of fluid dynamics. We give special attention to the lack of explicit examples in the higher-dimensional case.

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Velocity fluctuations of noisy reaction fronts

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The position of a propagating reaction front fluctuates because of the shot noise coming from the discreteness of reacting particles and stochastic character of reactions. What is the probability that the noisy front moves, during a given time interval, considerably slower or faster than its deterministic counterpart? Can the noise arrest the front motion for some time, or even make it move in the wrong direction? What is the most likely particle density profile of an unusual front realization? I will present a WKB theory that assumes many particles in the front region and, in some cases, answers these questions. The details strongly depend on whether the front propagates into a metastable or unstable state [1,2].

[1] B. Meerson, P.V. Sasorov and Y. Kaplan, Phys. Rev. E **84**, 011147 (2011).

[2] B. Meerson and P.V. Sasorov, Phys. Rev. E **84**, 030101(R) (2011).

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Particle growth in a subdiffusive medium

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During the last decades, the phenomenon of anomalous diffusion has attracted much attention of researchers. A subdiffusive transport has been observed in numerous physical and biological systems, specifically in gels. We investigate the growth of a solid nucleus due to the subdiffusive transport of a dissolved component towards the nucleus surface. The process is described by a subdiffusive version of the Stefan problem. In planar and spherical cases, exact self-similar solutions of the problem have been found in terms of the Wright function. An instability of the particle growth, which is similar to the Mullins-Sekerka instability of a crystallization front in the case of a normal diffusion, is revealed.

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Stationary solutions of the convective Cahn-Hilliard equation

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The convective Cahn-Hilliard equation (CCHE) has a number of physical applications, including the description of phase transitions and facetting by a non-equilibrium crystal growth. Stationary solutions of the CCHE are described by a third-order nonlinear ordinary differential equation, which is not integrable. Nevertheless, a family of explicit solutions can be found by means of multiple-scale expansions. Numerical analysis reveals a complex set of bifurcations leading to the appearance of patterns which include steep kink-like structures and sloppy oscillating fragments.

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Coarsening and the deep quench obstacle problem

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Phase separation occurs in a wide spectrum of contexts, from galaxy formation to biofilm formation, and many models have been proposed to describe the dynamics. Common to these processes is a linear regime dominated by a "most unstable mode" and a later coarsening regime during which larger components grow at the expense of the smaller components. We explore some of the features of the dynamics within the relatively simple context of the deep quench (low temperature) obstacle problem. We obtain new analytical bounds on the rate of coarsening, and present results of numerical simulations based on a number of benchmarks. By following the dynamics using a number of different benchmarks, we find that partial scaling can be verified, and that the transition between linear and coarsening behavior is in fact characterized by a number of sequential transitions which call for further analysis.

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A moment-preserving flow for surfaces and its applications

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Hele-Shaw flow is an incompressible viscous fluid flow in an experimental device which consists of two closely placed parallel plates. An interesting feature of Hele-Shaw flow is that the evolution of a fluid domain under the flow produced by injection of fluid does not change its geometric moments in time, while the area increases. We will introduce a new geometric flow which has an analogous property, that is, the moments of the boundary of the domain are preserved under the flow. Applications to other problems will also be mentioned.

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Born-Oppenheimer approximation and accuracy of molecular dynamics

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We present results used for estimating accuracy of molecular dynamics as an approximation of the evolution of heavy nuclei in a many-body quantum system. The presented approach is based on the study of the time-independent Schroedinger equation and thus differs from a more standard analysis derived from time-dependent Schroedinger equation. It gives a different perspective on the Born-Oppenheimer approximation, Schroedinger Hamiltonian systems and numerical simulations in molecular dynamics at micro-canonical ensemble. Results from joint work with A. Szepessy, H. Hoel and R. Tempone will be presented.

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Interface dynamics and singularities

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We will discuss some models arising in mathematical physics, including the Laplacian growth.

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Fingering in a channel and tripolar Loewner evolutions

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Loewner evolutions describes a rather general class of growth processes in two dimensions where a curve starts from a given point on the boundary of a domain $\mathbb{P}$ in the complex z-plane and grows into the interior of $\mathbb{P}$. More specifically, the Loewner equation is a first-order differential equation for the conformal mapping $g_t(z)$ from the “physical domain,” consisting of the region $\mathbb{P}$ minus the curve, onto a “mathematical domain” represented by $\mathbb{P}$ itself. In this work, a class of Laplacian growth models in the channel geometry is studied using the formalism of tripolar Loewner evolutions, in which three points, namely, the channel corners and the point at infinity, are kept fixed. Initially, the problem of fingered growth, where growth takes place only at the tips of slit-like fingers, is revisited and a class of exact solutions is presented. A model for interface growth is then formulated in terms of a generalized tripolar Loewner equation and several examples are presented. It is shown that the growing interface evolves into a steadily moving finger and that tip competition arises for nonsymmetric initial configurations with multiple tips. Possible extensions, including stochastic tripolar Loewner evolutions, will be briefly discussed.

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