Special Session 50: Mathematical Novelties in Inverse Problems in Imaging Sciences

Alexandru Tamasan, University of Central Florida, USA

The goal of this session is to bring together scientists working on the mathematics of Inverse Problems with applications to Imaging Science. To be presented are the recent advances on the mathematical methods in reconstructing/imaging various physical properties such as electrical conductivity in Electrical Impedance Tomography (EIT), Current Density Impedance Imaging (CDII), or Ultrasound Modulated EIT, or optical properties in Photo-acoustics, or elastic properties in Ultrasound Elastography.

Geometrical effects of the conductivity on the D-bar method procedure for the electrical impedance tomography

Toufic El Arwadi
Beirut Arab University, Lebanon
telarwadi@gmail.com

Several authors have treated the direct reconstructions methods for the Calderón inverse conductivity problem in dimension two for regular, less regular and discontinuous conductivities. These methods consist on two steps: calculate the scattering transform from Cauchy data, then recover the conductivity from the scattering data. The first step is an ill posed step. For this, many authors have treated several approximations of the scattering transform for regular case (like $\mathbf{t}^{\mathbf{exp}}$ and $\mathbf{t}^{\mathbf{B}}$) and full regularization strategy in 2009. But only one work treats the case of less regular conductivity . In this talk I will treat the $\mathbf{t}^{\mathbf{B}}$ approximation for Nachman's proof , and $S^{\mathbf{a}}$ approximation for Knudsen-Tamasan proof. I will present the geometrical effect of the conductivity on the reconstruction procedure, will present also the stability of $\mathbf{t}^{\mathbf{B}}$ approximation, relation between $S^{\mathbf{a}}$ and $\mathbf{t}^{\mathbf{B}}$ for radial symetric conductivities and a new numerical scheme for solving the D-bar equation. I will present some numerical examples which justify my theoretical results.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Identification of minimum phase preserving operators

Peter Gibson
York University, Canada
pcgibson@yorku.ca

Minimum phase functions are fundamental in a range of applications, including control theory, communication theory and signal processing. A basic mathematical challenge that arises in the particular context of geophysical imaging is to understand the structure of linear operators preserving the class of minimum phase functions. The heart of the matter is an inverse problem: to reconstruct an unknown minimum phase preserving operator from its value on a limited set of test functions. This entails, as a preliminary step, ascertaining sets of test functions that determine the operator, as well as the derivation of a corresponding reconstruction scheme. In the present paper we exploit a recent breakthrough in the theory of stable polynomials to solve the stated inverse problem completely. We prove that a minimum phase preserving operator on the half line can be reconstructed from data consisting of its value on precisely two test functions. And we derive an explicit integral representation of the unknown operator in terms of this data. A remarkable corollary of the solution is that if a linear minimum phase preserving operator has rank at least two, then it is necessarily bounded and injective.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Some results on the attenuated ray transform

Nicholas Hoell
University of Toronto, Canada
nhoell@math.utoronto.ca

We discuss several recent results on filtered backprojection inversion formulae for the attenuated ray transform on curves in 2-dimensional settings. The method is based on a particular complexification of the vector fields defining the initial particle transport. This problem first arose in the medical imaging modality SPECT and has more recently proven useful in other contexts such as the unique reconstruction of permittivity and permeability coefficients of a conductive body from boundary measurements.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Reinterpretation of the imaginary part of the complex potential In EIT

Sungwhan Kim
Hanbat National University, USA
sungwhan@hanbat.ac.kr

This work presents a mathematical analysis of the time harmonic electric potential which is a solution to an elliptic PDE with complex coefficient. In Electrical Impedance Tomography, the complex coefficient is the admittivity distribution of the interested object and hence the corresponding voltage potential is a complex-valued function. In this paper, we are interested in investigating a highly complicated interrelation between the real and imaginary parts

ofthe complex voltage potential. This investigation is indispensable for complete understanding of the complex elliptic PDE and its applications in EIT. In this work, we show that the imaginary part of the complex potential has a close relation with the difference of its real part and a real-valued voltage potential which is a solution to the standard elliptic PDE with conductivity coefficient. This observation implies that the imaginary part itself is a kind of frequency difference between two potentials at different frequencies and it can be used to image the admittivity changes with respect to frequency from single measurement of the imaginary part unlike typical frequency difference EIT. This sheds new light on the role of the imaginary part of the complex potential.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Partial data inverse problems in unbounded domains

Xiaosheng Li
Florida International University, USA
xli@fiu.edu

The inverse boundary value problems consist of the recovery of the coefficients of the partial differential equations in a medium from measurements of the solutions on its boundary. In this talk we present the recent developments on such problems in two unbounded domains, a half space and an infinite slab. The available measurements are only on part of the boundary hyperplane(s). The unique determination results will be discussed for the Schroedinger types of equations.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A convergent algorithm for the hybrid problem of reconstructing conductivity from minimal interior data

Amir Moradifam
University of Toronto, Canada
amir@math.toronto.edu

We consider the hybrid problem of reconstructing the isotropic electric conductivity of a body Ω from interior Current Density Imaging data obtainable using MRI measurements. We only require knowledge of the magnitude $|J|$ of one current generated by a given voltage f on the boundary $\partial\Omega$. As previously shown, the corresponding voltage potential u in Ω is a minimizer of the weighted least gradient problem

$$u = \operatorname{argmin}\{\int_\Omega a(x)|\nabla u| : u \in H^1(\Omega), \quad u|_{\partial\Omega} = f\},$$

with $a(x) = |J(x)|$. In this paper we present an alternating split Bregman algorithm for treating such least gradient problems, for $a \in L^2(\Omega)$ non-negative and $f \in H^{1/2}(\partial\Omega)$. We give a detailed convergence proof by focusing to a large extent on the dual problem. This leads naturally to the alternating split Bregman algorithm. The dual problem also turns out to yield a novel method to recover the full vector field J from knowledge of its magnitude, and of the voltage f on the boundary. We then present several numerical experiments that illustrate the convergence behavior of the proposed algorithm. This is a joint work with Adrian Nachman and Alexandre Timonov.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Inverse born series for the Calderon problem

Shari Moskow
Drexel University, USA
moskow@math.drexel.edu
Simon Arridge, John Schotland

We propose a direct reconstruction method for the Calderon problem based on inversion of the Born series. We characterize the convergence, stability and approximation error of the method and illustrate its use in numerical reconstructions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Quantitative thermo-acoustic imaging: an exact reconstruction formula

Loc Nguyen
Ecole Normale Superieure at Paris, France
lnguyen@dma.ens.fr
H. Ammari, J. Garnier, W. Jing

The quantitative thermo-acoustic imaging is considered in this talk. Given several data sets of electromagnetic data, we first establish an exact formula for the absorption coefficient, which involves derivatives of the given data up to the third order. However, because of the dependence of such derivatives, this formula is unstable in the sense that small measurement noises may cause large errors. Hence, with the presence of noise, the obtained formula, together with noise regularization, provides an initial guess for the true absorption coefficient. We next correct the errors by deriving a reconstruction formula based on the least square solution of an optimal control problem and show that this optimization step reduces the errors occurring.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Inverse problem of determining an absorbtion coefficient and a speed of sound in the wave equation by the BC method

Leonid Pestov
Immanuel Kant Baltic Federal University, Russia
lpestov@kantiana.ru

We consider inverse dynamical problem for the wave equation with unknown variable speed and absorbtion in a bounded domain Ω. A linear procedure based on the BC method for determining both coefficients is proposed. The procedure includes solution of boundary control problem for states $u(.,T), u_t(.,T)$ of special kind and determining unknowns fromlinear integral equations. The time of observation must be greater than optical diameter of Ω.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Reconstruction strategies in quantitative photoacoustic tomography

Kui Ren
University of Texas at Austin, USA
ren@math.utexas.edu

In quantitative photoacoustic tomography (qPAT) we aim at reconstructing physical parameters of biological tissues from "measured" data of absorbed radiation inside the tissues. Mathematically, qPAT problems can be regarded as inverse problems related to some elliptic partial differential equations. We present in this talk some new reconstruction strategies for inverse problems in qPAT with different types of available data.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence of a minimizer for the weighted least gradient problem

Greg Spradlin
Embry-Riddle University, USA
spradlig@erau.edu
Alexandru Tamasan

We consider the following problem: Given a bounded smooth domain Ω in $\mathbb{R}^n$, boundary data $f \in H^{\frac{1}{2}}(\partial\Omega)$, and a continuous, positive bounded function $f : \Omega \to \mathbb{R}$ bounded away from zero, show that the functional $\int_\Omega a(x)\|D(u)\|$, where u has trace f on $\partial\Omega$, has a minimizer. The problem has origins in medical imaging.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Viscoelasticity in magnetic resonance elastography

Jeong-Rock Yoon
Clemson University, USA
jryoon@clemson.edu

Magnetric resonance elastography aims to reconstruct the elasticity of human tissue for early stage cancer detection, using the interior displacement data acquired from MR signal associated with a vibrating transducer. For simplicity, one uses purely elastic model in applications. However, human tissue is viscoelastic that shows energy absorption of the propagating medium, especially in liver. In this talk, we discuss what properties have to be changed under this more realistic model.

$\longrightarrow \infty \diamond \infty \longleftarrow$